

Wm. Howden 1800
A R I T H M E T I C,

RATIONAL AND PRACTICAL:

W H E R E I N

The Properties of NUMBERS are clearly pointed out; the THEORY of the Science deduced from first Principles; the Methods of OPERATION demonstratively explained; and the whole reduced to PRACTICE in a great variety of useful RULES.

CONSISTING OF THREE PARTS, viz.

- I. VULGAR ARITHMETIC.
- II. DECIMAL ARITHMETIC.
- III. PRACTICAL ARITHMETIC.

By J O H N M A I R, A. M.
RECTOR OF THE ACADEMY AT PERTH.

T H E F O U R T H E D I T I O N.

E D I N B U R G H:

Printed for JOHN BELL and WILLIAM CREECH;
And sold by T. LONGMAN, G. G. J. & J. ROBINSON, T. CADELL,
C. DILLY, and RICHARDSON & URQUHART, LONDON.

M.DCC.LXXXVI.

1560 / 4574

Entered in Stationers Hall, according to Act
of Parliament.



Wm. Hornden

TO

1800

THE RIGHT HONOURABLE
THOMAS EARL OF KINNOUL,

THE FOLLOWING
TREATISE OF ARITHMETIC,

I S,
WITH GREAT RESPECT,
INSCRIBED,

BY
HIS LORDSHIP'S
MUCH OBLIGED,
MOST OBEDIENT, AND
VERY HUMBLE SERVANT,

J O H N M A I R.

THE BIBLE

THOMAS BIBLE OF MINNOLLI

REPRINT OF THE BIBLE

WITH GREAT CARE

PRINTED

BY

THE BIBLE

PRINTED

IN THE BIBLE

THE BIBLE

THE BIBLE

P R E F A C E.

AN encomium on the usefulness and excellency of Arithmetic would, in this place, be vain and idle. The world is already abundantly sensible of the worth and importance of this noble art. All, then, that seems necessary by way of preface, is to give some short account of the following Treatise, and point out the improvements the reader may expect to meet with in perusing it.

And, in general, I persuade myself it will be found fully to answer the title-page; the properties of numbers being exhibited in a clear and instructive manner, and the whole theory built on first principles; that is, on axioms, or such propositions as are self-evident. The rules of operation, deduced immediately from the theory, are conceived in a few expressive words, and illustrated by a great variety of suitable examples. Care too has been taken to range things every where in such order, that what is prior paves the way for what is to follow.

By connecting things in this manner, the rugged path is rendered smooth, and the learner will proceed with pleasure from what is more simple, to what is more complex; and rise by easy steps from the lowest to every superior part of the science. And to furnish him with materials for exercise on every branch, and at the same time save labour to the teacher, sets of unwrought examples, with their answers, are inserted in all places where this appeared to be any way necessary, or of advantage.
—But to be more particular:

THIS

THIS Treatise begins with a General Introduction ; wherein Arithmetic is defined, its limits stated, and the reader taught to distinguish between Arithmetic, and the practical purposes to which its operations may be applied.

THE Work then is divided into Three Parts, under the titles of *Arithmetic Vulgar*, *Decimal*, and *Practical*.

IN the First Part, Notation is explained, the various kind of numbers described, and the axioms, or first principles, laid down, on which are founded the theory and rules of operation. Addition, Subtraction, Multiplication, and Division, are taught at full length, with all the modern improvements, and the reason of the rules every where assigned.

The properties of proportional numbers, or numbers in geometrical proportion, are briefly pointed out ; and thence is derived the Rule of Three : In which a simple and easy method of stating the questions is laid down ; whereby the puzzling distinction of *direct* and *inverse* is precluded, or rendered needless, and the multiplicity of rules usually adhibited on this head, is avoided. Nor is this method confined to the simple Rule of Three, but extended to the Rule of Five, Seven, Nine, Eleven, &c. The doctrine of Vulgar Fractions, and the rules of Practice therewith connected, are delivered in a way simple and complete, without prolixity.

THE Second Part contains Decimal Arithmetic, or the doctrine of Decimal Fractions. Here the origin of decimals of every kind, *viz.* finite, repeating, and circulating, is inquired into, their properties pointed out, and described at large ; the various sorts and ways of reduction are minutely explained, and properly exemplified : Complete tables too are constructed for converting the parts of integers to decimals by inspection ; and also another

ther set of tables for reducing readily decimals of every sort to value. In Addition, Subtraction, Multiplication, and Division, the operations are conducted by a few plain easy rules ; and these illustrated by a copious set of proper examples.

Some lovers of Arithmetic have alleged, that authors, in treating of interminate decimals, are always curt and scanty in their explications, and leave this new part of Decimal Arithmetic so much involved in obscurity, as to be still, in their opinion, far above the reach of vulgar readers. But I flatter myself, that all complaints of this kind will now be removed ; the methods of operation in decimals of this sort being here set before the learner with such clearness and perspicuity, that one, though of a low capacity, may soon acquire skill enough, not only to work examples wherein repeaters or circulates occur, but may even understand the reason of the process.

This part concludes with a large Decimal Practice, setting forth the superior excellency of Decimal Arithmetic above every other method of numerical computation. Here is shewn the utility and conveniency of decimals in constructing tables of various kinds ; how they may be used with great advantage in multiplication and division of duodecimals and sexagesimals ; how they often rival vulgar fractions in point of accuracy, and surpass them in point of brevity ; and, finally, that the decimal operation is frequently the shortest and easiest in the Rule of Three, the work being done that way in least time, and with fewest figures.

THE Third Part contains the application of the other two to practical purposes, in a great variety of useful rules. Here facility is studied, the mercantile and fashionable methods of computation are introduced, and adapted to the several branches of commerce and business. Sometimes different ways of operation are exhibited,

bited, and the choice left to the practitioner. Large and correct tables on compound interest and annuities are inserted, with their construction and uses. Mensuration of surfaces and solids, artificers work, board and timber, &c. is taught in an easy manner, and at more than usual length.

To the above account I shall only add, that many things of importance, several small improvements, and some things curious, as well as useful, not hitherto taken notice of, will be found scattered here and there in different parts of the book.—To conclude, no pains have been spared in collecting materials, and working them up, with a view to render this system of Arithmetic instructive to the tyro, entertaining to the scholar, and useful to the man of business.

C O N -

C O N T E N T S.

INTRODUCTION	Pag. 1
--------------	--------

PART I.

VULGAR ARITHMETIC.

Chap. I. NOTATION.

Notation	5
Numeration	10
The species of Numbers	11
Arithmetical Axioms	12
Explanation of Characters	14

II. ADDITION

Addition of Integers	16
Addition of the parts of Integers	ib.
The proof of Addition	17
	37

III. SUBTRACTION

Subtraction of Integers	39
Subtraction of the parts of Integers	40
The proof of Subtraction	41
	48

IV. MULTIPLICATION

Multiplication of Integers	50
Contractions in Multiplication	52
Select methods of multiplying Integers	54
Multiplication of the parts of Integers	57
The proof of Multiplication	62
	69

V. DIVISION

Division of Integers	71
Contractions in Division	ib.
Select methods of dividing Integers	79
Division of the parts of Integers	88
The proof of Division	89
	92

VI. REDUCTION

Reduction descending and ascending	93
Mixt Reduction	94
	104

VII. The RULE OF THREE

The simple Rule of Three direct	114
The simple Rule of Three inverse	116
The compound Rule of Three	125
The Rule of Five direct	129
	131
b	The

	Pag.
<i>The Rule of Five inverse</i>	136
<i>The Rule of Seven, Nine, Eleven, &c.</i>	141
<i>Contractions in the Rule of Three</i>	144
VIII. VULGAR FRACTIONS	147
<i>Reduction of Vulgar Fractions</i>	149
<i>Addition of Vulgar Fractions</i>	162
<i>Subtraction of Vulgar Fractions</i>	165
<i>Multiplication of Vulgar Fractions</i>	169
<i>Division of Vulgar Fractions</i>	172
<i>The Rule of Three in Vulgar Fractions</i>	175
IX. RULES OF PRACTICE	178
PART II.	
DECIMAL ARITHMETIC.	
Chap. I. NOTATION OF DECIMALS	198
II. REDUCTION	200
1. <i>To reduce a vulgar fraction to a decimal</i>	<i>ib.</i>
2. <i>To reduce the parts of coin, weight, &c. to decimals</i>	210
3. <i>To reduce the remainder of a division to a decimal</i>	237
4. <i>To reduce a decimal to value</i>	238
5. <i>To reduce a decimal to a vulgar fraction</i>	264
6. <i>To reduce unlike circles to others similar and conterminous</i>	266
III. ADDITION	268
1. <i>To add finite or approximate decimals</i>	<i>ib.</i>
2. <i>To add repeating decimals</i>	270
3. <i>To add circulating decimals</i>	271
IV. SUBTRACTION	275
1. <i>To subtract finite or approximate decimals</i>	<i>ib.</i>
2. <i>To subtract repeaters or circulates</i>	276
V. MULTIPLICATION	277
1. <i>To multiply finite or approximate decimals</i>	278
<i>Contractions in multiplication</i>	279
2. <i>To multiply a repeating multiplicand</i>	283
3. <i>To multiply a circulating multiplicand</i>	<i>ib.</i>
4. <i>To multiply by an interminate multiplier</i>	285
<i>Practical questions</i>	301
VI. DIVISION	302
1. <i>To divide finite or approximate decimals</i>	303
<i>Contractions</i>	

C O N T E N T S.

xi

	Pag.
<i>Contractions in division</i>	308
<i>Divisors used instead of multipliers, &c.</i>	313
2. <i>To divide a repeating dividend</i>	314
3. <i>To divide a circulating dividend</i>	316
4. <i>To divide by an interminate divisor</i>	317
<i>Practical questions</i>	329
VII. DECIMAL PRACTICE	331

PART III.

PRACTICAL ARITHMETIC.

Chap. I. FELLOWSHIP	355
<i>Fellowship without time</i>	<i>ib.</i>
<i>Fellowship with time</i>	360
II. FACTORAGE	363
III. TARE AND TRET, &c.	367
IV. BARTER	379
V. LOSS AND GAIN	382
VI. MISCELLANIES	387
<i>Brokage, or Brokerage</i>	<i>ib.</i>
<i>Bankruptcy</i>	388
<i>Insurance</i>	390
<i>Purchase of Stocks</i>	393
<i>How to find the value of the short hundred</i>	395
<i>How to find the value of the long hundred</i>	396
<i>How to find the value of 1 C. or 112 lb.</i>	397
<i>How to find the value of the great gross</i>	<i>ib.</i>
<i>How to find the price by inverting the question</i>	398
VII. EXCHANGE	399
<i>Exchange with Holland</i>	<i>ib.</i>
<i>— with Hamburgh</i>	405
<i>— with France</i>	408
<i>— with Portugal</i>	410
<i>— with Spain</i>	412
<i>— with Venice</i>	414
<i>— with Genoa</i>	415
<i>— with Leghorn</i>	416
<i>— with America and the West Indies</i>	418
<i>— with Ireland</i>	420
<i>— betwixt London and other places in Britain</i>	422
	Simple

	Pag.
<i>Simple arbitration of exchange</i>	422
<i>Compound arbitration</i>	427
VIII. ALLIGATION	437
<i>Alligation Medial</i>	<i>ib.</i>
<i>Alligation Alternate</i>	441
IX. RULE OF FALSE	452
<i>Single Position</i>	453
<i>Double Position</i>	454
X. EXTRACTION OF ROOTS	459
<i>Extraction of the root of the square</i>	460
<i>of the cube</i>	467
<i>of the biquadrate</i>	472
<i>of the fifth power</i>	473
<i>of the sixth power</i>	<i>ib.</i>
<i>of the seventh power</i>	474
<i>of the eighth and ninth power</i>	475
XI. PROGRESSION	475
<i>Arithmetical progression</i>	476
<i>Geometrical progression</i>	485
XII. INTEREST	494
<i>Simple interest</i>	<i>ib.</i>
<i>Equation of payments</i>	513
<i>Compound interest</i>	515
XIII. ANNUITIES	529
<i>Annuities certain</i>	530
<i>Annuities perpetual, or freehold estates</i>	545
<i>Annuities for life</i>	547
XIV. MENSURATION	561
<i>Mensuration of surfaces</i>	562
<i>of solids</i>	573
<i>of land, or Surveying</i>	582
<i>of artificers work</i>	585
<i>of board and timber</i>	595
<i>of liquors, or Gauging</i>	603

ARITHMETIC,

RATIONAL and PRACTICAL.

INTRODUCTION.

ARITHMETIC is a science explaining the properties of numbers, and shewing the method or art of computing by them.

The division of arithmetic into theory and practice is natural and just. The theory, or speculative part, treats of the nature and affections of numbers, and this is properly *science*. The practical part consists in the application of the rules gathered from the theory to the solution of questions or problems; and arithmetic in this respect is rightly termed an *art*.

Number, which is the object of arithmetic, is that which answers directly to the question, How many? and is either an unit, or some part or parts of an unit, or a multitude of units.

To a person having the idea of number in his mind, the following questions naturally occur, *viz.* 1. How is such a number to be expressed or written? Hence we have Notation. 2. What is the sum of two or more numbers? Hence Addition. 3. What is the difference of two given numbers? Hence Subtraction. 4. What will be the result or product of a given number repeated or taken a certain number of times? Hence Multiplication. 5. How often is one given number contained in another? Hence Division.

A

These

These five, *viz.* Notation, Addition, Subtraction, Multiplication, and Division, are the chief parts, or rather the whole of arithmetic. Every arithmetical operation requires the use of some of these, and nothing but a proper mixture of them is necessary in any operation whatever; and, by an Arabic term, these are called the *algorithm*.

These may be fitly compared to the five mechanic powers, which, variously combined, produce engines of different forms, and of amazing force; and which, at first sight, are apt to strike us with surprise; but, upon examination, their effects are all found to flow from a proper combination of the simple powers. In like manner, we meet with a great variety of complex arithmetical operations, curious calculi, and astonishing solutions, which yet are all effected by a due application of the different parts of the algorithm.

Numerical computations are necessary almost in every affair; but the manner of the operations depends upon the nature of the subject to which they are applied. Arithmetic is a handmaid to most of her sister arts and sciences; and while she is employed in their service, she must be under their direction, and observe the laws they prescribe. Thus the calculation of a solar or lunar eclipse is performed by numbers; but the operation is directed by the precepts of astronomy: a reckoning at sea is wrought by numbers; but the steps of the work are conducted by the rules of navigation: the contents of a cask is cast up by numbers; but the art of gauging guides the operation: the perpendicular height of a hill is computed by numbers; but it is trigonometry that directs the manner of their application.

The canons directing the application of numbers in other sciences, are not to be reckoned arithmetical rules: no rule is to be esteemed such, but where the operation entirely depends on arithmetical principles, and proceeds without any foreign aid or direction,

Hence from the number of arithmetical rules is excluded the rule of Three, with its numerous train of dependents, such as,
Fellowship,

Fellowship, the rule of False, Interest, Annuities, Alligation, Exchange, Barter, &c. The doctrine of proportion is delivered in the elements of geometry. It is Euclid, *lib. 6. prop. 16.* who demonstrates, that if four quantities be proportional, the rectangle, or product of the extremes, will be equal to that of the means: it is this property of proportionals that gives birth to the rule of Three; it is this that directs the arithmetical operation. For the like reason, Involution and Extraction of roots are no arithmetical rules; it is algebra that teaches the manner of these operations. Reduction and the rules of Practice are likewise to be exterminated; as also, addition, subtraction, &c. of the parts of integers, such as, shillings, pence, farthings, ounces, &c. The statutes of a country, with respect to coins, weights, and measures, prescribe the method of operation on these heads. It remains, then, that the five parts of which the algorithm consists make the whole of arithmetic: here the limits of this science are to be fixed, otherwise it would prove endless and inexhaustible.

True it is, that writers on this subject go generally beyond these bounds: they frequently introduce into their printed treatises all the rules excluded above, and several others. Nor is this any fault: authors are at liberty, and in the right, to store their writings with all such numerical operations as may be useful to the public. The only thing blameable in their conduct is, their not stating the limits of the science, and teaching their readers to distinguish betwixt arithmetic, and the application of its operations to practical or useful subjects.

The neglect here taken notice of, has been attended with this remarkable consequence, that vulgar readers have generally mistaken these adventitious rules for parts of arithmetic; and as they observed new rules published every now and then in new books, they were apt to imagine, that arithmetic admitted of no limits, and could never be attained to any sort of perfection.

Arithmetic, as a science, ought to demonstrate her own rules; that is, the rules of operation in addition, subtraction, multiplication, and division, both of integers and fractions; but this is

all; to employ her in demonstrating the rules borrowed from other arts and sciences, would be vain and fruitless. An attempt of this kind would be making her act out of sphere, and putting her upon an office she is by no means fit for. Numerical demonstrations are successfully applied to subjects purely arithmetical, and they can be carried little further. The proper demonstrations of these foreign rules are only to be had from geometry, algebra, or some other branch of science; and as these are much beyond the reach of the young arithmetician, he cannot at present expect entire satisfaction on this head, but must wait with patience, till, in the course of his studies, he come to be acquainted with the superior sciences.

The following treatise is designed as a rational, and at the same time a practical system of arithmetic; wherein the five parts of this science are fully but succinctly handled, and all the arithmetical rules numerically demonstrated. The adventitious rules depending on other sciences are likewise explained, and exemplified in the way that appears best for answering the purposes of the merchant, the accountant, and mechanic. Some few things too are introduced for the entertainment of the scholar and the curious.

PART

P A R T I.

VULGAR ARITHMETIC.

CHAPTER I.

N O T A T I O N.

NOTATION is that part of arithmetic which explains the method of writing down, by characters or symbols, any number expressed in words; as also the way of reading or expressing, in words, any number given in characters or symbols. But the first of these is properly notation, and the last is more usually called numeration.

The ancient Romans, for this purpose, made choice of a few of the letters of their alphabet; and this is called the *literal*, or *Roman notation*. This way is still of considerable use, and necessary to be understood. But the method which now prevails, and is practised over a great part of the known world, is effected by characters or symbols, called *figures*; and is named *figural notation*, or *notation of numbers by figures*.

The things then proper to be comprised in this chapter are, 1. The literal or Roman notation. 2. The figural notation. 3. Numeration, or the way of reading numbers. 4. Descriptions of the kinds or species of numbers. 5. Arithmetical principles, or axioms. 6. The signification of such characters as are sometimes used for brevity's sake. 7. Some practical questions.

I. *The Roman Notation.*

The letters used by the ancient Romans, in their notation of numbers, were only these five, C, I, L, V, X.; whose values and order are as follows:

One, Five, Ten, Fifty, An hundred, Five hundred, A thousand.

I, V, X, L, C, IC, CIO.

By the repetition or combination of these they expressed any other number, according to the following rules:

I. The repetition of a letter repeats its value.

Thus, II signifies two, III three; and XX twenty, XXX thirty; and CC two hundred, CCC three hundred, &c.

II. The annexing a letter of a lower value to a letter of a higher value, adds their values, or denotes their sum.

Thus, VI signifies six, VII seven, VIII eight; and XI denotes eleven, XII twelve, XV fifteen, XVI sixteen; and LV fifty-five.

A 3

LX

LX sixty, LXX seventy, LXXX eighty; and CV one hundred and five, CX one hundred and ten, CL one hundred and fifty, CLV one hundred and fifty-five; and IOC six hundred, IOCC seven hundred; and CIO·IO one thousand five hundred, CIO·IOC one thousand six hundred, CIO·IOCC one thousand seven hundred, &c.

III. The prefixing a letter of a lower value to a letter of a higher value, subtracts their values, or denotes their difference.

Thus, IV signifies four, IX nine, XL forty, XC ninety, IC ninety-nine, &c.

IV. The annexing of O to the number of IO makes its value ten times greater.

Thus, IOO signifies five thousand, and IOOO fifty thousand.

V. The prefixing of C, together with the annexing of O, to the number CIO makes its value ten times greater.

Thus, CCIOO denotes ten thousand, and CCCIOOO an hundred thousand.

The ancients, as Pliny observes, carried this kind of notation no higher. If at any time they had occasion to express a greater number, they did it by repetition; thus, CCCIOOO, CCCIOOO signified two hundred thousand, and CCCIOOO, CCCIOOO, CCCIOOO denoted three hundred thousand, &c.

The Romans, in later times, instead of IO used D; and instead of CIO they introduced M.

They likewise sometimes expressed thousands by a line drawn over the head of numbers. Thus, $\overline{\text{III}}$ signifies three thousand, $\overline{\text{V}}$ five thousand, $\overline{\text{X}}$ ten thousand, $\overline{\text{LX}}$ sixty thousand, $\overline{\text{C}}$ an hundred thousand, $\overline{\text{M}}$ a thousand thousands, or a million, $\overline{\text{MM}}$ two millions, &c.

In some editions of Cæsar's Commentaries we find ∞ used, instead of CIO, to signify a thousand.

Any thing further necessary to be known in the Roman notation may be learned from the following table; in which, with a view to exercise the learner, as well as for the sake of brevity, I have chosen to express the value of the literal numbers by figures, rather than words.

T A B L E.

T A B L E.

I.	—	1	XC.	—	90
II.	—	2	C.	—	100
III.	—	3	CI.	—	101
IV. or IIII.	—	4	CII, &c.	—	102
V.	—	5	CC.	—	200
VI.	—	6	CCC.	—	300
VII.	—	7	CCCC. or CD.	—	400
VIII. or IIIX.	—	8	ICD. or D.	—	500
IX.	—	9	ICC. or DC.	—	600
X.	—	10	IDCC. or DCC.	—	700
XI.	—	11	IDCCC. or DCCC.	—	800
XII.	—	12	IDCCCC. or DCCCC. or CM.	—	900
XIII.	—	13	CID. or M.	—	1,000
XIV. or XIIII.	—	14	CID, C. or MC.	—	1,100
XV.	—	15	CID, CC. or MCC. &c.	—	1,200
XVI.	—	16	MM. or II.	—	2,000
XVII.	—	17	MMM. or III.	—	3,000
XVIII.	—	18	MMMM. or IV.	—	4,000
XIX.	—	19	IDD. or V.	—	5,000
XX.	—	20	IDDM. or VI.	—	6,000
XXI.	—	21	IDDMM. or VII.	—	7,000
XXII.	—	22	IDDDMM. or VIII.	—	8,000
XXIII.	—	23	IDDDMMM. or IX.	—	9,000
XXIV. or XXIIII.	—	24	CCIDC. or X.	—	10,000
XXV.	—	25	CCIDCM. or XI.	—	11,000
XXVI.	—	26	CCIDCCMM. or XII. &c.	—	12,000
XXVII.	—	27	IDCC. or L.	—	50,000
XXVIII.	—	28	IDCCM. or LI.	—	51,000
XXIX.	—	29	IDCCMM. or LII. &c.	—	52,000
XXX.	—	30	CCCIDCC. or C.	—	100,000
XL.	—	40	CID, ICC, LXXXV. or M, DCC, LXXXV.	1785	
L.	—	50			
LX.	—	60			
LXX.	—	70			
LXXX.	—	80			

II. Figural Notation.

An unit, or unity, is that number by which any thing is called one of its kind. It is the first number; and if to it be added another unit, we shall have another number, called *two*; and if to this last another unit be added, we shall have another number, called *three*; and thus, by the continual addition of an unit, there will arise an infinite increase of numbers. On the other hand, if from unity any part be subtracted, and again from that part another part be taken away, and this be done continually, we shall have an infinite decrease of numbers.

But though number, with respect to increase and decrease, be infinite, and knows no limits; yet ten figures, variously combined or repeated, are found sufficient to express any number whatsoever.

These, with the method of notation by them, were originally invented by some of the eastern nations, probably the Indians; afterwards improved by the Arabians; and at last brought over to Europe, particularly into Britain, betwixt the tenth and twelfth century. From the ten fingers of the hands, on which it had been usual to compute numbers, the figures were called *digits*. Their form, order, and value, are as follows:

1 one, an unit, or unity, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 cipher, nought, null, or nothing. Of these, the first nine, in contradistinction to the cipher, are called *significant figures*.

The value of the figures now assigned is called their *simple value*, as being that which they have in themselves, or when they stand alone. But when two or more figures are joined as in a line, the figures then receive also a local value from the place in which they stand, reckoning the order of places from the right hand towards the left, thus,

7	7	7	7	7	7	7	7	7	7	7	7	7
First place.	Second place.	Third place.	Fourth place.	Fifth place.	Sixth place.	Seventh place.	Eighth place.	Ninth place.	Tenth place.	Eleventh place.	Twelfth place.	&c.

A figure standing in the first place has only its simple value; but a figure in the second place has ten times the value it would have in the first place; and a figure in the third place has ten times the value it would have in the second place; and universally a figure in any superior place has ten times the value it would have in the next inferior place.

Hence it is plain, that a figure in the first place simply signifies so many units as the figure expresses; but the same figure advanced to the second place, will signify so many tens; in the third place, it will signify so many hundreds; in the fourth place, so many thousands; in the fifth place, so many ten thousands; in the sixth place, so many hundred thousands; and in the seventh place, so many millions, &c. Thus, 7 in the first place, will denote seven units; in the second place, seven tens, or seventy; in the third place, seven hundred, in the fourth place, seven thousand, &c.

Every three places, reckoning from the right hand, make a half-period; and the right-hand figures of these half-periods are termed *units* and *thousands* by turns; the middle figure is always tens, and the left-hand figure always hundreds. But here observe, that the right-hand figure of every half-period, properly and strictly speaking, is always units; for the place called *thousands*,

sands, when expressed at more length, is termed *units of thousands*, or the *units place of thousands*.

Two half-periods, or six places, make a full period; and the periods, reckoning from the right hand towards the left, are titled as follows, *viz.* the first is the period of *units*; the second, that of *millions*; the third is titled *bimillions*, or *billions*; the fourth, *trimillions*, or *trillions*; the fifth, *quadrillions*; the sixth, *quintillions*; the seventh, *sextillions*; the eighth, *septillions*; the ninth, *octillions*; the tenth, *nonillions*, &c.

Half-periods are usually distinguished from one another by a comma, and full periods by a point or colon; as in the table following:

T A B L E.

4th Period.		3d Period.		2d Period.		1st Period.	
Trillions.		Billions.		Millions.		Units.	
Hundred thousands.		Hundred thousands.		Hundred thousands.		Hundred thousands.	
Ten thousands.		Ten thousands.		Ten thousands.		Ten thousands.	
Thousands.		Thousands.		Thousands.		Thousands.	
Hundreds.		Hundreds.		Hundreds.		Hundreds.	
Tens.		Tens.		Tens.		Tens.	
Units.		Units.		Units.		Units.	
9	6	8	5	2	3	7	8
4	0	1	3	7	8	9	4
0	8	7	0	0	0	0	0
5	5	0	0	4	4	0	0

The table may be expressed in a more concise form, thus,

4.	3.	2.	1. Per.
Trillions.	Billions.	Millions.	Units.
$\overline{CXM,CXU}$	$\overline{CXM,CXU}$	$\overline{CXM,CXU}$	$\overline{CXM,CXU}$
9 6 4, 0 8 5	8 1 3, 7 0 0	2 3 7, 8 9 4	6 7 8, 0 4 0

From the table it is obvious, that though a cipher signify nothing of itself, yet it serves to supply vacant places, and raises the value of significant figures on its left hand, by throwing them into higher places. Thus, in the first period, by a cipher's filling the place of units, the figure 4 is thrown into the place of tens, and signifies forty. And a cipher likewise supplying the place of hundreds, the figure 8 is advanced to the next higher place, and signifies eight thousand, &c. But a cipher does not change the value of a significant figure on its right hand. Thus, 07, or 007, is the same as 7.

From the table it likewise appears how any number expressed in words,

words, may be written down by figures; in the doing of which observe the following

R U L E.

Beginning at the left hand, and writing towards the right, put every figure in such place and period as the verbal expression points out, supplying the omitted places with ciphers. Examples follow.

	Millions.	Units.
	$\overline{\text{CXM}}$, CXU :	$\overline{\text{CXM}}$, CXU.
Nine hundred and eighty seven millions	9 8 7	0 0 0, 0 0 0
Forty-five millions and seven hundred thousand	4 5	7 0 0, 0 0 0
Six millions four hundred and thirty-two thousand	6	4 3 2, 0 0 0
Eight hundred and five thousand and nine hundred	8	0 5, 9 0 0
Seventy-three thousand and ten	7 3	0 1 0
Five thousand and four	5	0 0 4
Four hundred and twenty	4	2 0
Ninety-five	9	5
Seven	7	

is written thus,

III. Numeration.

Notation and numeration are so nearly allied, that he who understands the former cannot fail soon to acquire the latter. The method of reading numbers when expressed by letters, is sufficiently clear from the table subjoined to the Roman notation; and the way of reading any number expressed by figures, may be easily learned from the table of the figural notation; in the doing of which observe the following

R U L E.

Beginning at the left hand, and reading toward the right; to the simple value of every figure join the name of its place, and conclude each period by expressing its title, every where omitting the ciphers. Examples follow.

Millions	Units.	
$\overline{\text{CXM}}$, CXU :	$\overline{\text{CXM}}$, CXU.	
9 0 0	0 0 0, 0 0 0	} is read thus.
7 8	0 0 9, 0 0 0	
4	0 0 0, 8 0 0	
9 0	3, 0 0 5	
2 0	0 3 4	
8	4 6 9	
7	0 8	
9	6	
3		
		{

Nine hundred millions.
 Seventy-eight millions and nine thousand.
 Four millions and eight hundred.
 Nine hundred and three thousand and five.
 Twenty thousand and thirty-four.
 Eight thousand four hundred and sixty-nine.
 Seven hundred and eight.
 Ninety-six.
 Three, or Three units.

N. B.

N. B. The name of the unit's place, in all periods, and the title of the right-hand period, are commonly omitted, or very rarely expressed.

IV. Descriptions of the kinds or species of numbers.

The species of numbers are very various and manifold: but in this place it will be sufficient to describe such as appear most useful and necessary; particularly those that will occur in the ensuing treatise.

1. An *integer*, or *whole number*, is an unit, or any multitude of units; as 1, 7, 48, 100, 125.

2. A *fraction*, or *broken number*, is any part or parts of an unit; and is expressed by two numbers, which are separated from one another by a line drawn betwixt them; the under number being called the *denominator*, and the upper one the *numerator*, of the fraction; as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$.

3. A *mixt number* is an integer with a fraction joined to it; as $4\frac{3}{4}$, $7\frac{3}{4}$, $48\frac{5}{8}$.

4. A number is said to *measure* another number, when it is contained in that other number a certain number of times, or when it divides that other number without any remainder. Thus, 3 measures 6, 9, or 12.

5. An *even number* is that which is measured by 2, or which 2 divides without any remainder; as 2, 4, 6, 8, 10, 12.

6. An *odd number* is that which 2 does not measure, or which cannot be divided by two, without a remainder; as 1, 3, 5, 7, 9, 11, 13.

7. A *prime number* is that which unity, or itself, only measures; as 3, 5, 7, 11, 13, 17, 19.

8. A *composite number* is that which is measured by some other number than itself, or unity; as 12, which is measured by 2, 3, 4, or 6.

9. Numbers are called *prime* to one another, when unity only measures them. Thus 13 and 36 are prime to one another; for no number, except unity, measures both.

10. Numbers are called *composite* to one another, when some number, besides unity, measures them. Thus 12 and 18 are composite to one another; for 3 or 6 measures both of them.

11. A number which measures another is called an *aliquot part* of that other. Thus 6 is an *aliquot part* of 18, and 3 of 12, and 5 of 20.

12. The number measured, or which contains the *aliquot part* a certain number of times, is called a *multiple* of that *aliquot part*. Thus 18 is a multiple of 6, and 12 of 3.

13. A number is called an *aliquant part* of another, when it does not divide that other without a remainder. Thus 7 is an *aliquant part* of 24.

14. Two,

14. Two, three, or more numbers, which, multiplied together, produce another number, are called the *component parts* of the number produced. Thus 3 and 4, or 2 and 6, are the *component parts* of 12; and 2, 3, and 4, are the component parts of 24.

15. The product of a number multiplied into itself is called the *square*, or *second power*, of that number; and the number itself is in this case called the *root*. And if the square be multiplied into the root, the product is called the *cube*, or *third power*, of that number. And if the cube be multiplied into the root, the product thence arising is called the *biquadrate*, or *fourth power*, &c.

V. Arithmetical Principles or Axioms.

An axiom or first principle in any science is a self-evident proposition which needs no demonstration. The following ones flow necessarily from the nature of numbers, the manner of their notation, or their specific properties. On them is founded the theory of arithmetic, and from them are deduced the rules to be observed in every operation.

I. Any given number may be increased or diminished at pleasure.

For there is no number so great, but a greater may be given; nor is there any number so small, but a smaller may be assigned.

II. The figures of any number increase in value from the right toward the left hand.

Hence in addition, subtraction, and multiplication, the operation begins at the units figure or lowest place on the right hand, and proceeds, with the flux of numbers, to the highest place on the left.

III. Ten in any inferior place makes one or an unit in the next higher place, and the reverse.

Hence when the sum of any column added, or the product of any two figures multiplied, amounts to or exceeds ten, we carry an unit for every ten to the next higher place; for the highest digit being 9, any number above 9 is compound, and requires more than one place to express it; which is done by removing or carrying away the tens, as so many units, to the next superior place.

IV. If the right-hand figure of any number be cut off, the remaining figure or figures are a just number of tens, and the right-hand figure so cut off is the overplus.

Hence in addition and multiplication, the right-hand figure of the sum of any column, or of the product of any two figures, is always set down, and the remaining figure or figures are carried on to the next higher place. Thus if a sum or product amount to 10, the 0 is set down, and 1 is carried; if it amount to 72, the 2 is set down, and 7 is carried; if it amount to 100, then 0 is set down, and 10 is carried; if it amount to 136, then 6 is set down, and 13 carried, &c.: but if the sum or product amount only

ly to a single digit, then the digit is set down, and nothing is carried.

V. A number, by having one, two, three, &c. ciphers annexed to it, becomes ten, a hundred, a thousand, &c. times greater.

Because the figures of the number are, by this means, thrown into higher places, thus 7 by a cipher annexed becomes 70, by two ciphers 700, and by three ciphers 7000, &c.

VI. Any number is naturally resolved into as many constituent parts as it has significant figures, by annexing to each significant figure as many ciphers as there are figures on its right hand.

Thus 345 is resolved into 300, 40, and 5; and 6082 is resolved into 6000, 80, and 2; and 70090 into 70000 and 90.

VII. The separate value of a single figure in any number is greater by one at least, than the value of all the figures on its right hand.

Thus, in the number 199, the separate value of 1, viz. 100, is greater by 1 than the figures on its right hand 99.

VIII. An unit is an aliquot part of every whole number, and every whole number is a multiple of unity:

For unity measures every whole number.

IX. Numbers equally augmented or diminished, continue to have the same difference.

Let any numbers, such as 6 and 8, be equally augmented, by the addition of 4 to each of them, the sums 10 and 12 will have the same difference as 6 and 8, viz. 2. Again, if from each of them 4 be taken away, the remainders 2 and 4 will have the same difference, viz. 2.

X. The difference of two unequal numbers added to the lesser, gives a sum equal to the greater; or subtracted from the greater, leaves a remainder equal to the lesser.

Let 3, the difference of two unequal numbers, 5 and 8, be added to 5, the lesser, and the sum will be 8, equal to the greater; or, if the difference 3 be subtracted from the greater, 8, the remainder will be 5, equal to the lesser.

XI. None but similar or like things can be added or subtracted.

That is, units only can be added to units, tens only to tens, hundreds only to hundreds, &c.; for if unlike things were added together, the sum would be false. Thus, if 4 units were added to 5 tens, the sum would be 9; but neither 9 units, nor 9 tens, nor 9 of any other name.

In like manner, farthings only can be added to farthings, pence only to pence, shillings only to shillings, pounds only to pounds, ounces only to ounces, &c.: for if 3 farthings were added to 4 pence, the sum would be 7, but neither 7 farthings, nor 7 pence.

For the same reason, sheep can only be added to sheep, cows only to cows, horses only to horses, &c.: for if 7 sheep were added to 8 cows, the sum would be 15; but neither 15 sheep, nor 15 cows.

The

The same way of reasoning may be used with respect to subtraction.

XII. Any whole is equal to all its parts.

COROLLARIES.

1. If a significant figure, by itself, or with a cipher or ciphers annexed, be divided by 9, the remainder will be equal to the significant figure taken in its simple value.

Thus, if 8, 80, 800, 8000, &c. be divided by 9, the remainder will be 8 : for if 8 be divided by 9, the quotient will be 0, and the remainder 8. Again, 80 is equal to 8 10's; that is, to 8 9's, and 8 1's, or 8. And, in like manner, 800 is equal to 80 10's; that is, to 80 9's, and 80 1's, or 80, whose remainder is already shown to be 8. The same way of reasoning may be applied to any other such number.

2. If any number be divided by 9, the remainder will be equal to the sum of the figures of the said number taken in their simple value, or to the excess above the 9's contained in the said sum.

For let the number be resolved by axiom VI. into its constituent parts, then, by the preceding corollary, the significant figure of each part will be the remainder of that part when divided by 9; and so the remainders of the several parts will be the figures of the given number; out of which, if need be, let the 9's be taken away, and the excess will be the remainder.

3. Hence the remainder arising from a number divided by 9 is found by adding the figures of the said number. Thus, the remainder of the number 231 divided by 9 is 6; for 2, 3, and 1, added together, make 6. Again, the remainder of 84632 is 5; for 8, 4, 6, 3, and 2, added together, make 23, whose excess above the 9's contained in it is 5, found by adding 2 to 3.

4. Numbers expressed by the same figures, however different the value of the numbers be, have the same remainder when divided by 9. Thus, 436, 463, 346, 364, 643, and 634, have all the same remainder when divided by 9, because the sum of their figures is the same.

VI. *An explanation of characters sometimes used for the sake of brevity.*

= Denoting equal to, is the sign or mark of an equation; and signifies, that the numbers betwixt which it stands are equal to one another.

+ Denoting plus, more, or added to, is the sign or mark of addition; and signifies, that the numbers it is placed between, are to be added together; as $6+2=8$; that is, 6 plus 2, or 6 having 2 added to it, is equal to 8.

— Denoting minus, or less, is the sign of subtraction; and shews, that

that the latter of the numbers betwixt which it stands is to be subtracted from the former; as $7-3=4$; that is, 7 minus 3, or 7 having 3 subtracted from it is equal to 4.

× Denoting into, or multiplied into, is the sign of multiplication, and signifies, that the numbers betwixt which it stands are to be multiplied together; as $3 \times 5 = 15$; that is, 3 into 5, or 3 multiplied into 5, is equal to 15.

÷ Denoting divided by, is the mark of division; and signifies, that the former of the numbers betwixt which it stands is to be divided by the latter; as $12 \div 4 = 3$; that is 12 divided by 4, is equal to 3. Or this mark may be considered as setting the dividend above and the divisor below the line; and so it becomes a fractional expression, signifying, that the numerator is to be divided by the denominator; as $\frac{12}{4} = 3$; that is, 12 divided by 4 is equal to 3.

Denoting division is also marked thus, $4)12(3$.

:: Denoting so is, is the sign of geometric proportion; and is placed betwixt the second and third terms of four proportional numbers; thus, $4 : 8 :: 12 : 24$; that is, as 4 is to 8, so is 12 to 24.

√ This mark is called the *radical sign*; and has a figure or index set over it, to denote such a root of the number before which it stands. Thus, $\sqrt{25} = 5$ signifies, that the square root of 25 is 5; and $\sqrt[3]{64} = 4$ denotes the cube root of 64 to be 4. When the square root is meant, the radical sign is usually left blank, or has no figure placed over it.

VII. Practical Questions.

Q. 1. Troy was taken and destroyed by the Greeks, two thousand eight hundred and twenty years after the creation, and one thousand one hundred and sixty-four years after the flood, and four hundred and thirty-six years before the building of Rome, and eleven hundred and eighty-four years before the birth of Christ, and two thousand nine hundred and forty-four years before the accession of King George III. to the throne of Britain: How are these dates expressed in letters?

2. The inhabitants of a certain country are twenty-five millions four thousand and seventy men, thirty-four millions seven hundred and five thousand eight hundred and two women, fifty millions and four hundred boys, fifty-six millions six thousand and eight girls: How are these numbers expressed in figures?

3. A farmer has in his granary 470900608 grains of wheat, 80769000240 of barley, 702000678000 of rye, and 27300845302912 of oats: How are these numbers read, or expressed in words?

CHAP. II.

A D D I T I O N.

Addition is the collecting of two or more numbers into one sum or total.

I. *Addition of Integers.*

R U L E S.

I. Set figures of like place under other, *viz.* units under units, tens under tens, &c. See axiom XI.

II. Beginning at the lowest place, set down the right-hand figure of the sum of every column, and carry the rest as so many units to the next superior place. See axioms II. III. IV.

E X A M P L E I.

Because similar or like things only can be added, I place the numbers as directed in Rule I. *viz.* units under units, tens under tens, &c. as in the margin. Then, beginning at the lowest place, *viz.* that of units, I say, 4 units and 3 units make 7 units, which I set below in the place of units; then 3 tens and 5 tens make 8 tens, which I set below in the place of tens; then 2 hundreds and 4 hundreds make 6 hundreds, which I set below in the place of hundreds, and find the sum or total to be 687.

From the repetition of the former example on the margin, it is plain, that it is a matter of indifference which of the numbers be placed uppermost.

$$\begin{array}{r} 453 \\ 234 \\ \hline 687 \end{array}$$

E X A M P L E II.

Having placed the numbers, units under units, &c. as in the margin, I say, 2 and 1 make 3, and 3 make 6, and 4 make 10; which being just 1 ten, and nothing over, I set the right-hand figure 0 in the place of units; and because ten in any lower place makes but one in the next superior place, I carry my 1 ten, as directed in Rule II. saying, 1 ten, collected out of the units, and 6 tens make 7 tens, and 4 make 11, and 0 makes but still 11, and 7 make 18; here again I set down the right-hand figure 8, in the place of tens, and carry the remaining figure 1, being 1 hundred, to the next place, *viz.* that of hundreds; and having in like manner added up this column, the amount is 31; so I set down the right-hand figure 1 in the place of hundreds, and carry the remaining

$$\begin{array}{r} 5974 \\ 9803 \\ 7541 \\ 862 \\ \hline 24180 \end{array}$$

maining figure 3 to the next place or column; which being also added, amounts to 24; I set the right-hand figure 4 below, in its proper place, and the remaining figure 2, which belongs to the next place, I set on the left hand, there being no figure in the next place to which it can be carried. So the sum or total is 24180.

The reason of the operation will still further appear, by taking the sum of each column separately, and then adding them into one total, as in the margin.

5974
9803
7541
862

Sum of the

10 units
17. tens
30.. hundreds
21... thousands
—
24180 total.

In adding integers, some teach young beginners to dot at every ten, and setting down the excess, to carry 1 for every dot. Thus, in adding the example in the margin, I say, 4 and 8 make 12; where I dot, set down the excess 2, and carry 1 for the dot to the next column; saying, 1 carried and 6 make 7, and 8 is 15; where I again dot, set down the excess 5, and carry 1 to the next column; saying, 1 carried and 9 is 10; where I dot, and set down the 7 as the excess. The 1 carried for the dot is placed on the left hand. This method is too low for any but a child.

7 8. 8.
9. 6 4
—
1 7 5 2

M O R E E X A M P L E S.

Ex. 3.
78943745
60738937
84976858
93543945
48732790
85476437
48763728
7845963
94769

Ex. 4.
8467452
7893968
47637
83745
4897
548763
674376
9848937
53700480

II. *Addition of the parts of integers, such as shillings, pence, farthings, ounces, &c.*

R U L E S.

I. Place like parts under other; viz. farthings under farthings, pence under pence, &c. See axiom XI.

B

II. Begin

II. Begin at the lowest of the parts, and carry according to the value of an unit of the next superior denomination; *viz.* for every four in the sum of farthings carry 1 to the pence, and for every twelve in the pence carry 1 to the shillings, &c. The reason of this rule is plain from the tables of coin, weights, and measures.

III. If you carry at 20, 30, 40, 60, or any just number of tens, as in adding shillings, degrees, poles, minutes, seconds, &c. proceed with the column of units as in addition of integers, and from the sum of the column of tens carry 1 for every two, or 1 for every three, &c. according as 20 or two tens, 30 or three tens, &c. make an unit of the next superior denomination. The reason appears plain in the following operations.

I. M O N E Y.

T A B L E.

4 farthings	}	make	{	1 penny.
12 pence				1 shilling.
20 shillings				1 pound.

Marked thus :

$$l. \quad s. \quad d. \quad f. \text{ or } q.$$

$$1 = 20 = 240 = 960$$

l. is put for *libra*, a pound; *d.* for *denarius*, a penny; and *q.* for *quadrans*, a fourth-part; but *f.* is now the more usual mark for farthings.

There are a few other denominations of money, but never used in keeping accounts, *viz.* a groat, in value 4d. a tester 6d. a crown 5s. half a crown 2s. 6d. a noble 6s. 8d. an angel 10s. a mark 13s. 4d. a guinea 21s. half a guinea 10s. 6d. and a quarter of a guinea 5s. 3d.

That the learner may proceed in addition of money with the greater ease, it will be proper he get the following table by heart.

M O N E Y - T A B L E.

<i>f.</i>	<i>d.</i>	<i>d.</i>	<i>s.</i>	<i>s.</i>	<i>l.</i>
4 =	1	12 =	1	20 =	1
8 =	2	24 =	2	40 =	2
12 =	3	36 =	3	60 =	3
16 =	4	48 =	4	80 =	4
20 =	5	60 =	5	100 =	5
24 =	6	72 =	6	120 =	6
28 =	7	84 =	7	140 =	7
32 =	8	96 =	8	160 =	8
36 =	9	108 =	9	180 =	9
40 =	10	120 =	10	200 =	10

E X.

E X A M P L E.

Having, according to Rule I. placed like (10)(20)(12)(4) parts under other, *viz.* farthings under farthings, pence under pence, &c. and in each of these denominations, units under units, tens under tens, as in the margin, I begin with the lowest of the parts, *viz.* the farthings; and say, 2 farthings and 1 farthing make 3 farthings, and 2 make 5, and three make 8; which, by the money-table, is 2 fours, or 2 pence, and nothing over; wherefore I place 0 below in the place of farthings, or rather leave that place blank, and carry my 2 pence to the place of pence, as directed in Rule II. saying, 2 pence, collected out of the farthings, and 9 make 11, and 8 make 19, and 1 (passing the 0) make 20; to this sum of my units I add my tens. Thus, 20 and 1 ten make 30, and 1 ten more make 40 pence; which, by the money-table, is 3 twelves, or 3 shillings, and 4 pence over; these 4 pence I set below in the place of pence, and carry my 3 shillings to the place of shillings. Thus, 3 shillings, collected out of the pence, and 1 shilling make 4, and 7 make 11, and 9 make 20, and 8 make 28; and because in shillings we carry at a just number of tens, *viz.* at 20, I set my right-hand figure 8 below in the place of units, as directed in Rule III. and carry my 2 tens to the place of tens. Thus, 2 tens collected out of the units, and 1 ten make 3 tens, and 1 make 4, and 1 make 5 tens, or 2 twenties, and 1 ten over; and because 2 tens, or 1 twenty, make an unit in the next place, *viz.* that of pounds, I set my 1 ten below in the place of tens, and carry my 2 twenty shillings, or 2 pounds, to the place of pounds; which, being integers, are added as taught in addition of integers.

It is usual to subjoin the farthings to the pence by way of fraction, as in the margin, where the former example is transcribed in this form for the learner's instruction; in which $\frac{1}{4}$ denotes one farthing, $\frac{1}{2}$ two farthings, and $\frac{3}{4}$ three farthings.

L.	s.	d.
74	18	11 $\frac{3}{4}$
96	9	10 $\frac{1}{2}$
58	17	8 $\frac{3}{4}$
63	11	9 $\frac{1}{2}$
293	18	4

The method of adding money explained above, is the most approved, and generally practised by men of business; but yet other methods are sometimes used; such as,

1. Some, in instructing young children, teach them to dot at the figure which makes an unit of the next superior denomination; and, after setting down the overplus, they carry 1 for every dot.

B 2.

Thus,

Thus, in adding the example in the margin, (10) (20) (12) (4)
 I begin with the farthings, and say, 1 and 3 make 4; where I dot, and, setting down 47 17. 10. 2
 the 2 as an overplus, I carry 1 for the dot, 98 14. 9. 3.
 and say, 1 carried and 8 make 9, and 9 is 45 13 8 1
 18; where I dot, and say, 6 of overplus and 10 make 16; where I again dot, and, setting down 192 6 4 2
 the overplus 4, I carry two for the dots, and proceed, saying, 2 carried and 13 make 15, and 4 make 19, and 1 ten on the left make 29; where I dot, and go on, saying, 9 of overplus and 7 make 16, and 1 ten on the left make 26; where I again dot, and, setting down the overplus 6, I carry 2 for the dots to the pounds, which are integers, and may be added either by dotting at every ten, or by the usual method of adding integers. This method of dotting in the addition of money is childish, and ought not to be too far indulged.

2. Some follow the common method in adding the farthings and pence; but in adding the shillings proceed thus: (10) (20) (12)
 2 shillings carried from the pence and 4 make 6, L. s. d.
 and 6 make 12, and 8 make 20; and to this sum 84 18 11 $\frac{1}{4}$
 of the units I add the tens on the left hand, say- 63 16 10 $\frac{1}{4}$
 ing, 20 and 1 ten is 30, and 1 is 40, and 1 is 50; 78 10 9 $\frac{3}{4}$
 which, by the money-table, is 2l. 10s.; where- 95 4
 fore I set down the 10s. and carry the 2l. to the 322 10 7 $\frac{1}{2}$
 pounds, which are integers, and added accord-
 ingly.

3. In adding up large accounts, some dot at 60 in the pence, and for every dot carry 5 to the shillings; and in adding the shillings they dot likewise at 60, (10) (20) (12)
 and for every dot carry 3 to the pounds. Thus, L. s. d.
 in adding the example in the margin, I begin 45 19 6 $\frac{1}{4}$
 with the farthings, which I add in the usual man- 48 17 10 $\frac{3}{4}$
 ner, and find they amount to 23; which, by the 83 14. 11 $\frac{1}{4}$
 money-table, is 5 $\frac{3}{4}$ d.; wherefore I set down the 72 15 9 $\frac{1}{2}$
 $\frac{3}{4}$, and carry 5 to the pence, which I proceed to 84 13 4
 add, by saying, 5 carried and 8 is 13, and 10 is 86 9 10 $\frac{1}{2}$
 23, and 11 is 34, and 10 is 44, and 11 is 55, and 78 11 $\frac{1}{4}$
 10 is 65; where I dot, and go on, saying, the 64 10
 overplus 5 and 4 is 9, and 9 is 18, and 11 is 29, 56 18. 10 $\frac{1}{2}$
 and 10 is 39, and 6 is 45; which, by the money- 79 16 11 $\frac{3}{4}$
 table, is 3s. 9d.; accordingly I set down the 9d. 80 11 10 $\frac{1}{2}$
 and 3s. added to 5s. for the dot make 8 to 85 10 8 $\frac{3}{4}$
 be carried to the shillings; which I proceed to 868 9 $\frac{3}{4}$
 add, by saying, 8 carried and 10 is 18, and 11 is 29, and 6 is 35, and 1 ten on the left is 45, and 8 is 53, and 1 ten on the left is 63; where I dot, and go on, say-
 ing, 3 of overplus and 10 is 13, and 9 is 22, and 3 is 25, and 1
 ten on the left is 35, and 5 is 40, and 1 ten is 50, and 4 is 54, and
 1

1 ten is 64; here again I dot, and go on, saying, 4 of overplus and 7 is 11, and 1 ten is 21, and 9 is 30, and 1 ten is 40, which is just 21. and no overplus to be set down; and these 21. with 61. for the two dots, make 8 to be carried to the pounds, which are integers, and may be either added in the usual method, or by dotting at every 100.

4. In casting up large accounts or bills, some chuse to divide them into parcels, and then, by any of the methods taught above, they cast up each parcel separately, and afterwards add the sums of the several parcels into one total; as is done by the following example :

(10)	(20)	(12)			
L.	s.	d.			
42	14	8			
36	15	4 $\frac{1}{2}$			
38	17	10 $\frac{3}{4}$	L.	s.	d.
			118	7	11 $\frac{1}{4}$
32	18	9 $\frac{1}{4}$			
48	16	11 $\frac{1}{2}$			
50	19	10			
			132	15	6 $\frac{3}{4}$
56	8	6 $\frac{1}{2}$			
54	6				
57		8 $\frac{1}{4}$			
52	14	7			
			220	9	10
Total			471	13	4

It remains now to be observed, with respect to the empty places, whether in shillings, pence, or farthings, or those in the left-hand columns of the shillings and pence, that, instead of filling them up with ciphers, as the common practice was some time ago, the general fashion now, in most cases, is to leave them blank; and this remark holds, not only in addition of money, but in every other kind of addition; nay, extends to every part of arithmetic, whether in addition, subtraction, multiplication, or division.

M O R E E X A M P L E S.

Ex. 1.

(10)	(20)	(12)
L.	s.	d.
789	17	10 $\frac{1}{2}$
476	14	11 $\frac{3}{4}$
894	18	9
438	13	10 $\frac{1}{4}$
643	12	8
784	15	
435	6	10 $\frac{1}{2}$
947	8	6 $\frac{3}{4}$

Ex. 2.

(10)	(20)	(12)
L.	s.	d.
483	15	11 $\frac{1}{4}$
724	13	4
865	18	7 $\frac{1}{2}$
978	14	8 $\frac{3}{4}$
685	19	6
538	17	4 $\frac{1}{4}$
945	9	8
782	5	10

Ex. 3.			Ex. 4.		
(10)	(20)	(12)	(10)	(20)	(12)
L.	s.	d.	L.	s.	d.
5748	19	11 $\frac{1}{4}$	4786	17	10 $\frac{1}{2}$
4875	17	10 $\frac{1}{2}$	8490	14	9 $\frac{1}{4}$
9387	13	4	5786	13	4 $\frac{3}{4}$
6453	14	8	8327	15	8
5874	10	7 $\frac{1}{2}$	7500	9	7 $\frac{1}{2}$
9482	9	11 $\frac{1}{2}$	6742	8	6
6093		10 $\frac{1}{4}$	894		10 $\frac{1}{4}$
7800	17	6	672	18	
508	15	10 $\frac{1}{4}$	95	16	11 $\frac{1}{2}$
75			8	19	10 $\frac{1}{4}$

2. AVOIRDupois WEIGHT.

T A B L E.

16 drams	}	make	1 ounce.
16 ounces			1 pound.
28 pounds			1 quarter.
4 quarters			1 hundred.
20 hundreds			1 tun.

Marked thus :

T.	C.	℥.	lb.	oz.	dr.
1	= 20	= 80	= 2240	= 35840	= 573440
1	= 4	= 112	= 1792	= 28672	

By Avoirdupois weight are weighed butter, cheese, rosin, wax, pitch, tar, tallow, soap, salt, hemp, flax, beef, brags, iron, steel, tin, copper, lead, alum, and all grocery wares.

Note, 19 $\frac{1}{2}$ C. of lead make a fodder.

In adding the following example, I begin with the ounces, and say, 15 and 10 make 25; which being above 16, I dot, and carry away the excess 9, saying, 9 of excess and 6 make 15, and 8 make 23; where I again dot, and carry away the excess 7, saying, 7 and 2 is 9, and 1 ten on the left is 19; where I dot, and proceed with the excess 3, saying, 3 and 4 is 7, and 1 ten on the left is 17; where I dot, and carry the excess 1, saying, 1 and 5 is 6, and 1 ten on the left is 16; where I again dot, and there being no excess, I have nothing to set down.

(10)	(20)	(4)	(28)	(16)
T.	C.	℥.	lb.	oz.
74	19	3	27.	15.
85	17	2	24.	14.
68	13	1	20	12.
52	18	3	19.	8.
50	10	2	18.	6
48	9	3	16	10.
97	5	1	3	15
478	15	3	20	

I now proceed to add the pounds; saying, 5 carried from the ounces, *viz.* one for every dot, and 3 make 8, and 6 make 14, and 1 ten on the left is 24, and 8 make 32; which being above 28, I dot, and go on, saying, 4 of excess and 1 ten on the left is 14, and 9 is 23, and 1 ten on the left is 33; where I again dot, and go on, saying, 5 of excess and 20 is 25, and 4 is 29; where I dot, and proceed, saying, 1 of excess and 2 tens on the left make 21, and 7 make 28; where I dot, and the 2 tens, or 20, on the left, I set below.

I should now proceed to add the quarters; saying, 4 carried from the pounds and 1 make 5, &c.; but as you carry here 1 for every four, the quarters are added exactly as the farthings in addition of money. In the hundreds you carry at 20; which, therefore, are added as shillings. The tuns are integers; and added accordingly.

The excess may sometimes be known without actual addition. Thus, in casting up the ounces of the above example, it is easy to perceive, that 15 ounces want only 1 to complete the pound; accordingly I imagine 1 to be taken from 10; and, after dotting, I go on with the excess 9, saying, 9 and 6 make 15; and again, as 15 wants only 1, I imagine this 1 to be taken from 8, and so I dot, and proceed with the excess 7, saying, 7 and 2 make 9, &c.

The accounts may be kept clean by making the dots on a blotter, or piece of waste paper laid under your hand.

Men of business frequently add both the ounces and the pounds as integers; divide the sum by 16 or 28; and, setting down the remainder as the excess, carry the quotient.

Retailers of silk, worsted, thread, and such other small wares, have occasion only for pounds, ounces, and drams, and this they call *avoirdupois small weight*; and in retailing some of these, it is usual to divide the ounce into four quarters instead of sixteen drams.

The denomination on the left hand, whether tuns, hundreds, or pounds, is always integers, and added as such; which in the following examples is pointed out by (10) superinscribed, being the number at which you carry in integers. This remark may be extended to other sorts of addition; as will appear in what follows:

M O R E E X A M P L E S.

Ex. 1.

(10)	(20)	(4)	(28)
T.	C.	℥.	lb.
34	14	13	15
36	18	1	14
32	16	2	12
30	12	1	10
28	15	3	23
25	19	2	20
26	13	3	24
22	10	1	18
38	15	3	12

Ex. 2.

(10)	(4)	(28)	(16)
C.	℥.	lb.	oz.
25	1	18	12
42	3	14	14
45	2	24	15
36	1	18	10
64	3	22	13
62	1	20	11
48	3	25	9
52	1	19	8
54	2	8	6

Ex. 3.

(10)	(16)	(16)
lb.	oz.	dr.
3	10	6
4	14	11
5	13	14
6	12	10
7	10	8
8	15	14
4	8	12
2	6	10
5	7	9

Ex. 4.

(10)	(16)	(4)
lb.	oz.	q.
4	12	3
7	10	2
3	14	1
5	13	3
6	11	2
8	10	3
4	13	2
8	15	3
3	8	1

3. T R O Y W E I G H T.

T A B L E.

24 grains	}	make	{	1 penny-weight.
20 penny-weights				1 ounce.
12 ounces				1 pound Troy.

Marked thus:

lb.	oz.	dw.	gr.
1	= 12	= 240	= 5760

By Troy weight are weighed gold, silver, jewels, amber, electuaries, and liquors.

Note. That 1 lb. Avoirdupois is equal to 14 oz. 11 dw. 15½ gr. Troy.

In adding the example in the margin, I (10) (12) (20) (24) begin with the grains, and say, 13 and 4 lb. oz. dw. gr. make 17, and 1 ten on the left make 27, 48 11 18 21. which being above 24, I dot, and go on with 42 10 14 18. the excess 3, saying, 3 and 2 make 5, and 40 9 16 20. 2 tens, or 20, on the left, make 25; where 36 8 15 22. I again dot, and proceed with the excess 1, 38 10 10 14. saying, 1 and 20 make 21, and 8 make 29; 53 6 17 13. where I dot, and go on with the excess 5, 261 10 14 12. saying, 5 and 1 ten on the left make 15, and 1 makes 16, and 20 on the left make 36. I set down the excess 12, and carry away 4 for the dots, to the penny-weights.

In casting up the penny-weights you carry at 20, which therefore are added exactly like shillings. In the ounces you carry at 12, which are therefore added as pence. The pounds are integers, and added as such.

Some

Some men of business add the grains as integers, and, setting down the sum on a separate paper, divide it by 24, and then place the remainder as the excess under the grains, and carry the quotient to the penny-weights.

M O R E E X A M P L E S.

Ex. 1.

(10)	(12)	(20)	(24)
<i>lb.</i>	<i>oz.</i>	<i>dw.</i>	<i>gr.</i>
72	11	19	23
74	10	13	21
70	9	16	18
54	8	12	22
56	6	10	20
63	10	8	16
62	11	6	10
60	10	18	14

Ex. 2.

(10)	(12)	(20)	(24)
<i>lb.</i>	<i>oz.</i>	<i>dw.</i>	<i>gr.</i>
42	10	14	22
44	9	16	18
40	11	19	21
36	8	15	17
32	6	8	12
35	10	6	13
38	11	17	8
40	11	18	23

4. APOTHECARIES WEIGHT.

T A B L E.

20 grains	}	make	}	1 scruple.
3 scruples				1 dram.
8 drams				1 ounce.
12 ounces				1 pound.

Marked thus:

$$\text{lb} \quad \text{℥} \quad \text{ʒ} \quad \text{℥} \quad \text{ʒ} \quad \text{gr.}$$

$$1 = 12 = 96 = 288 = 5760$$

By this weight apothecaries compound their medicines, their pound being the same as the pound Troy, but differently divided; but yet they buy and sell their drugs by Avoirdupois.

In adding the example in the margin, I begin with the grains; and because in grains we carry at 20, they are added as shillings; you add the scruples as integers, and carry 1 for every three of that sum; you add the drams in the same manner, and carry 1 for every 8; in the ounces you carry at 12, which are therefore added as pence; the pounds are integers, and added as such,

(10)	(12)	(8)	(3)	(20)
<i>lb</i>	<i>℥</i>	<i>ʒ</i>	<i>℥</i>	<i>gr.</i>
44	11	7	2	19
43	10	5	1	15
48	9	3	2	14
36	8	2	1	18
32	10	6	2	12
30	11	4	1	17
238	2	7	1	15

M O R E

MORE EXAMPLES.

Ex. 1.					Ex. 2.		
(10)	(12)	(8)	(3)	(20)	(10)	(3)	(20)
lb	3	3	9	gr.	3	9	gr.
36	10	6	2	18	14	1	18
32	11	7	1	14	28	2	14
18	10	5	2	12	32	1	16
14	11	3	1	19	41	2	19
23	9	4	1	18	35	1	12
28	8	7	2	17	26	2	15
45	7	3	1	14	25	1	17
53	6	2		18	53	2	13

5. WOOL WEIGHT.

7 pounds	}	make	1 clove.
2 cloves			1 stone.
2 stones			1 todd.
$6\frac{1}{2}$ todods			1 wey.
2 weys			1 sack.
12 sacks			1 last.

Marked thus:

Last. sack. wey. todd. stone. clove. lb.
 $1 = 12 = 24 = 156 = 312 = 624 = 4368$

Ex. 1.					Ex. 2.		
(10)	(12)	(2)	($6\frac{1}{2}$)	(2)	(10)	(2)	(7)
<i>Last.</i>	<i>sack.</i>	<i>wey.</i>	<i>tod.</i>	<i>sto.</i>	<i>Ston.</i>	<i>clov.</i>	<i>lb.</i>
72	11	1	6	1	14	1	6
70	10	1	3	1	28	1	5
74	9	1	5	1	23	1	4
63	8	1	3	1	18	1	3
82	11	1	5	1	17	1	2
86	10		4	1	15	1	5

In casting up the todods, I say, 3 carried from the stones, and 4 is 7, and 5 is 12, and 3 is 15, and 5 is 20, and 3 is 23, and 6 is 29; in which I find $6\frac{1}{2}$ four times, making 26, and 3 of excess: accordingly I set down the excess 3, and carry 4 to the weys. The manner of adding the other denominations is obvious from the table, or from the superinscribed numbers.

Note. If the sum of the todods contain an odd number of weys, as 1, 3, 5, 7, &c. then the excess will be either $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, or $5\frac{1}{2}$.

6. DRY

6. DRY MEASURE.

T A B L E.

2 pints	}	make	1 quart.
2 quarts			1 pottle.
2 pottles, or 8 pints			1 gallon.
2 gallons			1 peck.
4 pecks, or 8 gallons			1 bushel.
4 bushels			1 coomb.
2 coombs, or 8 bushels			1 quarter.
5 quarters, or 40 bushels			1 load.

Marked thus :

Lo. qrs. coom. bush. peck. gal. pot. qrts. pts.
 $1 = 5 = 10 = 40 = 160 = 320 = 640 = 1280 = 2560$

By this is measured grain ; as wheat, barley, pease, &c. ; also salt, sand, fruit, oysters, &c.

Note. That $33\frac{1}{4}$ cubic inches make a corn-pint, $268\frac{1}{4}$ cubic inches make a corn-gallon, and $2150\frac{3}{4}$ cubic inches a Winchester bushel, which is a cylindrical vessel $18\frac{1}{4}$ inches wide, and 8 inches deep.

Ex. 1.

(10)	(5)	(8)	(4)
Load.	qrs.	bu.	pec.
28	4	7	3
18	3	6	2
14	2	5	1
15	3	4	3
42	4	7	2
48	2	6	1
36	3	5	3

Ex. 2.

(10)	(40)	(8)
Load.	bu.	gal.
34	39	7
36	32	6
24	28	5
18	36	4
25	38	3
20	35	5
32	34	6

In casting up the bushels in *Ex. 2.* because we carry at 40, a just number of tens, I add up the right-hand column as integers, set down the right-hand figure of the sum, and carry the rest to the left-hand column ; which I also add up as integers, and carry 1 for every four of the sum.

In measuring coals the following table takes place :

4 pecks	}	make	1 bushel.
9 bushels			1 quarter.
4 quarters			1 chaldron.

Ex. 1.

Ex. 1.

(10)	(4)	(9)
<i>Chal.</i>	<i>qrs.</i>	<i>bu.</i>
54	3	8
28	2	7
52	1	6
46	3	5
48	2	7
32	3	8
36	2	5

Ex. 2.

(10)	(36)	(4)
<i>Chal.</i>	<i>bu.</i>	<i>pec.</i>
36	35	3
32	30	2
28	34	3
25	28	2
26	32	3
24	25	1
18	16	2

In casting up the bushels in *Ex. 2.* where we carry at 36, I say, 4 carried from the pecks and 16 make 20, and 25 make 45; which being above 36, I dot, and carry the excess 9, saying, 9 and 32 is 41; where I again dot, and go on with the excess 5, saying, 5 and 28 make 33, and 4 make 37; where I dot, and proceed with the excess 1, saying, 1 and 3 tens, or 30, on the left, make 31, and as 31 wants only 5 to complete the chaldron, I imagine that 5 taken from 30; where I dot again, and proceed with the excess 25; and as the next number 35 wants only 1, I suppose that 1 taken from 25, and dotting again, I set down the excess 24, and carry 5 for the dots to the chaldrons, which are integers.

The man of business may either follow the above method; or he may add up the bushels as integers, divide their sum by 36, set down the remainder as the excess, and carry the quotient to the chaldrons.

The dry measures in most parts of *Scotland* differ from those used in *England*, and are divided as in the following table:

4 lippies or forpats	} make	1 peck.
4 pecks		1 firlo.
4 firlots		1 boll.
16 bolls		1 chalder.

Ex. 1.

(10)	(16)	(4)	(4)
<i>Ch.</i>	<i>b.</i>	<i>f.</i>	<i>p.</i>
25	15	3	2
18	14	2	1
35	12	3	2
28	10	1	3
30	11	3	1
42	13	1	3
40	10	2	1

Ex. 2.

(10)	(4)	(4)	(4)
<i>B.</i>	<i>f.</i>	<i>p.</i>	<i>l.</i>
28	2	3	2
34	3	1	3
30	1	2	1
26	3	1	2
28	1	3	3
45	2	1	1
33	3	2	2

The bolls in *Ex. 1.* are added as ounces or drams in Avoirdupois weight; but the bolls in *Ex. 2.* are integers, and added as such.

7. WINE-MEASURE.
T A B L E.

2 pints	}	make	{	1 quart.
4 quarts				1 gallon.
10 gallons				1 anchor.
18 gallons				1 runlet.
31½ gallons				1 barrel.
42 gallons				1 tierce.
63 gallons				1 hoghead.
2 hogheads				1 pipe or butt.
2 pipes	}		{	1 tun.

Marked thus :

<i>Tun.</i>	<i>pipe.</i>	<i>hhd.</i>	<i>gall.</i>	<i>qrt.</i>	<i>pt.</i>
1	= 2	= 4	= 252	= 1008	= 2016

A puncheon, strictly speaking, is 4 anchors, or 80 gallons ; but any cask betwixt a hoghead and a pipe is called a puncheon.

All spirits, mead, perry, cyder, vinegar, oil, and honey, are measured as wine.

The wine-pint contains $28\frac{7}{8}$ cubical inches, and the wine-gallon contains 231 cubical inches.

Ex. 1.				Ex. 2.		
(10)	(2)	(2)	(63)	(10)	(63)	(8)
<i>Tuns.</i>	<i>pipe.</i>	<i>hhd.</i>	<i>gall.</i>	<i>Hhd.</i>	<i>gall.</i>	<i>pts.</i>
48	1	1	36	45	23	7
64	1	1	48.	52	38	6
52	1	1	30.	43	46	5
43	1	1	23	25	18	7
32	1	1	56.	73	25	6
45	1	1	38	65	30	7

In casting up the gallons, where we carry at 63, I proceed as follows, viz. in Ex. 1. I readily perceive that 56 wants only 7 to complete the hoghead ; accordingly I imagine 7 taken from 38, and, after dotting at 56, I go on with the excess 31, saying, 31 and 3 make 34, and 2 tens, or 20, on the left, make 54 ; and as 54 wants only 9, I imagine 9 taken from 30 ; so I dot at 30, and proceed with the excess 21, saying, 21 and 8 make 29, and 40 on the left make 69 ; which being above 63, I dot again, and go on with the excess 6, saying, 6 and 36 make 42 ; which I set down, and carry 3 for the dots to the hogheads. The manner of operation is the same in Ex. 2.

The man of business may either cast up the gallons as directed above ; or he may add them as integers, divide the sum by 63, set down the remainder as the excess, and carry the quotient.

8. BEER

8. BEER and ALE MEASURE.

T A B L E.

2 pints	}	make	1 quart.
4 quarts			1 gallon.
9 gallons			1 firkin.
2 firkins			1 kilderkin.
2 kilderkins			1 barrel.
1½ barrel			1 hoghead.
2 hogheads			1 butt.
2 butts			1 tun.

Marked thus :

<i>Tun.</i>	<i>butt.</i>	<i>bbd.</i>	<i>barr.</i>	<i>kild.</i>	<i>firk.</i>	<i>gall.</i>	<i>grt.</i>	<i>pt.</i>								
1	=	2	=	4	=	6	=	12	=	24	=	216	=	864	=	1728

The beer or ale pint contains $35\frac{1}{4}$ cubic inches, and the gallon 282.

In some places, 8 gallons of ale is esteemed a firkin ; and in some places 34 gallons make a barrel.

Ex. 1.				Ex. 2.		
(10)	(3)	(2)	(9)	(10)	(3)	(36)
<i>Hbd.</i>	<i>kild.</i>	<i>firk.</i>	<i>gal.</i>	<i>But.</i>	<i>bar.</i>	<i>gall.</i>
32	2	1	8	68	2	35
30	1	1	7	42	1	32
28	1	1	5	40	2	28
40	2	1	6	34	1	15
42	1	1	7	22	2	12
36	2	1	6	18	1	24
37	1	1	7	9	2	18

The gallons in Ex. 2. are added like the bushels in Ex. 2. of coal-measure.

In *Scotland* the denominations of liquid measure are as in the following

T A B L E:

4 gills	}	make	1 mutchkin.
2 mutchkins			1 chopin.
2 chopins			1 pint.
2 pints			1 quart.
4 quarts			1 gallon.
16 gallons			1 hoghead.

The *Scotch* mutchkin is somewhat less than the *English* pint.

Ex. 1.

Ex. 1.				Ex. 2.			
(10)	(16)	(4)	(2)	(10)	(2)	(2)	(4)
Hbd.	gall.	qrt.	pt.	Pt.	chop.	mut.	gil.
42	14	3	1	14	1	1	3
48	12	2	1	23	1	1	2
50	15	1	1	25	1	1	3
52	10	3	1	26	1	1	2
45	13	1	1	15	1	1	1
42	14	2	1	18	1	1	3
28	12	3	1	20	1	1	2
22	10	3	1	24	1	1	3

9. CLOTH-MEASURE.

T A B L E.

4 nails	}	make	1 quarter.
4 quarters			1 yard.
3 quarters			1 ell Flemish.
5 quarters			1 ell English.

A nail is equal to $2\frac{1}{4}$ inches, and consequently a quarter is equal to 9 inches.

Hollands are generally measured by the ell English; tapistry by the ell Flemish; and most other things by the yard.

Ex. 1.			Ex. 2.			Ex. 3.		
(10)	(4)	(4)	(10)	(3)	(4)	(10)	(5)	(4)
Yds.	qrs.	n.	El.Fl.	qrs.	n.	El.Eng.	qrs.	n.
42	3	3	63	2	3	69	4	3
36	2	1	52	1	3	62	3	2
30	1	3	62	2	2	40	1	3
28	2	2	48	1	1	45	4	2
24	1	3	42	2	1	36	2	1
25	3	1	40	2	2	34	1	3

10. LONG MEASURE.

T A B L E.

3 barley-corns, or 12 lines	}	make	1 inch.
12 inches			1 foot.
3 feet			1 yard.
2 yards, or 6 feet,			1 fathom.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet,			1 pole, perch, or rod.
4 poles, or 66 feet,			1 chain.
10 chains, or 40 poles,			1 furlong.
8 furlongs, or 80 chains,			1 mile.
3 miles			1 league.

8. BEER and ALE MEASURE.

T A B L E.

2 pints	}	make	{ 1 quart.
4 quarts			{ 1 gallon.
9 gallons			{ 1 firkin.
2 firkins			{ 1 kilderkin.
2 kilderkins			{ 1 barrel.
1 $\frac{1}{2}$ barrel			{ 1 hoghead.
2 hogheads			{ 1 butt.
2 butts			{ 1 tun.

Marked thus :

Tun. butt. bhd. barr. kild. firkin. gall. qrt. pt.
 1 = 2 = 4 = 6 = 12 = 24 = 216 = 864 = 1728

The beer or ale pint contains $35\frac{1}{4}$ cubic inches, and the gallon 282.

In some places, 8 gallons of ale is esteemed a firkin ; and in some places 34 gallons make a barrel.

Ex. 1.

(10)	(3)	(2)	(9)
Hbd.	kild.	firkin.	gal.
32	2	1	8
30	1	1	7
28	1	1	5
40	2	1	6
42	1	1	7
36	2	1	6
37	1	1	7

Ex. 2.

(10)	(3)	(36)
But.	bar.	gall.
68	2	35
42	1	32
40	2	28
34	1	15
22	2	12
18	1	24
9	2	18

The gallons in *Ex. 2.* are added like the bushels in *Ex. 2.* of coal-measure.

In *Scotland* the denominations of liquid measure are as in the following

T A B L E:

4 gills	}	make	{ 1 mutchkin.
2 mutchkins			{ 1 chopin.
2 chopins			{ 1 pint.
2 pints			{ 1 quart.
4 quarts			{ 1 gallon.
16 gallons			{ 1 hoghead.

The *Scotch* mutchkin is somewhat less than the *English* pint.

Ex. 1.

<i>Ex. 1.</i>				<i>Ex. 2.</i>			
(10)	(16)	(4)	(2)	(10)	(2)	(2)	(4)
<i>Hbd.</i>	<i>gall.</i>	<i>qrt.</i>	<i>pt.</i>	<i>Pt.</i>	<i>chop.</i>	<i>mut.</i>	<i>gil.</i>
42	14	3	1	14	1	1	3
48	12	2	1	23	1	1	2
50	15	1	1	25	1	1	3
52	10	3	1	26	1	1	2
45	13	1	1	15	1	1	1
42	14	2	1	18	1	1	3
28	12	3	1	20	1	1	2
22	10	3	1	24	1	1	3

9. CLOTH-MEASURE.

T A B L E.

4 nails	}	make	1 quarter.
4 quarters			1 yard.
3 quarters			1 ell Flemish.
5 quarters			1 ell English.

A nail is equal to $2\frac{1}{4}$ inches, and consequently a quarter is equal to 9 inches.

Hollands are generally measured by the ell English; tapistry by the ell Flemish; and most other things by the yard.

<i>Ex. 1.</i>			<i>Ex. 2.</i>			<i>Ex. 3.</i>		
(10)	(4)	(4)	(10)	(3)	(4)	(10)	(5)	(4)
<i>Yds.</i>	<i>qrs.</i>	<i>n.</i>	<i>El.Fl.</i>	<i>qrs.</i>	<i>n.</i>	<i>El.Eng.</i>	<i>qrs.</i>	<i>n.</i>
42	3	3	63	2	3	69	4	3
36	2	1	52	1	3	62	3	2
30	1	3	62	2	2	40	1	3
28	2	2	48	1	1	45	4	2
24	1	3	42	2	1	36	2	1
25	3	1	40	2	2	34	1	3

10. LONG MEASURE.

T A B L E.

3 barley-corns, or 12 lines	}	make	1 inch.
12 inches			1 foot.
3 feet			1 yard.
2 yards, or 6 feet,			1 fathom.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet,			1 pole, perch, or rod.
4 poles, or 66 feet,			1 chain.
10 chains, or 40 poles,			1 furlong.
8 furlongs, or 80 chains,			1 mile.
3 miles			1 league.

A hand, or hand's breadth, in horfemanship, is 4 inches.

A common pace is 2 feet 6 inches.

A geometrical pace is 5 feet.

The chain is divided into 100 equal parts, or links; each link being $7\frac{92}{100}$ inches.

Ex. 1.			
(10)	(3)	(8)	(40)
Leag.	miles.	fur.	poles.
20	2	7	38
18	2	5	24
25	1	6	35
24	2	7	32
18	1	4	26
12	2	3	20
17	2	3	33

Ex. 2.			
(10)	(5 $\frac{1}{2}$)	(3)	(12)
Poles.	yds.	feet.	inches.
36	4	2	11
28	3	1	10
25	1	2	11
16	2	1	9
24	3	1	7
36	2	2	8
23	4	2	5

In casting up the poles in Ex. 1. where we carry at 40, a just number of tens, you add as in integers; and from the sum of the left-hand column carry 1 for every four, setting down the excess. The poles in Ex. 2. are integers, and added as such.

The denominations of long measure used in *Scotland* are as in the following

T A B L E:

37 inches	} make	1 ell.
6 ells, or $18\frac{1}{2}$ feet,		1 fall.
4 falls, or 74 feet,		1 chain.
10 chains, or 40 falls,		1 furlong.
8 furlongs, or 80 chains,		1 mile.

The chain is divided into 100 links; each link being $8\frac{88}{100}$ inches.

Ex. 1.			
(10)	(8)	(10)	(4)
Miles.	fur.	ch.	falls.
24	7	9	3
22	5	8	2
18	4	6	3
14	6	5	3
28	3	7	2
23	2	8	3

Ex. 2.			
(10)	(4)	(6)	(37)
Ch.	falls.	ells.	inches.
36	3	5	36.
32	2	4	32.
45	1	3	25.
23	2	4	18
18	3	5	16.
24	2	2	24

In casting up the chains in Ex. 1. because you carry at 10, you work as in addition of integers. In Ex. 2. the chains are integers.

In adding the inches in Ex. 2. I say, 24 and 6 make 30, and 1 ten on the left make 40; which, being above 37, I dot at 16, and go on with the excess 3, saying, 3 and 18 make 21, and 25 make

make 46; so I dot at 25, and proceed with the excess 9, saying, 9 and 32 make 41; accordingly I dot at 32, and go on with the excess 4, saying, 4 and 36 make 40; so I dot at 36, set down the excess 3, and carry 4 for the dots to the ells.

Men of business are at liberty to add the inches as above directed; or they may cast them up as integers, divide the sum by 37, and, setting down the remainder as the excess, carry the quotient.

II. LAND-MEASURE.

T A B L E.

30 $\frac{1}{4}$ square yards	}	make	{	1 square pole.
40 square poles				1 rood of land.
4 roods				1 acre.

Note, Customary measure in some places differs from the statute measure contained in the above table.

Land is usually measured by a chain of 100 links, whose length is 4 poles, or 66 feet; and so 10 square chains is equal to an acre.

Ex. 1.				Ex. 2.			
(10)	(4)	(40)	(30 $\frac{1}{4}$)	(10)	(4)	(40)	
<i>Acres.</i>	<i>roods.</i>	<i>sq.p.</i>	<i>sq.y.</i>	<i>Acres.</i>	<i>roods.</i>	<i>sq.p.</i>	
25	3	39	15	48	2	35	
23	2	25	24.	34	1	27	
48	1	32	18	30	3	25	
64	3	35	14.	46	3	38	
52	1	31	22.	42	1	34	
43	2	36	26	39	2	32	

In adding the square yards in Ex. 1. I say, 26 and 22 make 48; which being above 30 $\frac{1}{4}$, I dot at 22, and go on with the excess 17 $\frac{3}{4}$, saying, 17 $\frac{3}{4}$ and 14 make 31 $\frac{3}{4}$; so I dot again at 14, and proceed with the excess 1 $\frac{1}{2}$, saying, 1 $\frac{1}{2}$ and 18 make 19 $\frac{1}{2}$, and 24 make 43 $\frac{1}{2}$; so I dot at 24, and go on with the excess 13 $\frac{1}{4}$, saying, 13 $\frac{1}{4}$ and 15 make 28 $\frac{1}{4}$; which I set down as the excess, and carry 3 for the dots to the poles.

In casting up the square poles, whether in Ex. 1. or 2. you carry at 40, a just number of tens, which therefore are added as integers; and from the sum of the left-hand column you carry 1 for every four.

The denominations of land-measure used in *Scotland* are contained in the following

T A B L E:

36 square ells	}	make	{	1 square fall.
40 square falls				1 rood of land.
4 roods				1 acre.

C

Note,

Note, That customary measure in some places differs from this table.

The *Scotch* chain consists of 100 links, being in length 4 fells, or 74 feet; and consequently 10 square chains make an acre.

Four *Scotch* acres are somewhat more than five *English* acres.

Ex. 1.				Ex. 2.		
(10)	(4)	(40)	(36)	(10)	(4)	(40)
Acr.	r.	sq. f.	sq. el.	Acr.	r.	sq. f.
72	3	38	34	42	2	34
68	1	35	26	28	3	32
62	2	24	30	23	1	35
43	3	37	35	46	3	26
41	1	33	28	35	2	28
45	2	28	32	53	1	25

The square ells in Ex. 1. are added like the bushels in coal-measure.

12. SUPERFICIAL MEASURE.

T A B L E.

144 square inches } make { 1 square foot.
9 square feet } { 1 square yard.

By this measure are measured board, glass, pavements, plastering, wainscoting, tiling, flooring, &c.

Ex. 1.		Ex. 2.		Ex. 3.	
(10)	(9)	(10)	(9)	(10)	(9)
Yds.	feet.	Yds.	feet.	Yds.	feet.
72	8	42	5	38	5
43	7	36	3	32	6
34	5	22	7	74	7
68	6	18	8	85	4
35	4	24	6	42	8
32	7	35	4	26	7

If there be inches, they must be added as integers, then dividing their sum by 144, the remainder will be the excess, and the quotient must be carried to the feet.

13. SOLID MEASURE.

1728 solid inches } make { 1 solid foot.
27 solid feet } { 1 solid yard.

By this is measured timber, stone, digging, &c.

Ex. 1.

Ex. 1.

(10) (27)

Yds. feet.

52 26.

48 22.

34 18.

25 23

68 12.

32 16

Ex. 2.

(10) (27)

Yds. feet.

43 25

42 18

34 14

26 23

38 20

45 16

Ex. 3.

(10) (27)

Yds. feet.

67 23

25 24

32 18

27 23

16 15

14 14

In adding the feet, I say (*Ex. 1.*) 16 and 12 make 28; which being above 27, I dot at 12, and go on with the excess 1, saying, 1 and 23 make 24, and 8 make 32; where I dot again, and proceed with the excess 5, saying, 5 and 1 ten on the left make 15, and as 22 wants only 5, I take that 5 from the 15, and, dotting at 22, I go on with the excess 10; and as 26 wants only 1, I take that 1 from 10; so I dot at 26, and, setting down the excess 9, I carry 4 for the dots to the yards.

If inches be given, they must be added as integers, and their sum being divided by 1728, the remainder will be the excess, and the quotient must be carried to the feet.

14. A C I R C L E.

T A B L E.

60 seconds	}	make	{	1 minute.
60 minutes				1 degree.
30 degrees				1 sign.
12 signs				1 circle.

Marked thus :

Circ. fig.

1 = 12 = 360 = 21600 = 1296000

Ex. 1.

(10) (12) (30) (60)

Circ. fig.

15 11 29 59

24 5 24 56

36 7 18 42

52 6 20 35

19 10 15 28

28 4 12 45

Ex. 2.

(10) (30) (60) (60)

Sig.

10 25 48 54

9 23 40 36

5 18 34 48

2 22 32 40

7 20 30 25

3 18 15 18

In casting up the seconds, minutes, and degrees, where you carry at 60 or 30, a just number of tens, you add as in integers, and from the sum of the left-hand column, you carry 1 for every 6 in the seconds and minutes, and 1 for every three in the degrees.

C 2

15. TIME.

15. T I M E.

T A B L E.

60 seconds	} make	1 minute.
60 minutes		1 hour.
24 hours		1 day.
7 days		1 week.
4 weeks		1 month.
13 months		1 year of 364 days.

But the year complete consists of 365 days and 6 hours ; and is divided into twelve unequal calendar months ; whose names, with the number of days they contain, are as in the margin.

The 6 hours, in the space of 4 years, make a day ; and is accordingly every fourth year added to February, which then consists of 29 days ; and this year is called *Bissexile* or *Leap-year*, and contains 366 days.

Names.	Days.
January	31
February	28
March	31
April	30
May	31
June	30
July	31
August	31
September	30
October	31
November	30
December	31

365

Ex. 1.

(10)	(13)	(4)	(7)
Year	mon.	wee.	da.
25	11.	3	6
26	10.	2	4
32	12.	3	2
34	11.	1	5
38	9	2	6
42	7.	3	3
45	4	2	4

Ex. 2.

(10)	(24)	(60)	(60)
Da.	ho.	min.	sec.
38	23	56	48
34	20	45	54
32	18	42	36
25	12	25	18
18	16	15	45
20	21	38	42
15	14	50	25

In adding the months in Ex. 1. I say, 5 carried from the weeks, and 4 make nine, and 7 make 16 ; which being above 13, I dot at 7, and go on with the excess 3, saying, 3 and 9 make 12, and 1 make 13 ; where I dot, and proceed, saying, 1 ten on the left, and 12 make 22 ; where I again dot, and go on with the excess 9, saying, 9 and 10 make 19 ; where I dot, and proceed with the excess 6, saying, 6 and 11 make 17 ; where I dot, and setting down the excess 4, I carry 5 for the dots to the years.

In

In Ex. 2. the hours are added like the grains in Troy weight, and the minutes and seconds are added exactly like those of a circle.

The learner, by this time, may be supposed to be pretty well skilled in addition; and will henceforth, in order to his casting up any account, want only to know, how many of any inferior denomination makes an unit of the next superior; and this he will acquire partly by reading books, and partly by conversation and practice; and as a large detail of this kind would be tedious, and does not properly belong to arithmetic, I shall only add two or three things more.

12 things of any kind }
12 dozen } make { 1 dozen,
12 small gross } { 1 small gross.
12 small gross } { 1 great gross.

24 sheets of paper }
20 quires } make { 1 quire.
20 quires } { 1 ream.

20 things of any sort }
5 score } make { 1 score.
5 score } { 100, or the short hundred.
6 score, } { 120, or the long hundred.

III. *The Proof of Addition.*

Addition may be proved several ways.

1. Merchants and men of business usually add each column first upwards and then downwards, and upon finding the sum to be the same both ways, they conclude the work to be right: and this is all the proof that their time, or the hurry of business, will admit of.

2. It is a common practice in schools, to prove the work by a second summing without the top-line; and if this sum added to the top-line make the first total, the work is supposed to be right; as in the two following examples.

Ex. 1.

8745	Top-line
9378	
4764	
5876	
28763	Total
20018	Total without the top-line
28763	Proof
	C 3

Ex. 2.

L.	s.	d.
748	15	10 $\frac{3}{4}$
674	13	11 $\frac{1}{2}$
835	17	9 $\frac{1}{4}$
90	18	8
2350	6	3 $\frac{1}{2}$
1601	10	4 $\frac{1}{4}$
2350	6	3 $\frac{1}{2}$
		3. Addition

3. Addition is also proved by casting out the 9's : for if the excess above the 9's in the total be the same as the excess in the items, the work may be presumed right. Thus, to prove the example in the margin, I begin with the items, and say, $3 + 4 = 7$, and $7 + 7 = 14 = 1 + 4 = 5$: with this 5 I pass to the next item, and say, $5 + 6 = 11 = 1 + 1 = 2$, and $2 + 8 = 10 = 1$, and $1 + 4 = 5$: which 5, being the excess of the items, I place at the top of the cross, and proceed to cast the 9's out of the total, saying, $1 + 3 = 4$, and $4 + 1 = 5$: which 5, being the excess of the total, I place at the foot of the cross ; and because it is the same with the figure at the top, I conclude the work to be right.

$$\begin{array}{r} 347 \\ 684 \\ \hline 1031 \end{array} \quad \begin{array}{r} 5 \\ X \\ 5 \end{array}$$

If the items are of different denominations ; as pounds, shillings, pence, &c. ; you must begin with the highest denomination ; and, after casting out the 9's, reduce the excess to the next inferior denomination ; and then casting out the 9's, reduce the excess to the next inferior denomination ; proceed in like manner with this and all the other lower denominations, placing the last excess at the top of the cross ; then, in the same manner, cast the 9's out of the total, placing the excess at the foot of the cross ; and if the figure at the foot and top be the same, the work may be presumed right.

Thus, to prove the example in the margin, I begin with the pounds, and say, $4 + 8 = 12 = 1 + 2 = 3$, and $3 + 5 = 8$, and $8 + 5 = 13 = 1 + 3 = 4$; and the excess 4 reduced to shillings is $4 \times 20 = 80 = 8$, and $8 + 1 = 9 = 0$; then $7 + 1 = 8$, and $8 + 8 = 16 = 1 + 6 = 7$; and the excess 7 reduced to pence is $7 \times 12 = 84 = 8 + 4 = 12 = 1 + 2 = 3$, and $3 + 1 = 4$, and $4 + 7 = 11 = 1 + 1 = 2$; and the excess 2 reduced to farthings is $2 \times 4 = 8$, and $8 + 1 = 9 = 0$, and $0 + 2 = 2$: so this 2, being the excess of the items, I place at the top of the cross, and proceed to cast the 9's out of the total, saying, $1 + 4 = 5$, and the excess 5 reduced to shillings is $5 \times 20 = 100 = 1$, and $1 + 1 = 2$, and $2 + 6 = 8$; and this excess 8 reduced to pence is $8 \times 12 = 96 = 6$, and $6 + 5 = 11 = 1 + 1 = 2$; and this excess 2 reduced to farthings is $2 \times 4 = 8$, and $8 + 3 = 11 = 1 + 1 = 2$: which excess 2 I place at the foot of the cross ; and because it is the same with the figure at the top, I conclude the work to be right.

$$\begin{array}{r} L. \quad s. \quad d. \quad f. \\ 48 \quad 17 \quad 10 \quad 1 \\ 55 \quad 18 \quad 7 \quad 2 \\ \hline 104 \quad 16 \quad 5 \quad 3 \end{array} \quad \begin{array}{r} 2 \\ X \\ 2 \end{array}$$

The method of proof by casting out the 9's is founded on the corollaries deduced from the axioms ; and if any operation, whether in addition, subtraction, multiplication, or division, be right, this kind of proof will always show it to be so ; but if an operation be wrong, by a figure or figures being misplaced, or by miscounting 9, or any just number of 9's, this kind of proof will not discover the mistake.

IV. *Practical Questions.*

1. A person dying, left for the use of his widow L. 4000; to each of his four sons L. 4256; to each of his five daughters L. 3678; to each of six near relations L. 245: What was his estate? *Ans.* L. 40,884.

2. H of Hamburg is debtor to L of London: For broad cloth, L. 428 : 14 : 10; for kerseys, L. 273 : 17 : $6\frac{1}{2}$; for fustians, L. 364, 19s. 8d.; for druggets, L. 568 : 18 : $4\frac{3}{4}$; for muslin, L. 392 : 12 : $8\frac{1}{2}$; for grocery-wares, L. 683 : 15 : $8\frac{1}{4}$; the factorage came to L. 104, 6s.; custom, wharfage, and incident charges, L. 4, 15s. For what sum must L draw on H? *Ans.* L. 2821 : 19 : 10.

3. B buys six bags of hops, of which N^o 1. weighed C. 2 : 3 : 14; N^o 2. C. 2 : 1 : 24; N^o 3. C. 2 : 2 : 20; N^o 4. C. 2 : 1 : 22; N^o 5. C. 2 : 0 : 26; N^o 6. C. 2 : 3 : 12; as also a couple of pockets ditto, that weighed $54\frac{1}{2}$ lb. each. How many hundred weight did he purchase? *Ans.* C. 16 : 2 : 3.

4. A goldsmith sells five dozen of silver spoons, weighing 12 lb. 10 oz. 14 dw.; two tankards, weighing 1 lb. 8 oz. 18 dw.; ten salts, weighing 4 lb. 11 oz. 16 dw.; forty-two plates, weighing 38 lb. 6 oz. 10 dw.; a pair of jugs, weighing 2 lb. 4 oz. 13 dw.; two tea kettles, weighing 13 lb. 5 oz. 12 dw. What quantity did he sell? *Ans.* 74 lb. 3 dw.

5. A wine merchant imports 12 tuns 2 hhds 45 gallons of claret; 14 tuns 3 hhds 48 gallons of Malaga; 16 tuns 1 hhd 54 gallons of Port; 20 tuns 2 hhds 56 gallons of Canary; 18 tuns 3 hhds of Madeira; and 10 tuns 2 hhds 42 gallons of sherry. What quantity did he import in all? *Ans.* 94 tuns 56 gallons.

6. A certain farm consists of six inclosures, whereof the first contains 42 acres 3 roods 36 poles; the second 36 a. 2 r. 24 p.; the third, 45 a. 1 r. 38 p.; the fourth, 52 a. 2 r. 28 p.; and each of the other two contains 26 a. 3 r. 14 p. How many acres in all? *Ans.* 231 a. 1 r. 34 p.

7. A gentleman had seven sons: from the birth of the first to the birth of the second there intervened 384 days 18 hours 50 minutes; from the birth of the second to that of the third, 582 d. 22 h. 58 m.; from the birth of the third to that of the fourth, 623 d. 12 h. 48 m.; from the birth of the fourth to that of the fifth, 592 d. 16 h. 36 m.; from the birth of the fifth to that of the sixth, 745 d. 19 h. 45 m.; from the birth of the sixth to that of the seventh, 864 d. 17 h. 54 m. What was the age of the eldest when the youngest was born? *Ans.* 3794 days, 12 hours, and 51 minutes.

C H A P. III.

S U B T R A C T I O N.

SUBTRACTION is the taking a lesser number from a greater, in order to discover their difference, or the remainder.

C 4

I. Sub-

I. *Subtraction of Integers.*

R U L E S.

I. Set figures of like place under other, *viz.* units under units, tens under tens, &c. and the greater of the given numbers uppermost. See axiom XI.

II. Beginning at the place of units, take the lower figures from those above, borrowing and paying ten, as need requires, and write the remainders below. See axioms II. III. IX.

E X A M P L E I.

Because similar or like things only can be subtracted, I place the numbers as directed in Rule I. *viz.*
 units under units, tens under tens, &c. and the greater uppermost, as in the margin. Then, beginning at the place of units, I say, 2 units from 7 units, and 5 units remain; which I set below, in the place of units; then 6 tens from 6 tens, and nothing remains; wherefore I set 0 below, in the place of tens; then 5 hundred from 8 hundred, and 3 hundred remain; which I set below in the place of hundreds; and find the total difference or remainder to be 305.

867	major or minuend.
562	minor or subtrahend.
305	difference or remainder.

E X A M P L E II.

Having placed the numbers, units under units, &c. as in the margin, I say, 5 units from 2 units I cannot, but, because an unit in the next superior place makes ten in this place, I borrow 1, *viz.* 1 ten, from the said next place, as directed in Rule II.; which 1 ten being added to 2 makes 12; then I say, 5 from 12, and 7 remains; which 7 I set below, in the place of units: then I proceed, and pay the unit borrowed, either by esteeming 3, the next figure in the major, to be only 2, or, which is more usual, and the same in effect, by adding 1 to the next figure in the minor, thus, 1 that I borrowed and 8 make 9, from 3 I cannot, but, borrowing as before, I say, 9 from 13, and 4 remains; which 4 I set below: I proceed, and say, 1 that I borrowed and 7 make 8, from 4 I cannot, but from 14 and 6 remains; which 6 I set below: I go on, and say, 1 borrowed and 2 make 3, from 7, and 4 remains; which 4 I set below. So the difference or remainder is 4647. By borrowing and paying in this manner the major and minor are equally augmented, or have the same number added to each of them; and consequently continue to have the same difference, by axiom IX.

7432
2785
4647

E X-

EXAMPLE III.

Some, instead of adding 10 to the upper lesser figure, subtract directly from 10, and add the difference to the upper figure for the remainder, thus, 3 from 2 I cannot, but 3 from 10, and 7 remains; which 7 added to 2 gives 9 for a remainder: then I go on, saying, 1 I borrowed and 7 make 8, which from 5 I cannot, but from 10, and 2 remains; which, 2 added to five gives 7 for a remainder: so I proceed, saying, 1 borrowed and 8 make 9, which from 4 I cannot, but from 10, and 1 remains; which 1 added to 4 gives 5 for a remainder: so I go on, and say, 1 borrowed from 7, and 6 remains, and 0 from 1, and 1 remains. This method is the same in effect with the other; but not so usual, and somewhat childish.

MORE EXAMPLES.

	<i>Ex. 4.</i>	<i>Ex. 5.</i>	<i>Ex. 6.</i>
From	84765	94307	742680
Take	20857	5768	8794

II. Subtraction of the parts of integers; such as shillings, pence, farthings, ounces, &c.

RULES.

I. Place like parts under other, viz. farthings under farthings, pence under pence, &c. and the greater of the given numbers uppermost. See axiom XI.

II. Begin at the lowest of the parts, and borrow according to the value of an unit of the next superior denomination; viz. in farthings borrow 4, in pence borrow 12, &c. as the tables of coin, weights, and measures direct.

III. If you borrow 20, 30, 40, 60, or any just number of tens, as in subtracting shillings, degrees, poles, minutes, seconds, &c. proceed with the right-hand column, as in subtraction of integers; and then subtract your tens, borrowing, if need be, the number of tens contained in an unit of the next superior denomination. The reason appears plain in the following operations:

I. MONEY.

Having, according to Rule I. placed like parts under other, viz. farthings under farthings, pence under pence, &c. and in each of these denominations, units under units, tens under tens, and the greater of the given numbers uppermost, as in the margin, I begin with the

	(10)	(20)	(12)	(4)	
	L.	s.	d.	f.	
under farthings,	73	15	10	2	major.
&c. and in each of these denominations,	48	12	6	2	minor.
units under units, tens under tens, and					
the greater of the given numbers uppermost, as in the margin, I begin with the	25	3	4		rem.

farthings,

farthings, and say, 2 from 2, and 0 remains; so, having nothing to set down, I leave the place blank, and proceed to the pence, saying, 6 from 10 and 4 remains; which 4 I set down, and go on to the shillings, saying, 2 from 5, and 3 remains, and 1 from 1, and 0 remains; or I may say at once, 12 from 15, and 3 remains; which 3 being set down, I proceed to the pounds, which are integers, and subtracted as such.

Here I say, 3 farthings from 1 farthing I cannot, but, as directed in Rule II. I say, 3 from 4, the

number of farthings in 1 penny borrowed, and 1 remains; which I added to 1 in the major gives 2 farthings for a remainder; which I set down, and proceed to the pence, saying, 1 penny borrowed and 10 make 11, which from 6 I cannot, but from 12, the number	(10)	(20)	(12)	
	L.	s.	d.	
	708	14	6 $\frac{1}{4}$	major.
	278	17	10 $\frac{3}{4}$	minor.
	429	16	7 $\frac{1}{2}$	rem.

of pence in 1 shilling, and 1 remains; which I added to 6 in the major gives a remainder of 7; which I set down, and go on to the shillings; and because in subtracting shillings we borrow a just number of tens, viz. 2 tens, or 20, I work as directed in Rule III. and in the right-hand column say, 1 borrowed and 7 make 8, which from 4 I cannot, but from 14, and 6 remains; which being set down, I go on to the left-hand column, and say, 1 borrowed and 1 make 2, which from 1 I cannot, but from 2, the number of tens in 1 pound, and 0 remains, which 0 added to 1 in the major gives 1 for a remainder; which I set down, and proceed to the pounds, saying, 1 borrowed and 8 make 9, which from 8 I cannot, but from 18, &c.

Note 1. Some add the number borrowed to the figure or number in the major, and then subtract from their sum. Thus, in the farthings they add the 4 borrowed to 1 in the major, and then from the sum 5 they subtract the 3 in the minor; and in the pence they add the 12 borrowed to 6 in the major, and subtract from the sum 18, &c.; but the method taught above is the easiest and most usual.

Note 2. A great many people, in subtracting the shillings, instead of working as directed in Rule III. proceed thus: saying, 1 borrowed and 17 in the minor make 18, which from 14 I cannot, but from 20, the number borrowed, and 2 remains; which 2 added to 14 in the major gives 16 for a remainder. The learner may chuse any of the two methods he likes best.

MORE EXAMPLES.

	Ex. 1.			Ex. 2.			Ex. 3.			Ex. 4.		
	(10)	(20)	(12)	(10)	(20)	(12)	(10)	(20)	(12)	(10)	(20)	(12)
	L.	s.	d.	L.	s.	d.	L.	s.	d.	L.	s.	d.
From	743	16	4	82	13	4½	95	8	7	34	10	6¾
Sub.	375	15	10	54	14	7¾	68	13	6½	20	12	7
Rem.												

Ex. 5.

<i>Ex. 5.</i>		<i>L.</i>		
A borrowed of B				150
		<i>L.</i>	<i>s.</i>	<i>d.</i>
A paid B at one time	-	20	10	6
At another time	-	18	13	4
At another	-	12	16	8
Sold him goods to the value of	-	25	14	7½
In all				

Balance due to B

<i>Ex. 6.</i>		<i>L.</i>		
A lent B				200
		<i>L.</i>	<i>s.</i>	<i>d.</i>
A received at one time	-	36	14	8
At another time	-	40	10	
At another a bill of	-	34	13	4
At another a draught of	-	25	16	10
Goods to the value of	-	16	14	6½
In all				

Balance due to A

<i>Ex. 7.</i>		<i>L.</i>		
A steward collected of rents	-			2000
Remitted to his master at one time	-	300		
At another	-	250		
Paid small bills amounting to	-	68	13	4
Paid accounts amounting to	-	45	16	8½
Paid of taxes and repairs	-	47	15	6½
In all				

Balance due to the master

2. AVOIRDUPOIS WEIGHT.

I begin with the pounds, and say, 24 (10) (4) (28)
 from 22 I cannot, but from 28, the number *C.* 2 *lb.*
 of pounds in 1 quarter, and 4 remains, 84 1 22 major.
 which added to 22 in the major, gives 26 49 3 24 minor.
 for a remainder; which I set below, and
 proceed to the quarters, saying, 1 quarter 34 1 26 rem.
 borrowed and 3 make 4, which from 1 I
 cannot, but from 4, the number of quarters in 1 *C.* and 0 remains,
 which 0 added to 1 in the major gives 1 for a remainder; which I
 set down, and go on to the *C.* which are integers, saying, 1 *C.*
 borrowed and 9 make 10, which from 4 I cannot, but from 14, *C.*

M O R E

MORE EXAMPLES.

Ex. 1.

	(10)	(20)	(4)	(28)
	<i>T.</i>	<i>C.</i>	<i>℥.</i>	<i>lb.</i>
From	38	14	2	18
Take	25	14	3	21

Rem.

Ex. 2.

	(10)	(4)	(28)	(16)
	<i>C.</i>	<i>℥.</i>	<i>lb.</i>	<i>oz.</i>
	48	3	16	10
	23	3	19	13

The *C.* in *Ex. 1.* are subtracted like shillings; that is, either as directed in Rule III. or by the method mentioned in note 2. above.

Ex. 3.

	<i>C.</i>	<i>℥.</i>	<i>lb.</i>
A merchant buys of sugar	-	-	80 1 14
	<i>C.</i>	<i>℥.</i>	<i>lb.</i>
	15	2	14
	16	2	14
Sells at different times	15	3	14
	16	3	
Sold in all			
Remains on hand			

3. TROY WEIGHT.

Ex. 1.

	(10)	(12)	(20)	(24)
	<i>lb.</i>	<i>oz.</i>	<i>dw.</i>	<i>gr.</i>
From	74	10	12	16
Sub.	28	11	15	19

Rem.

Ex. 2.

	(10)	(12)	(20)	(24)
	<i>lb.</i>	<i>oz.</i>	<i>dw.</i>	<i>gr.</i>
	63	8	14	10
	54	10	17	14

Ex. 3.

	<i>lb.</i>	<i>oz.</i>	<i>dw.</i>	<i>gr.</i>
Bought of silver	684	10	12	18
	<i>lb.</i>	<i>oz.</i>	<i>dw.</i>	<i>gr.</i>
	25	8	14	12
	14	10	16	10
Sold at several times	26	11	18	14
	35	10	12	15
	42	6	15	17
	38	11	14	16
Sold in all				
Remains unfold				

4. APOTHECARIES WEIGHT.

Ex. 1.						Ex. 2.					
(10) (12) (8) (3) (20)						(10) (3) (20)					
lb 3 3 9 gr.						3 9 gr.					
From	42	8	4	1	15	24	1	13			
Sub.	22	7	5	2	19	13	2	17			
Rem.											

Ex. 3.						Ex. 4.			
	lb	3	3	9	gr.		3	9	gr.
From	58	5	3	1	10		37	2	14
Sub.	37	10	5	1	16		28	2	18
Rem.									

5. WOOL-WEIGHT.

Ex. 1.						Ex. 2.		
(10) (12) (2) (6½) (2)						(10)	(2)	(7)
Last. sac. wey. tod. ston.						Ston.	clo.	lb.
From	84	7	1	3	1	25	1	3
Sub.	73	4	1	4	1	13	1	5
Rem.								

6. DRY MEASURE.

Ex. 1.					Ex. 2.				
(10) (5) (8) (4)					(10) (40) (8)				
Load. qrs. bu. pec.					Load. bu. gal.				
From	36	3	4	2	83	25	5		
Sub.	24	3	6	3	63	34	6		
Rem.									

In subtracting the bushels in Ex. 2. because you borrow a just number of tens, viz. 4 tens, or 40, work as directed in Rule III.; that is, proceed with the right-hand column, as in subtraction of integers, and in the left-hand column borrow 4, when the figure in the major is less than the figure to be subtracted.

COAL-MEASURE.

Ex. 1.				Ex. 2.			
(10) (4) (9)				(10) (36) (4)			
Chal. qrs. bu.				Chal. bu. pec.			
From	68	1	3	78	23	1	
Sub.	38	2	5	49	27	3	
Rem.							

SCOTCH

SCOTCH DRY MEASURE.

Ex. 1.				Ex. 2.			
(10)	(16)	(4)	(4)	(10)	(4)	(4)	(4)
<i>Chal. bo. fir. pec.</i>				<i>Bo. fir. pec. lip.</i>			
From	36	11	1 2	28	1	2	1
Sub.	25	13	2 3	17	2	3	2
Rem.							

7. WINE-MEASURE.

Ex. 1.				Ex. 2.			
(10)	(2)	(2)	(63)	(10)	(63)	(8)	
<i>Tun. pip. bhd. gall.</i>				<i>Hbd. gall. pts.</i>			
From	56	1	1 35	63	45	3	
Sub.	48	1	1 47	38	53	7	
Rem.							

8. BEER and ALE MEASURE.

Ex. 1.				Ex. 2.			
(10)	(3)	(2)	(9)	(10)	(3)	(36)	
<i>Hbd. kil. fir. gall.</i>				<i>But. bar. gal.</i>			
From	43	1	1 3	57	1	21	
Sub.	35	2	1 5	47	1	30	
Rem.							

SCOTCH LIQUID MEASURE.

Ex. 1.				Ex. 2.			
(10)	(16)	(4)	(2)	(10)	(2)	(2)	(4)
<i>Hbd. gal. qrt. pt.</i>				<i>Pt. cho. mu. gil.</i>			
From	37	11	2 1	47	1	1	2
Sub.	28	12	2 1	33	1	1	3
Rem.							

9. CLOTH-MEASURE.

Ex. 1.			Ex. 2.			Ex. 3.		
(10)	(4)	(4)	(10)	(3)	(4)	(10)	(5)	(4)
<i>Yds. qrs. n.</i>			<i>E. Fl. qr. n.</i>			<i>E. En. qr. n.</i>		
From	38	1 2	48	1 2		78	3 2	
Sub.	23	2 3	29	1 3		57	4 3	
Rem.								

10. LONG

10. LONG MEASURE.

Ex. 1.				Ex. 2.				
(10)(3)(8)(40)				(10)(5 $\frac{1}{2}$)(3)(12)				
<i>Lea. mi. fur. po.</i>				<i>Po. yd. ft. inch.</i>				
From	33	1	4	29	25	3	1	10
Sub.	25	1	5	37	17	4	2	11
Rem.								

SCOTCH LONG MEASURE.

Ex. 1.				Ex. 2.				
(10)(8)(10)(4)				(10)(4)(6)(37)				
<i>Mi. fur. cb. fall.</i>				<i>Ch. fal. el. inch.</i>				
From	35	3	7	1	45	2	3	24
Sub.	23	5	7	2	27	3	5	32
Rem.								

11. LAND-MEASURE.

Ex. 1.				Ex. 2.			
(10)(4)(40)(30 $\frac{1}{4}$)				(10)(4)(40)			
<i>Acr. roo. sq. p. sq. y.</i>				<i>Acr. roo. sq. p.</i>			
From	38	1	27	17	44	2	32
Sub.	28	3	33	25	27	2	36
Rem.							

SCOTCH LAND-MEASURE.

Ex. 1.				Ex. 2.			
(10)(4)(40)(36)				(10)(4)(40)			
<i>Acr. roo. sq. f. sq. el.</i>				<i>Acr. roo. sq. f.</i>			
From	72	2	24	30	48	1	33
Sub.	35	3	27	31	28	2	34
Rem.							

12. SUPERFICIAL MEASURE.

Ex. 1.				Ex. 2.			
(10)(9)(144)				(10)(9)(144)			
<i>Yds. ft. inch.</i>				<i>Yds. ft. inch.</i>			
From	723	3	128	654	5	96	
Sub.	478	5	132	96	7	118	
Rem.							

When

When the number of inches in the minor is greater than in the major, subtract, as in integers, from 144, and the difference added to the inches in the major gives the remainder.

13. SOLID MEASURE.

<i>Ex. 1.</i>				<i>Ex. 2.</i>			
(10)(27)(1728)				(10)(27)(1728)			
<i>Tds. ft. inch.</i>				<i>Tds. ft. inch.</i>			
From	76	18	378	48	16	1000	
Sub.	48	21	964	35	22	1432	

Rem.

The last direction takes place here, with this variation, that you subtract the number of inches in the minor from 1728.

14. A CIRCLE.

<i>Ex. 1.</i>					<i>Ex. 2.</i>				
(10)(12)(39)(60)					(10)(30)(60)(60)				
<i>Circ. fig.</i>					<i>Sig.</i>				
From	13	5	18	46	7	14	43	50	
Sub.	9	7	28	52	4	19	59	54	

Rem.

In subtracting the degrees, minutes, and seconds, proceed with the right-hand column as in subtraction of integers, and in the left-hand column of degrees borrow 3, and in that of minutes and seconds borrow 6, as directed in Rule III.

15. TIME.

<i>Ex. 1.</i>					<i>Ex. 2.</i>				
(10)(13)(4)(7)					(10)(24)(60)(60)				
<i>Year.mon.wk.da.</i>					<i>Da. ho. min. sec.</i>				
From	24	7	1	4	48	14	36	28	
Sub.	13	10	3	5	32	18	43	37	

Rem.

III. The Proof of Subtraction.

Merchants and men of business use no other proof besides a revival of the work, or running over it a second time; but it is usual in schools to put the learner upon proving the operation, by some of the three methods following, viz.

i. The

1. The work may be proved by addition; for if you add the remainder to the minor, the sum, by Axiom X. will be equal to the major, as in the two following examples :

Ex. 1.

5847	major
2569	minor
3278	rem.
5847	proof

Ex. 2.

L.	s.	d.
73	15	10
48	12	6
25	3	4
73	15	10

2. By subtraction; for if you subtract the remainder from the major, the difference, by Axiom X. will be equal to the minor, as follows :

L.	s.	d.
5847	major	
2569	minor	
3278	rem.	
2569	proof	

L.	s.	d.
73	15	10
48	12	6
25	3	4
48	12	6

3. By casting out the 9's; for the major being equal to the sum of the minor and remainder, if you cast the 9's out of the major, and place the excess at the top of the cross, and then cast the 9's out of the minor and remainder, as if they were items in addition, and place the excess at the foot of the cross, it is plain, the figure at the top and foot, if the work be right, will be the same. Only, in proving subtraction of money, Avoirdupois weight, &c. care must be taken to begin with the highest denomination, reducing always the excess to the next inferior denomination, as taught in the proof of addition. See the following examples :

6	5847	major	L.	s.	d.	7
X	2569	minor	73	15	10	X
6	3278	rem.	48	12	6	7
			25	3	4	

IV. Practical Questions.

1. America was discovered by Columbus in the year 1492: How long is it since, this being the year 1785? *Ans.* 293 years.

2. The sum of two numbers is 8764; the lesser is 795: What is the greater? *Ans.* 7969.

D

3. The

3. The greater of two numbers is 4832; their difference is 967: What is the lesser? *Ans.* 3865.

4. A borrowed of B L. 347 : 13 : 4, and afterwards paid him L. 218 : 16 : 8: What is the balance due? *Ans.* L. 128 : 16 : 8.

5. What sum added to L. 638 : 14 : $7\frac{1}{4}$ will make L. 1000? *Ans.* L. 361 : 5 : $4\frac{3}{4}$.

6. A grocer buys C. 17 : 1 : 21 sugar, of which he sells C. 13, 2 q. 24 lb.: How much remains on hand? *Ans.* C. 3 : 2 : 25.

7. The difference of the weight of two silver bowls is 3 lb. 9 oz. 18 dw. 20 gr.; the largest bowl weighs 13 lb. 5 oz. 13 dw. 17 gr.: What is the weight of the lesser? *Ans.* 9 lb. 7 oz. 14 dw. 21 gr.

8. A vintner had in his cellar 18 tuns 2 hhds 48 gallons claret; but having sold 13 tuns 2 hhds 54 gallons, how much remains? *Ans.* 4 tuns 3 hhds 57 gallons.

9. A gentleman's estate consisted of 7648 acres 1 rood and 26 poles; but, to answer the demands of dunning creditors, he was obliged to sell off several large inclosures, consisting of 3278 acres 2 roods 32 poles: What is he now possessed of? *Ans.* 4369 acres 2 roods 34 poles.

10. The planet Venus revolves round the sun in 224 days 16 hours 49 minutes 24 seconds; and Mercury in 87 days 23 hours 15 minutes 53 seconds: What is the difference of their periodical times? *Ans.* 136 days 17 hours 33 minutes 31 seconds.

11. A merchant, on balancing his books, finds, that he has in ready money L. 348 : 13 : $4\frac{1}{2}$; goods to the value of L. 2635, 16s. $8\frac{1}{4}$ d.; debts due to him L. 1784 : 18 : $6\frac{1}{4}$: at the same time he owes to A, L. 275 : 14 : 10; and to B, L. 384 : 18 : $7\frac{1}{2}$: What is his neat stock? or what will he be worth after all his debts are paid? *Ans.* L. 4108 : 15 : 2.

12. The weight of two hhds tobacco when packed is C. 9 : 2 : 16 each; the weight of the two empty hhds is $26\frac{1}{2}$ lb. each: What is the neat weight of the tobacco? *Ans.* C. 18 : 3 : 7.

C H A P. IV.

M U L T I P L I C A T I O N.

IN multiplication there are two numbers given, *viz.* one to be multiplied, called the *multiplicand*; and another that multiplies it, called the *multiplier*; these two go under the common name of *factors*; and the number arising from the multiplication of the one by the other is called the *product*, and sometimes the *fact*, or the *rectangle*. If a multiplier consists of two or more figures, the numbers arising from the multiplication of these several figures into the multiplicand, are called *particular* or *partial products*; and their sum is called the *total product*.

Multiplication

Multiplication then is the taking or repeating of the multiplicand, as often as the multiplier contains unity. Or,

Multiplication, from a multiplicand and a multiplier given, finds a third number, called the *product*, which contains the multiplicand as often as the multiplier contains unity.

Hence multiplication supplies the place of many additions; for if the multiplicand be repeated or set down as often as there are units in the multiplier, the sum of these, taken by addition, will be equal to the product by multiplication. Thus, $5 \times 3 = 15 = 5 + 5 + 5$.

The first and lowest step in multiplication is, to multiply one digit by another; and the fact or number thence arising is called a *single product*. This elementary step may be learned from the following table, commonly called *Pythagoras's table of multiplication*: which is consulted thus: seek one of the digits or numbers on the head, and the other on the left side, and in the angle of meeting you have their product. The learner, before he proceed further, ought to get the table by heart.

To Pythagoras's table are here added, on account of their usefulness, the products of the numbers 10, 11, 12.

T A B L E.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

I. *Multiplication of Integers.*

R U L E S.

I. Set the multiplier below the multiplicand, so as like places may stand under other, *viz.* units under units, tens under tens, &c. : but if either or both of the factors have ciphers on the right hand, set their first significant figures under other.

The order prescribed in this rule is not absolutely necessary, but very convenient, as will appear in the examples.

II. Beginning at the right hand, multiply each figure of the multiplier into the whole multiplicand, carrying as in addition, and placing the right-hand figure of each particular product directly under the multiplying figure. See Axioms II. III. IV. VI.

III. Add the particular products, and their sum will be the total product. See Axiom XII.

E X A M P L E I.

Having placed the multiplier under the multiplicand, as directed in Rule I. I proceed to the operation, and say, 7 times 4 make 28 ; I set the 8 below

Factors {	94 multiplicand.
	7 multiplier.
	658 product.

in the place of units, and carry the 2 tens to the next place, as directed in Rule II. saying, 7 times 9 make 63, and 2 that I carried, make 65 ; I set 5 below in the place of tens, and the 6, which belongs to the next place, I set on its left hand, there being no further place to which it can be carried ; so the product is 658.

E X A M P L E II.

Here I first multiply my right-hand figure 8 into the whole multiplicand, as in the former example ; then I proceed, and multiply likewise my 6 tens into the whole multiplicand, saying, 6 times 2 make 12 ; I set the 2 below under the multiplying figure, *viz.* in the place of tens, and carry my 1 to the next place, as directed in Rule

742 multiplicand.
68 multiplier.
5936 } particular
4452 } products.
50456 Total product.

II. The reason why I set the 2 under the multiplying figure, or in the place of tens, is, because the multiplying figure 6 by Axiom VI. is really 6 tens or 60, and 60 times 2 make 120 ; so that by carrying the 1 to the next place, and setting down 20, the 0 would fall into the place of units, and throw the 2 into the place of tens ; but as 0 can make no alteration in the addition of the partial products, the setting of it down is safely and justly omitted.

From

From the repetition of the former example on the margin, it appears, that though any of the factors may be made the multiplier, the product being the same in both cases, yet the operation becomes easier and shorter by making that factor multiplier which consists of the fewest significant figures. Here likewise observe, that in multiplying 7 hundreds into 8 units of the multiplicand, I set the right-hand figure of the product under the multiplying figure, *viz.* in the place of hundreds, because the multiplying figure 7 is really 700.

68
742
—
136
272
476
—
50456

EXAMPLE III.

When the multiplier has ciphers on the right hand, as it would be evidently lost labour to multiply by the ciphers, their only use being to throw figures on their left hand into higher places, I set the first significant figures of the factors under other; and, after the operation is finished, I annex the ciphers of the multiplier to the right hand of the product.

853
72000
—
1706
5971
—
61416000

EXAMPLE IV.

When the multiplicand has ciphers on the right hand, the case is in effect the same; wherefore I proceed in the operation as before, and annex the ciphers to the product.

853000
72
—
1706
5971
—
61416000

EXAMPLE V.

When both multiplicand and multiplier have ciphers on the right hand; as the case is plainly a compound of the two former, I annex to the product the ciphers of both factors.

853000
7200
—
1706
5971
—
6141600000

EXAMPLE VI.

When the multiplier has ciphers intermixed with significant figures, I omit the ciphers, because the multiplying by them would only produce so many lines of ciphers, and so be labour in vain; wherefore I multiply by the significant figures only; but I take care to place the right-hand figure of each particular product directly under the multiplying figure.

29601847
300905
—
148009235
266416623
88805541
—
8907343771535

The reason of setting the right-hand figure of each particular product directly under the multiplying figure, will still further appear by resolving the multiplier into its constituent parts, as in the margin.

29601847	multiplicand.
5	
900	partial multipliers.
300000	
148009235	prod. by 5.
26641662300	prod. by 900.
8880554100000	prod. by 300000.
8907343771535	total product.

MORE EXAMPLES.

87694×358	$= 31394452$
59387×796	$= 47272052$
78464×4207	$= 330098048$
978×7800	$= 7628400$
673000×89	$= 59897000$
58470×900700	$= 52663929000$

Contractions, and simple ways of working multiplication of integers.

1. To multiply any number by 10, by 100, by 1000, &c. to the given number annex one, two, three ciphers, &c. Thus, $23 \times 10 = 230$; and $384 \times 100 = 38400$; and $745 \times 1000 = 745000$.

2. To multiply any number by 9, by 99, by 999, &c. multiply the given number first by 10, by 100, by 1000, &c. that is, annex one, two, three, &c. ciphers to it; from this subtract the given number, and the remainder is the product; as in the following examples:

<i>Ex. 1.</i>	<i>Ex. 2.</i>	<i>Ex. 3.</i>
Mult. 47 by 9	Mult. 627 by 99	Mult. 999 by 999
470	62700	999000
Sub. 47	Sub. 627	Sub. 999
Prod. 423	Prod. 62073	Prod. 998001

From Ex. 3. we may learn, in general, that to multiply any number consisting entirely of 9's by itself, is to set 1 in the place of units, then as many ciphers, save one, as there are 9's in the given number; then 8, and on the left hand of 8 as many 9's as there are ciphers on its right.

This method of compendizing may be further extended, thus:

If the multiplier be all 9's, except the right-hand figure, or except the two or the three figures next the right hand, annex as many

many ciphers to the multiplicand as there are figures in the multiplier; from which subtract the product of the multiplicand into the complement of the right-hand figure to 10, *viz.* what it wants of 10; or into the complement of the two figures next the right hand to 100; or into the complement of the three figures next the right hand to 1000, &c.

Ex. 1.

$$\begin{array}{r} 7846 \times 996 \\ 7846000 \\ 31384 = 7846 \times 4 \\ \hline 7814616 \text{ prod.} \end{array}$$

Ex. 2.

$$\begin{array}{r} 54786 \times 9988 \\ 547860000 \\ 657432 = 54786 \times 12 \\ \hline 547202568 \text{ prod.} \end{array}$$

Again, if the multiplier be all 9's except the left-hand figure, add unity to the said left-hand figure, and multiply the sum into the multiplicand; to the product annex a cipher for each of the other figures in the multiplier; from which subtract the multiplicand; and the remainder will be the product sought.

Ex. 1.

$$\begin{array}{r} 2846 \times 799 \\ 2846 \\ 8 = 7 + 1 \\ \hline 2276800 \\ 2846 \\ \hline 2273954 \text{ prod.} \end{array}$$

Ex. 2.

$$\begin{array}{r} 4327 \times 1999 \\ 4327 \\ 2 = 1 + 1 \\ \hline 8654000 \\ 4327 \\ \hline 8649673 \text{ prod.} \end{array}$$

3. To multiply any number by 5; first multiply it by 10, that is, annex a cipher to it, and then halve it: and to multiply any number by 15, use the same method; and add both numbers together, as in the following examples:

Multiply 7439 by 5

$$\begin{array}{r} 74390 \\ \hline \text{Product } 37195 \end{array}$$

Multiply 9856 by 15

$$\begin{array}{r} 98560 \\ 49280 \\ \hline \text{Product } 147840 \end{array} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ add}$$

4. To multiply any number by 11, 12, 13, 14, 15, 16, &c. multiply by the unit's figure, and add the back-figure of the multiplicand to the product; and to multiply by 21, 22, 23, 24, 25, 26, 27, &c. add the double of the back-figure; and to multiply by 31, 32, 33, 34, &c. add the triple of it; and to multiply by 112, 113, 114, &c. add the two back-figures; and to multiply by 101,

D 4

102,

102, 103, 104, &c. add the next back-figure save one : as in the following examples :

<i>Ex. 1.</i> Mult. 876 by 11 11 9636 	Or thus: 876 876 9636 	Or thus: 876 876 9636 	<i>Ex. 2.</i> 694 14 9716
	<i>Ex. 3.</i> 435 27 11745 	<i>Ex. 4.</i> 241 34 8194 	
<i>Ex. 5.</i> 7234 112 810208	Or thus: 7234 7234 7234 7234 810208	<i>Ex. 6.</i> 263 119 31297	<i>Ex. 7.</i> 745 103 76735

In multiplying by 12, as in Ex. 8. it is more usual, and equally easy, to proceed by saying, twelve times 8 make 96, and, setting down the 6, I say, twelve times 4 is 48, and 9 carried is 57; which I set down, and the product is 576.

5. If the multiplier consist of the same figure repeated, as 111, 222, 333, 777, &c. multiply by the unit's figure, and out of that product make up the total product, thus: Begin at the right hand, and first take one figure, then the sum of two, then the sum of three, &c. repeating the operation still from the right hand, as often as there are figures in the multiplier; then, neglecting the right-hand figure, or figure in the first place, take the sum of as many figures toward the left hand as the multiplier has places; and if there be not so many, take the sum of all the figures there are; then, neglecting the figures in the first and second place, begin at the figure in the third place, proceed as before; and thus go on till the last or left-hand figure is taken in alone; as in the following examples:

<i>Ex. 1.</i> 7645 33 22935 prod. by 3. 252285 total.	<i>Ex. 2.</i> 4983 666 29898 prod. by 6. 3318678 total.	<i>Ex. 3.</i> 38 4444 152 prod. by 4. 168872 total.
---	---	---

6. The

6. The operation may frequently be rendered shorter or easier, either by addition, subtraction, or a more simple multiplication; and the cases of this kind are so numerous and various, that they admit of no limitation. Consult the following examples and directions:

<i>Ex. 1.</i>	<i>Ex. 2.</i>	<i>Ex. 3.</i>
438	374	746
87	56	84
—	—	—
3066	2244	2984
3504	1870	5968
—	—	—
38106	20944	62664
<i>Ex. 4.</i>	<i>Ex. 5.</i>	<i>Ex. 6.</i>
824	685	789
642	147	328
—	—	—
1648	4795	6312
3296	9590	25248
4944	—	—
—	100695	258792
529008		

I work the above examples as follows:

Ex. 1. I multiply by 7, and add that product to the multiplicand, instead of multiplying by 8.

Ex. 2. I multiply by 6, and out of that product I subtract the multiplicand, instead of multiplying by 5.

Ex. 3. I multiply by 4, and double that product for 8.

Ex. 4. I multiply by 2; then I double, or multiply that product by 2, for 4; and then add these two products (the right-hand figure of the one to the right-hand figure of the other, &c.) for 6.

Ex. 5. I multiply by 7, and double this product for 14.

Ex. 6. I multiply by 8; and, because 8 times 4 make 32, I multiply that product by 4, for 32.

Several other contractions might be added, but they are rather curious than very useful.

Select methods of multiplying Integers.

1. Instead of multiplying by the multiplier, you may multiply by its component parts, or by the component parts of the nearest composite number; and to or from the last product add or subtract the product of the multiplicand into the difference betwixt the multiplier and the nearest composite number.

Thus, instead of multiplying by 72, you may multiply by 9, and that

that product by 8; and instead of multiplying by 56, you may multiply by 8 and 7, or by 7, 4, 2; for the choice of the component parts is arbitrary; and instead of multiplying by 37, you may multiply by 4 and 9, adding the multiplicand to the last product, for the unit wanting in the product of the component parts; and instead of multiplying by 34, you may multiply by 5 and 7, subtracting the multiplicand from the last product, for the unit of excess in the product of the component parts; and instead of multiplying by 68 in Ex. 5. you take the component parts of 64, being the nearest composite number, which is 8; and 8 or 4, 2 and 8, to the last product, adding the product of the multiplicand into 4, being the difference betwixt the multiplier and the nearest composite number gives the answer.

Here observe, that when three or more numbers, as in Ex. 6. are given to be multiplied into one another, the operation is called *continual multiplication*. And it is of no importance which of the component parts you make the first multiplier; for the last product will be the same, in whatever order the multipliers are taken: but it is convenient that the component parts be all digits, or at least but small numbers, not above 10, 11, or 12. See the following examples:

$$\begin{array}{r}
 \text{Ex. 1.} \\
 \text{Mult. } 436 \text{ by } 72 \\
 \quad 9 \\
 \hline
 3924 \\
 \quad 8 \\
 \hline
 31392
 \end{array}$$

$$\begin{array}{r}
 \text{Or thus:} \\
 436 \text{ by } 72 \\
 \quad 8 \\
 \hline
 3488 \\
 \quad 9 \\
 \hline
 31392
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 2.} \\
 \text{Mult. } 351 \text{ by } 56 \\
 \quad 8 \\
 \hline
 2808 \\
 \quad 7 \\
 \hline
 19656
 \end{array}$$

$$\begin{array}{r}
 \text{Or thus:} \\
 351 \text{ by } 56 \\
 \quad 7 \\
 \hline
 2457 \\
 \quad 4 \\
 \hline
 9828 \\
 \quad 2 \\
 \hline
 19656
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 3.} \\
 \text{Mult. } 642 \text{ by } 37 \\
 \quad 4 \\
 \hline
 2568 \\
 \quad 9 \\
 \hline
 23112 \\
 \text{Add } 642 \\
 \hline
 23754 \text{ prod.}
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 4.} \\
 \text{Mult. } 275 \text{ by } 34 \\
 \quad 5 \\
 \hline
 1375 \\
 \quad 7 \\
 \hline
 9625 \\
 \text{Sub. } 275 \\
 \hline
 9350 \text{ prod.}
 \end{array}$$

Ex. 5.

$$\begin{array}{r}
 \text{Ex. 5.} \\
 \text{Mult. } 348 \text{ by } 68 \\
 \begin{array}{r}
 4 \\
 \hline
 1392 \\
 2 \\
 \hline
 2784 \\
 8 \\
 \hline
 22272 \\
 \text{add } 1392 \\
 \hline
 23664 \text{ prod.}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 6.} \\
 \text{Mult. } 324 \text{ by } 252. \\
 \begin{array}{r}
 9 \\
 \hline
 2916 \\
 7 \\
 \hline
 20412 \\
 4 \\
 \hline
 81648 \text{ prod.}
 \end{array}
 \end{array}$$

This method may be extended to pretty high numbers. For,

$$\begin{array}{l}
 10 = 2 \times 5. \\
 50 = 2 \times 5 \times 5. \\
 100 = 4 \times 5 \times 5. \\
 500 = 4 \times 5 \times 5 \times 5. \\
 1000 = 8 \times 5 \times 5 \times 5. \\
 1500 = 4 \times 5 \times 5 \times 5 \times 3. \\
 10000 = 8 \times 5 \times 5 \times 5 \times 2 \times 5. \\
 100000 = 10 \times 10 \times 10 \times 10 \times 10.
 \end{array}$$

And the component parts of the intermediate numbers may easily be discovered. The conveniency and proper use of this method of multiplying will appear in Section 2. following.

2. Instead of beginning with the right-hand figure of the multiplier, you may begin with the left; only take care to place the right-hand figure of every particular product directly under the multiplying figure, as in the following examples.

$$\begin{array}{r}
 \text{Ex. 1.} \\
 \text{Mult. } 4568 \\
 \text{by } 234 \\
 \begin{array}{r}
 9136 \\
 13704 \\
 18272 \\
 \hline
 \text{Prod. } 1068912
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 2.} \\
 \text{Mult. } 6374 \\
 \text{by } 7408 \\
 \begin{array}{r}
 44618 \\
 25496 \\
 50992 \\
 \hline
 \text{Prod. } 47218592
 \end{array}
 \end{array}$$

3. Multiplication may be performed without any burden to the memory, by setting down every single product, as in the following examples.

Here

Here I say, 8 times 2 is 16, which I set down; then 8 times 4 is 32, which I likewise set down, *viz.* 3 before 1 and 2 under it; then 8 times 7 is 56, I set 5 before 3 and 6 under it; lastly, 8 times 8 is 64, which I set down in the same manner: These added as they stand, give the product.

Here observe, that when any single product is under 10, to prevent mistakes by misplacing, you must set a cipher in the place which a second product figure would have possessed.

Ex. 1.
 Mult. 8742
 by 8

 65316
 462

 Prod. 69936

Ex. 2.
 Mult. 3748
 by 26

 14248
 824
 1016
 648

 Prod. 97448

The disadvantage attending this method of multiplying is, that the addition is tedious.

4. Multiplication may be performed by addition in this manner: Set the digits 1, 2, 3, 4, &c. under one another, and opposite to 1 place your multiplicand; double it for 2; and to this sum add the multiplicand for 3; and again to this sum add the multiplicand for 4: and thus go on till you have a table of the products of the multiplicand by all the digits, or at least as many of them as your multiplier requires; then transfer your particular products out of this table, and their sum will be the total product. This method is convenient in large operations. See the following example.

T A B L E.

1	3785946
2	7571892
3	11357838
4	15143784
5	18929730
6	22715676
7	26501622
8	30287568
9	34073514

Mult. 3785946
 by 659847

 26501622
 15143784
 30287568
 34073514
 18929730
 22715676

Prod. 2498145110262

5. The Noble and ingenious Lord Napier, Baron of Merchiston in Scotland, invented a method of performing multiplication by rods; the separate form of which, with the figures inscribed upon them, are as in the plate, fig. 1.

The rods, excluding the index on the left hand, and the rod of ciphers

on
py
on
of
ers

To front p. 78

Index

Napiers Rods

Fig. 1

1	1	2	3	4	5	6	7	8	9	0
2	$\frac{2}{2}$	$\frac{4}{4}$	$\frac{6}{6}$	$\frac{8}{8}$	$\frac{10}{10}$	$\frac{12}{12}$	$\frac{14}{14}$	$\frac{16}{16}$	$\frac{18}{18}$	$\frac{0}{0}$
3	$\frac{3}{3}$	$\frac{6}{6}$	$\frac{9}{9}$	$\frac{12}{12}$	$\frac{15}{15}$	$\frac{18}{18}$	$\frac{21}{21}$	$\frac{24}{24}$	$\frac{27}{27}$	$\frac{0}{0}$
4	$\frac{4}{4}$	$\frac{8}{8}$	$\frac{12}{12}$	$\frac{16}{16}$	$\frac{20}{20}$	$\frac{24}{24}$	$\frac{28}{28}$	$\frac{32}{32}$	$\frac{36}{36}$	$\frac{0}{0}$
5	$\frac{5}{5}$	$\frac{10}{10}$	$\frac{15}{15}$	$\frac{20}{20}$	$\frac{25}{25}$	$\frac{30}{30}$	$\frac{35}{35}$	$\frac{40}{40}$	$\frac{45}{45}$	$\frac{0}{0}$
6	$\frac{6}{6}$	$\frac{12}{12}$	$\frac{18}{18}$	$\frac{24}{24}$	$\frac{30}{30}$	$\frac{36}{36}$	$\frac{42}{42}$	$\frac{48}{48}$	$\frac{54}{54}$	$\frac{0}{0}$
7	$\frac{7}{7}$	$\frac{14}{14}$	$\frac{21}{21}$	$\frac{28}{28}$	$\frac{35}{35}$	$\frac{42}{42}$	$\frac{49}{49}$	$\frac{56}{56}$	$\frac{63}{63}$	$\frac{0}{0}$
8	$\frac{8}{8}$	$\frac{16}{16}$	$\frac{24}{24}$	$\frac{32}{32}$	$\frac{40}{40}$	$\frac{48}{48}$	$\frac{56}{56}$	$\frac{64}{64}$	$\frac{72}{72}$	$\frac{0}{0}$
9	$\frac{9}{9}$	$\frac{18}{18}$	$\frac{27}{27}$	$\frac{36}{36}$	$\frac{45}{45}$	$\frac{54}{54}$	$\frac{63}{63}$	$\frac{72}{72}$	$\frac{81}{81}$	$\frac{0}{0}$

Index

Fig. 2.

1	9	5	8	7
2	$\frac{18}{18}$	$\frac{10}{10}$	$\frac{16}{16}$	$\frac{14}{14}$
3	$\frac{27}{27}$	$\frac{15}{15}$	$\frac{24}{24}$	$\frac{21}{21}$
4	$\frac{36}{36}$	$\frac{20}{20}$	$\frac{32}{32}$	$\frac{28}{28}$
5	$\frac{45}{45}$	$\frac{25}{25}$	$\frac{40}{40}$	$\frac{35}{35}$
6	$\frac{54}{54}$	$\frac{30}{30}$	$\frac{48}{48}$	$\frac{42}{42}$
7	$\frac{63}{63}$	$\frac{35}{35}$	$\frac{56}{56}$	$\frac{49}{49}$
8	$\frac{72}{72}$	$\frac{40}{40}$	$\frac{64}{64}$	$\frac{56}{56}$
9	$\frac{81}{81}$	$\frac{45}{45}$	$\frac{72}{72}$	$\frac{63}{63}$

ciphers on the right, are just the several columns of the multiplication table separated or cut asunder from head to foot; each of the little squares in the table being divided on the rods by diagonal lines into two triangles. The right-hand figure of every single product in the table is placed on the rods in the lower triangle, and the other in the upper: and such products as consist but of one digit are always set in the lower triangle, and the upper one left blank.

The rods are distinguished from one another by their top-figures, 1, 2, 3, 4, 5, &c. making in all ten different rods, besides the index: but as the same figure may occur several times in a multiplicand, it is necessary to have three or four rods of each kind; or to have the four sides of each rod inscribed with a different set of figures.

The rods are fitted for operation thus. To the right side of the index apply a rod on whose top is the left-hand figure of the multiplicand; next to this set the rod on whose top is the following figure of the multiplicand; and so on to the right-hand figure: then right against any figure on the index, you have the product arising from the multiplication of that figure into the multiplicand; but to be taken out by the help of an easy addition, as follows.

The figure in the lower triangle on the right-hand rod is the right-hand figure of the product; the figure in the upper triangle on this rod, added to the figure in the lower triangle on the next rod, gives the second figure of the product. Again, the figure in the upper triangle on the second rod, added to the figure in the lower triangle on the rod following, gives the third figure of the product, &c.; and the figure in the upper triangle on the rod next the index, is the last figure of the product. In this manner are the particular products taken from the rods, the sum whereof is the total product. See the plate, fig. 2. in which the rods are fitted or set together for the number 9587.

	Mult. 9587	
	by 347	
Against 7 on the index I find	67109	
Against 4 I find	- - 38348	
Against 3 I find	- - 28761	
Total product	3326689	

6. If you make a table of the multiplicand for the digits 1, 2, 3, 5, the particular products may be made out from this small table, almost with the same ease as from the rods, thus.
Suppose for a multiplicand 7894.

TABLE.

1	7894
2	15788
3	23682
5	39470

When the figure of the multiplier is 2, 3, or 5, you have the product by inspecting the table; when the multiplying figure is 4, double the number against 2, or add the numbers against 3 and 1; when it is 6, double the number against 3, or add the numbers against 5 and 1; when it is 7, add the numbers against 5 and 2; when it is 8, add the numbers against 5 and 3; when it is 9, triple the numbers against 3, or add the numbers against 5, 3, and 1.

II. *Multiplication of the parts of Integers.*

Here there are three cases.

1. If your multiplier is a single digit, set it under the units figure of the lowest denomination, multiply it into all the parts of the multiplicand, beginning at the lowest, and carrying always as in addition, or according to the value of the next superior place.

E X A M P L E.

What is the price of 7 packs of cloth, at L. 64 : 8 : 10½ per pack?

Here I say, 7 times 2 is 14, which is 3 pence and L. s. d.
2 farthings over; I set down the 2 farthings, and 64 8 10½
carry 3 to the place of pence, saying, 7 times 10 is 70, and 3 that I carried makes 73, which is 6 shil-
lings and 1 penny; I set down the 1 penny, and 451 2 1½
carry 6 to the place of shillings, saying, 7 times 8 is 56, and 6 that I carried is 62, which makes 3 pounds and 2 shil-
lings; I set down the 2 shillings, and carry 3 to the place of pounds,
which are integers.

2. If your multiplier consists of two or more figures, multiply continually by its component parts, or by the component parts of the composite number that comes nearest to it; and then multiply the given multiplicand by the difference of the multiplier, and the nearest composite number: the sum or difference of these two products is the answer.

E X A M P L E I.

What is the price of 56 C. tobacco, at L. 2 : 14 : 9¾ per C.?

Here the component parts are 8 and 7; for $8 \times 7 = 56$: therefore,

Multiply first by 8, and that product by 7; L. s. d.
or, which will give the same answer, multiply first 2 14 9¾
by 7, and then that product by 8. 8

21	18	6
		7
153	9	6
		E X.

EXAMPLE II.

What is the price of 126 yards of velvet, at L. 3 : 8 : 4 *per* yard?

Here I multiply first by 6, that product by 7, and that product again by 3 : but as the component parts are various, and may be chosen at pleasure, I would have had the same answer, had I multiplied by $9 \times 7 \times 2$; or by $7 \times 3 \times 3 \times 2$.

L.	s.	d.
3	8	4
20	10	
	7	
143	10	
	3	
430	10	

EXAMPLE III.

What is the price of 67 tuns of iron, at L. 18 : 16 : $8\frac{1}{2}$ *per* tun?

Here the nearest composite number is 64, whose component parts are 8×8 ; by which I multiply continually, as in the margin : then I multiply the given multiplicand by 3, the difference betwixt 64 and 67; and because the composite number is less than the multiplier, I add these two products for the answer.

L.	s.	d.
18	16	$8\frac{1}{2}$
150	13	8
1205	9	4
56	10	$1\frac{1}{2}$
1261	19	$5\frac{1}{2}$

EXAMPLE IV.

What is the weight of 77 chefts of goods, each cheft weighing C. 2 : 3 : 14 Avoirdupois?

Here the nearest composite number is 81; and accordingly I multiply by its component parts 9×9 : then I multiply the given multiplicand by 4, the difference betwixt 77 and 81; and because the composite number is greater than the multiplier, I subtract the one product from the other, for the answer.

C.	℥.	lb.
2	3	14
		9
25	3	14
		9
232	3	14
11	2	
221	1	14

EXAMPLE V.

What is the price of 274 yards of linen, at 3s. 4d. *per* yard?

Here

Here I multiply continually by the component parts $10 \times 10 \times 2$, which gives the price of 200 yards: and then, for the price of the rest, I work as follows, *viz.* for the 70 yards, I multiply the price of 10 yards by 7; and for the 4 yards, I multiply the price of 1 yard by 4; and these three products added gives the answer.

L.	s.	d.	
	3	4	price of 1 yard.
		10	
1	13	4	price of 10 yards.
		10	
16	13	4	price of 100 yards.
		2	
33	6	8	price of 200 yards.
11	13	4	price of 70 yards.
	13	4	price of 4 yards.
45	13	4	price of 274 yards.

From the above example may be deduced a general and easy rule for working all questions of this kind; and is of excellent use when the multiplier happens to be a high number; *viz.*

Multiply continually so many times by 10 as there are figures in the multiplier, save one; then multiply the given price by the right-hand figure of the multiplier; and again, the first product of 10 by the following figure of the multiplier; and so on, till you have multiplied by all the figures in the multiplier. The sum of these products is the answer.

EXAMPLE VI.

What is the price of 8604 yards of cloth, at 19s. 6½d. per yard?

L.	s.	d.	Price of	L.	s.	d.	Price of
	19	6½	1 yard, $\times 4 =$	3	18	2	4 yards.
		10					
9	15	5	10 yards, $\times 0 =$				
		10					
97	14	2	100 yards, $\times 6 =$	586	5		600 yards.
		10					
977	1	8	1000 yards, $\times 8 =$	7816	13	4	8000 yards.
			Price of 8604 yards,	8406	16	6	

MORE EXAMPLES.

1. What is the price of 8 yards of cloth, at 13s. 4d.? *Anf.* L. 5 : 6 : 8.
2. What comes 12 reams of paper to, at 8s. 6½d.? *Anf.* L. 5, 2s. 6d.
3. What cost 96 barrels, at L. 1 : 14 : 7½? *Anf.* L. 166, 2s.
4. What

4. What comes 123 gallons to, at 7 s. $8\frac{1}{4}$ d. ? *Ans.* L. 47, 10 s. $8\frac{1}{4}$ d.

5. A gentleman, whose yearly income is L. 250, spends daily 9 s. $6\frac{1}{2}$ d. : How much does he spend in a year, or 365 days ? and how much does he save yearly ? *Ans.* He spends L. 174 : 2 : $8\frac{1}{2}$, and saves L. 75 : 17 : $3\frac{1}{2}$.

6. What comes 9760 tuns to, at 18 s. $7\frac{1}{4}$ d. ? *Ans.* L. 9078, 16 s. 8 d.

3. If your multiplier consists of integers and parts, the operation is performed by a cross multiplication of the several parts of the multiplier into all the parts of the multiplicand.

The contents of mason and joiners work are frequently cast up by this kind of multiplication ; for understanding of which observe, that

The superficial content of any rectangle is found by multiplying the length into the breadth ; and the content of a right-angled triangle is found by multiplying the base into half the perpendicular or height.

The dimensions are usually taken in lineal feet, inches, and lines ; and the operation is performed by the following

R U L E S :

I. Any lineal measure multiplied into the same lineal measure produces squares of that name. Thus, lineal feet multiplied into lineal feet produce square feet ; lineal inches into lineal inches produce square inches, &c.

II. Lineal feet into lineal inches produce rectangles, 1 foot long and 1 inch broad, which divided by 12 quote square feet ; and the remainder multiplied by 12, produces square inches.

III. Lineal feet into lineal lines produce rectangles, 1 foot long and 1 line broad, which divided by 144 quote square feet ; and the remainders are rectangles equal to square inches.

IV. Lineal inches into lineal lines produce small rectangles, 1 inch long and 1 line broad, which divided by 12 quote square inches ; and the remainder, multiplied by 12, produces square lines.

E X A M P L E I.

In an area, pavement, or piece of plaister-work, in length 24 feet 7 inches, and in breadth 18 feet 5 inches, how many square feet ?

F.	in.				
24	7		$18 \times 7 = 126$		
18	5		$24 \times 5 = 120$		
432	35		12)246(20		
20	72		24		
452	107		$12 \times 6 = 72$		

} by Rule II.

E

Here

Here I multiply 18 lineal feet into 24 lineal feet, and the product is 432 square feet; then I multiply 5 lineal inches into 7 lineal inches, and the product is 35 square inches, by Rule I.; then I multiply 18 lineal feet into 7 lineal inches, and the product is 126; and again I multiply 24 lineal feet into 5 lineal inches, and the product is 120; which added to the former product gives 246 rectangles, each being 1 foot in length, and 1 inch in breadth; these divided by 12 quote 20 square feet; and the remainder 6 multiplied by 12 produces 72 square inches, according to Rule II.; these I add to the former square feet and inches, and find the answer or total product to be 452 square feet, and 107 square inches.

EXAMPLE II.

In an area or floor, in length 38 feet 9 inches 6 lines, and in breadth 23 feet 8 inches 6 lines, how many square feet?

F.	in.	li.
38	9	6
23	8	6
874	72	36
42	84	
2	78	
	8	72
919	98	108

$$\begin{array}{r} 23 \times 9 = 207 \\ 38 \times 8 = 304 \\ \hline 12 \overline{) 511(42} \\ \underline{48} \\ 31 \\ \underline{24} \\ 12 \times 7 = 84 \end{array}$$

} By Rule II.

$$\begin{array}{r} 38 \times 6 = 228 \\ 23 \times 6 = 138 \\ \hline 144 \overline{) 366(2} \\ \underline{288} \\ 78 \end{array}$$

} By Rule III.

$$\begin{array}{r} 8 \times 6 = 48 \\ 9 \times 6 = 54 \\ \hline 12 \overline{) 102(8} \\ \underline{96} \\ 12 \times 6 = 72 \end{array}$$

} By Rule IV.

Because the sum of the inches exceeds 144, I carry 1 from them to the column of feet, and set down the overplus, viz. 98.

The operation may be rendered easier and shorter by previously reducing the factors to two denominations, viz. inches and lines. Thus the former example may be proposed and wrought as follows.

In an area or floor, in length 465 inches 6 lines, and in breadth 284 inches 6 lines, how many square inches and feet?

Inch.	lin.
465	6
284	6
132060	36
374	72
132434	108

$$\begin{array}{r} 465 \times 6 = 2790 \\ 284 \times 6 = 1704 \\ \hline 12 \overline{) 4494(374 \text{ in.}} \\ \underline{12 \times 6 = 72 \text{ li.}} \end{array}$$

} By Rule IV.

The answer here is 132434 square inches and 108 square lines ; and if the inches be divided by 144, you will have 919 square feet, and a remainder of 98 square inches, as before.

Or the factors may be reduced to the lowest denomination, *viz.* lines, and then the product will be square lines, which, divided by 144, will quote square inches, and the remainder will be square lines ; and the square inches divided by 144, will quote square feet, and the remainder will be square inches. Again, the square feet divided by 9 will quote square yards, and the remainder will be square feet ; and the square yards divided by 36, will quote square roods, and the remainder will be square yards.

If this cross multiplication be extended to the mensuration of solids, the content of which is found by multiplying the superficial content of the base into the height, depth, length, or thickness, the operation must be conducted by the following

R U L E S :

V. Any superficial measure multiplied into the same lineal measure produces a solid of the same name. Thus superficial feet multiplied into lineal feet produce solid feet ; superficial inches multiplied into lineal inches produce solid inches, &c.

VI. Superficial feet into lineal inches produce parallelopipeds, whose base is 1 square foot, and their height 1 inch ; which divided by 12, quote solid feet ; and the remainder multiplied by 144, produces solid inches.

VII. Superficial feet into lineal lines produce parallelopipeds, whose base is 1 square foot, and their height 1 line ; which divided by 144, quote solid feet ; and the remainder multiplied by 12, produces solid inches.

VIII. Superficial inches into lineal lines produce parallelopipeds, whose base is 1 square inch, and their height 1 line ; which divided by 12 quote solid inches ; and the remainder multiplied by 144, produces solid lines.

IX. Lineal feet into superficial inches produce parallelopipeds, whose base is 1 square inch, and their height 1 foot ; which divided by 144, quote solid feet ; and the remainder multiplied by 12, produces solid inches.

X. Lineal feet into superficial lines produce parallelopipeds, whose base is 1 square line, and their height 1 foot ; which divided by 12, quote solid inches ; and the remainder multiplied by 144, produces solid lines.

XI. Lineal inches into superficial lines produce parallelopipeds, whose base is 1 square line, and their height 1 inch ; which divided by 144, quote solid inches ; and the remainder multiplied by 12, produces solid lines.

EXAMPLE III.

In a piece of timber, whose length is 18 feet 6 inches, breadth 2 feet 4 inches, and thickness 2 feet 3 inches; how many solid feet?

F.	in.	
18	6	$2 \times 6 = 12$
2	4	$18 \times 4 = 72$
		—
36	24	$12)84(7 \text{ F.}$
7		
43	24	superficial
2	3	lineal
86	72	
10	1296	
	576	
97	216	solid

By Rule II.

$$\begin{array}{r} 43 \times 3 = 129 \\ 12)129(10 \text{ F.} \\ 144 \times 9 = 1296 \text{ in.} \end{array} \quad \left. \vphantom{\begin{array}{r} 43 \times 3 = 129 \\ 12)129(10 \text{ F.} \\ 144 \times 9 = 1296 \text{ in.} \end{array}} \right\} \text{By Rule VI.}$$

$$\begin{array}{r} 2 \times 24 = 48 \\ 12 \times 48 = 576 \text{ in.} \end{array} \quad \left. \vphantom{\begin{array}{r} 2 \times 24 = 48 \\ 12 \times 48 = 576 \text{ in.} \end{array}} \right\} \text{By Rule IX.}$$

Here I first multiply 18 feet 6 inches into 2 feet 4 inches, as formerly, and the product is 43 feet 24 inches superficial; which I next multiply into 2 feet 3 inches lineal, thus, 43 superficial feet into 2 lineal feet produce 86 solid feet, and 24 superficial inches into 3 lineal inches produce 72 solid inches, by Rule V.; then 43 superficial feet into 3 lineal inches produce 129 parallelopeds, whose base is 1 square foot, and their height 1 inch; which divided by 12, quotes 10 solid feet; and the remainder 9 multiplied into 144, produces 1296 solid inches, by Rule VI. Again, 2 lineal feet into 24 superficial inches produce 48; which, being less than 144, I esteem a remainder, and multiplying it into 12, I have a product of 576 solid inches, by Rule IX.

Because the sum of the inches exceeds 1728, I carry 1 from them to the feet, and the overplus 216 I set down.

EXAMPLE IV.

How many solid feet in a polished stone that is 8 feet 9 inches 5 lines long, 7 feet 3 inches broad, and 3 feet 5 lines thick.

F.

F.	in.	l.	$7 \times 9 = 63$	} By Rule II.
8	9	5	$8 \times 3 = 24$	
7	3		—	
			12)87(7 F.	} By Rule III.
56	27		$12 \times 3 = 36 \text{ in.}$	
7	36		$7 \times 5 = 35 \text{ in.}$ by Rule III.	
	35		$3 \times 5 = 15$	} By Rule IV.
	1	36	12)15(1 in.	
			$12 \times 3 = 36 \text{ li.}$	
63	99	36 ^{sup.}		} By Rule VII.
3		5 ^{lin.}	$63 \times 5 = 315$, and 144)315(2 F.	
			$12 \times 27 = 324 \text{ in.}$	
189		180	$3 \times 99 = 297$, and 144)297(2 F.	} By Rule IX.
2	324		$12 \times 9 = 108 \text{ in.}$	
2	108		$3 \times 36 = 108$, and 12)108(9 in.	
	9		$5 \times 99 = 495$, and 12)495(41 in.	} By Rule VIII.
	41	432	$144 \times 3 = 432 \text{ lines.}$	
193	482	612	solid.	

The operation may be facilitated by previously reducing the three factors to two denominations, *viz.* inches and lines, as was done in Example II. on superficial measure.

Or the three factors may be reduced to the lowest denomination, *viz.* lines, which being multiplied continually, will produce solid lines; which divided by 1728 will quote solid inches, the remainder being solid lines; and the solid inches divided by 1728 will quote solid feet, the remainder being solid inches: and the solid feet divided by 27 will quote solid yards, the remainder being solid feet; and the solid yards divided by 216 will quote solid roods, the remainder being solid yards.

I shall only further observe, that as the rules for working questions by cross multiplication are numerous, and the operation tedious, it is easier to convert the parts into a decimal fraction of their integer, and then work as taught in multiplication of decimals.

III. The proof of Multiplication.

Multiplication may be proved several ways, *viz.* by multiplication, by division, and by casting out the 9's.

1. By multiplication: Change the places of the factors, and make that the multiplier which before was the multiplicand; and if the work be right, you will have the same product as before: but this method is tedious.

2. By division: When the work is right, the product divided by the multiplier quotes the multiplicand; or, divided by the multiplicand, quotes the multiplier. But this supposes the learner acquainted with division.

3. The most usual method therefore of proving multiplication is by casting out the 9's; which is done thus: Cast the 9's out of the multiplicand and multiplier, and place the excesses on the right and left sides of a cross; multiply these two figures into one another, casting the 9's out of their product, if need be, and place the excess at the top of the cross; then casting the 9's also out of the product of your multiplication, place its excess at the bottom; and if the work be right, the figures at top and bottom will agree, or be the same.

EXAMPLE I.

Here I cast the 9's out of the multiplicand, and place the excess 7 on the right side of the cross; then I cast the 9's out of the multiplier, and place the excess 2 on the left side of the cross; next I multiply these excesses 2 and 7 into one another, cast the 9's out of their product, and place the excess 5 at the top of the cross: lastly, I cast the 9's out of the product, and place the excess 5 at the foot of the cross: which being the same with the figure at the top, I conclude the work to be right.

$$\begin{array}{r}
 754 \\
 \times 38 \\
 \hline
 6032 \\
 2262 \\
 \hline
 28652
 \end{array}
 \begin{array}{c}
 5 \\
 \times 2 \\
 \hline
 5
 \end{array}$$

EXAMPLE II.

Here in casting the 9's out of the multiplicand, and out of the product, I begin with the pounds, and reduce the excess to shillings, and in like manner the excess of the shillings is reduced to pence, and that of the pence to farthings. The multiplier being an abstract number, needs no reduction; but if a multiplier be a mixt number, or consist of integers and parts, as feet and inches, &c. the excess of the higher denomination must always be reduced to the lower.

$$\begin{array}{r}
 L. \quad s. \quad d. \\
 43 \quad 8 \quad 4\frac{1}{4} \\
 \hline
 347 \quad 6 \quad 10
 \end{array}
 \begin{array}{c}
 7 \\
 \times 2 \\
 \hline
 7
 \end{array}$$

IV. Practical Questions.

1. The continual multiplication of the nine digits will give the number of changes that may be rung on nine bells: How many changes are there? *Ans.* 362880.
2. What is the sum, and what the difference of six dozen dozen, and half a dozen dozen? *Ans.* Sum 936. Diff. 792.
3. What number taken from the square of 86 will leave 17 times 34? *Ans.* 6818.
4. The lesser of two numbers is 284, their difference is 132; what is the square of their product, and what the cube of their sum? *Ans.* Square of their product is 13958004736; cube of their sum 343000000.

5. Each

5. Each of nine purses contains L. 81 : 13 : 4; how much money in all? *Ans.* L. 735.

6. What is the price of 1000 hhds tobacco, at L. 12 : 3 : 6 per hhd? *Ans.* L. 12175.

7. The wainscotting of a rectangular room measures 156 feet 4 inches about, and is 14 feet 3 inches high; the door is 7 feet by 3 feet 8 inches; eight window-shutters are 7 feet 2 inches by 4 feet 6 inches; the door and window-shutters, being wrought on both sides, are reckoned as work and half; the chimney, 3 feet 9 inches by 3 feet 3 inches, not being inclosed, is to be deducted: How many yards of wainscotting in the room? *Ans.* 261 sq. yards; 8 sq. feet, and 57 sq. inches.

8. How many solid yards of digging in a cellar that is 27 feet 4 inches long, 17 feet 8 inches broad, and 8 feet 2 inches deep? *Ans.* 146 solid yards, 1 solid foot, and 1024 solid inches.

C H A P. V.

D I V I S I O N.

DIVISION discovers how often one number is contained in another: or,

Division, from two numbers given, finds a third, which contains unity as often as the one given number contains the other.

The number to be divided, or which contains the other, is called the *dividend*; the number by which we divide, or which is contained in the dividend, is called the *divisor*; and the number found by division, or which expresses how often the dividend contains the divisor, is called the *quotient*, or *quot.*

As multiplication supplies the place of many additions, so division, which is the reverse of multiplication, serves instead of many subtractions; as will thus appear: Suppose it were required to divide 18 by 6, that is, to find how often 6 is contained in 18, the work by subtraction will stand as in the margin: by which it appears, that 6 is contained 3 times in the number 18. But this, by division, may be found at one trial, thus:

18
6
—
12
6
—
6
6
—
0

I set the divisor on the left of the dividend, leaving room on the right hand for the quotient, as in the margin; and then I say, How often 6 in 18? *Ans.* 3 times: this 3 I set in the quotient: then I multiply the quotient figure 3 into the divisor 6, saying, three times 6 make 18; which I set down below the dividend, and subtract it from the dividend, and 0 remains.

6)18(3
18
—
0

I. *Division of Integers.*

R U L E S.

I. From the left-hand part of the dividend point off the first dividend, *viz.* so many figures as will contain the divisor.

E 4

II. Ask

II. Ask how often the divisor is contained in the dividual, and put the answer in the quotient.

III. Multiply the divisor by the figure set in the quotient, and subtract the product from the dividual.

IV. To the right of the remainder bring down the next figure of the dividend for a new dividual; and then proceed as before.

The substance of these rules is briefly expressed in the following monoslich:

Dic quot? multiplica, subduc, transferque sequentem.

First ask how oft? in quot the answer make;

Then multiply, subtract, and down a figure take.

E X A M P L E I.

Here, because the divisor 7 is contained in 8, the left-hand figure of the dividend, I point it off, as my first dividual, according to Rule I.; and then I say, How often 7 in 8?

Ans. 1 time; which 1 I set in the quotient, as directed in Rule II.; then I multiply the divisor 7 by this quotient figure 1, and subtract the product 7 from the dividual 8, as directed in Rule III.; to the remainder 1 I bring down the following figure of the dividend, for my second dividual, as directed in Rule IV.; then I proceed as before, and say, How often

7 in 17? *Ans.* 2 times; wherefore, setting 2 in the quotient, I multiply and subtract, and find the next remainder to be 3; to which I bring down the following figure of the dividend, and have 35 for my third dividual; then I say, How often 7 in 35? *Ans.* 5 times; which 5 being placed in the quotient, I multiply and subtract, and 0 remains; so the quotient is 125.

By reviewing the steps of the preceding operation, and reducing, by Axiom VI. the dividuals and quotient-figures to their separate values, the reason of the rules will be obvious; for,

The separate value of the first dividual 8 is 800; and the separate value of 1, the first figure put in the quot, is 100; for as 8 contains 7, the divisor, 1 time, so 800 contains it 100 times, and 100 remains; to which I bring down the following figure of the dividend 7, whose separate value is 70; and my second dividual is 170; and as 7 is contained 2 times in 17, so it is contained 20 times in 170, and 30 remains; to which

	7)875(100	
1st dividual 800	20	} partial quots.
	700	
rem. 100	5	} total quot.
add. 70		
2d dividual 170		
	140	
rem. 30		
add. 5		
3d dividual 35		
	35	
	(0)	

which I bring down the next or last figure of the dividend 5; and my third dividual is 35, in which the divisor 7 is contained 5 times. Now it is evident, that the sum of the partial quots, 125, is the total quot, or a number expressing how often the dividend 875 contains the divisor 7.

From the above example we may learn, that there are always just so many figures in the quotient as there are dividuals; or the first dividual, with the number of subsequent figures in the dividend, is equal to the number of places or figures in the quotient.

Hence likewise may be inferred, that no divisor is contained in any dividual oftener than 9 times; for the dividual, excluding the right-hand figure, is always less than the divisor by 1 at least; and if both be multiplied by 10, or have a cipher annexed to each of them, the product of the dividual will be less than the product of the divisor by 10 at least; but no right-hand figure can supply this defect of 10; therefore the divisor is not contained ten times in any dividual, and consequently not oftener than 9 times.

Here too observe, that the right-hand figure of the first dividual, and all the subsequent figures of the dividend, have a point or dot set below them, as they are brought down; which is done to prevent mistakes, by distinguishing them, in this manner, from the figures not yet brought down.

E X A M P L E II.

Here, because 8 is not contained in 5, I point off 56 as my first dividual, and say, How often 8 in 56? *Ans.* 7; which I put in the quotient; then I multiply 7 into the divisor 8, and subtract the product 56 from the dividual; and as nothing remains, I bring down the next figure of the dividend, which happens to be a cipher; and as I cannot have 8 in 0, I put 0 in the quotient; and, as multiplying and subtracting is in this case needless, I bring down the next figure of the dividend 3; and as I cannot have 8 in 3, I put another 0 in the quotient, and bring down the next figure of the dividend 2; then I say, How often 8 in 32? *Ans.* 4; which I put in the quotient: then I multiply and subtract; and as nothing remains, I bring down the next figure of the dividend 8, and say, How often 8 in 8? *Ans.* 1; which I put in the quotient: then I multiply

$$\begin{array}{r}
 \text{1 num.} \\
 8) 56032897 (70041128 \text{ denom.} \\
 \underline{} \\
 56 \\
 \underline{} \\
 032 \\
 \underline{} \\
 32 \\
 \underline{} \\
 8 \\
 \underline{} \\
 8 \\
 \underline{} \\
 0 \\
 \underline{} \\
 9 \\
 \underline{} \\
 8 \\
 \underline{} \\
 17 \\
 \underline{} \\
 16 \\
 \underline{} \\
 (1)
 \end{array}$$

ply and subtract; and as nothing remains, I bring down the next figure of the dividend 9, and say, How often 8 in 9? *Ans.* 1; which I put in the quotient: then I multiply and subtract; and to the remainder 1 bring down the next and last figure of the dividend 7, and say, How often 8 in 17? *Ans.* 2; which I put in the quotient: then I multiply and subtract, and 1 remains.

To complete the quotient, I draw a line on the right hand, and set the remainder above the line, and the divisor 8 below it, signifying that 1 remains to be divided by 8; or this part of the quotient may be considered as a fraction, whose numerator is 1, and its denominator 8; and the quotient thus completed shews, that the dividend contains the divisor 7004112 times, and one eighth part of a time.

Here observe, that not only the last remainder, but every other remainder, must be less than the divisor; for if it be either greater or equal, the divisor might have been oftener got, and the quotient-figure is too little. And should any one in this case attempt to continue the operation, the quotient-figures would all be 9's, the dividuials would prove inexhaustible, and the remainders would constantly increase.

Hence also learn, that if any dividuial happen to be less than the divisor, you must put 0 in the quotient, and bring down the next figure of the dividend; and if it be still less than the divisor, you must put another 0 in the quotient, and bring down the following figure of the dividend, &c.

E X A M P L E III.

Here the divisor consists of two figures; and because it is contained in the two left-hand figures of the dividend 78, I point them off as my first dividuial; and say, How often 3 in 7? *Ans.* 2, and 1 remains; which I placed, or conceived as placed, on the left hand of the following figure 8, makes 18: then I say, Can I have the following figure of the divisor 6 also 2 times in 18? *Ans.* Yes; consequently I get 36, the divisor, 2 times in 78 the dividuial; wherefore I put 2 in the quotient, and multiply that 2 into the divisor 36, and the product 72 I subtract from the dividuial 78; and to the remainder 6 I bring down the following figure of the dividend 9, for a new dividuial; then I say, How often 3 in 6? *Ans.* 2, and 0 remains; again I say, Can I have 6 also 2 times in 9?

Ans. No; therefore I can have 36 in 69 only 1 time, which 1 I put in the quotient: then I multiply and subtract as before; and to the remainder 33 I bring down the next figure 4 for a new dividuial: then,

$$\begin{array}{r}
 36 \overline{) 789426} \quad (21928\frac{18}{36}) \\
 \underline{72} \\
 69 \\
 \underline{36} \\
 334 \\
 \underline{324} \\
 102 \\
 \underline{72} \\
 306 \\
 \underline{288} \\
 (18)
 \end{array}$$

then, because the dividual consists of a figure more than the divisor, I say, How often the first figure of the divisor 3 in the first two figures of the dividual 33? *Ans.* 9, and 6 remains; which 6 placed on the left hand of the following figure 4 makes 64: again I say, Can I have 6 also 9 times in 64? *Ans.* Yes; consequently 36 can be had 9 times in 334; wherefore I put 9 in the quotient: then I multiply and subtract; and to the remainder 10 I bring down the next figure 2 for a new dividual; here likewise, because the dividual has a figure more than the divisor, I say, How often 3 in 10? *Ans.* 3, and 1 remains; which 1 placed on the left hand of the following figure 2 makes 12: again I say, Can I have 6 also 3 times in 12? *Ans.* No; consequently 36 cannot be had 3 times in 102; wherefore I try if I can have it 2 times; saying, 2 times 3 is 6 from 10, and 4 remains; which 4 placed on the left hand of the next figure 2 makes 42: and I again say, Can I have 6 also 2 times in 42? *Ans.* Yes; consequently 36 can be had 2 times in 102; accordingly I put two in the quotient, multiply and subtract; and to the remainder 30, I bring down the next and last figure of the dividend 6, for a new dividual: then, because the dividual has a figure more than the divisor, I say, How often 3 in 30? *Ans.* 9, and 3 remains; which 3 placed on the left hand of the following figure 6 makes 36: and I again say, Can I have 6 also 9 times in 36? *Ans.* No; consequently 36 cannot be had 9 times in 306; therefore I try if it can be had 8 times, saying 8 times 3 is 24 from 30, and 6 remains; which 6 placed on the left hand of the following figure 6 makes 66: I again say, Can I have 6 also 8 times in 66? *Ans.* Yes; consequently 36 can be had 8 times in 306; wherefore I put 8 in the quotient, and multiply and subtract as before; the last remainder 18 is the numerator of a fraction, and the divisor its denominator, to be annexed to the integral part of the quotient; as was taught in the former example.

The preceding operation points out the manner of procedure when the divisor consists of more figures than one, *viz.* you must take the first figure of the divisor out of the first figure of the dividual, or out of the first two figures of the dividual, in case the dividual have a figure more than the divisor: then imagine the remainder to be prefixed to the next figure of the dividual, and try if you can have the second figure of the divisor as often out of this number; if you can, imagine again the remainder to be prefixed to the following figure of the dividual, and try if you can have the third figure of the divisor as often out of this number, &c.; but if you find you cannot have some subsequent figure of the divisor so often as you took the first, you must go back, and take the first figure of the divisor 1 time less, or some number of times less, out of the first, or out of the first two figures of the dividual: then proceed as before, repeating the trial, till you find you can have the second, and all the subsequent figures of the divisor, as often as you took the first.

But

But here observe, that if, in trying how often the divisor can be had in the dividend, either 9, or a number greater than 9, any where remain, you may conclude, without further trial, that all the subsequent figures of the divisor can be had as often as you took the first; as may be thus demonstrated.

Suppose the subsequent figures of the divisor to be the highest possible, that is, all 9's, and the following figures of the dividend the lowest possible, that is, all 0's; again, imagine the remainder 9 prefixed to the following figure of the dividend 0, and it will make 90; now it is plain, that the subsequent figure of the divisor 9 can be had in 90, the highest number of times possible, *viz.* 9 times, and 9 will remain; which prefixed to the next figure of the dividend 0 makes 90, in which the subsequent figure of the divisor 9 can again be had 9 times, and 9 will remain as before; therefore all the subsequent figures of the divisor can be had as often as you took the first; and if they can be had in this case, much more can they be had when a number greater than 9 remains.

E X A M P L E IV.

Let it be required to divide 1709481. among 234 men.

Here the divisor consists of three places; and because it is not contained in the three left-hand figures of the dividend, I point off 1709 as my first dividend, and say, How often 2 in 17? *Ans.* 8, and 1 remains; which 1, placed on the left of the following figure 0, makes 10: then I say, Can I have the following figure of the divisor 3 also 8 times in 10? *Ans.* No; consequently 234 cannot be had 8 times in 1709; wherefore I try if I can have it 7 times, saying, 7 times 2 is 14, from 17, and 3 remains; which 3, placed on the left of the following figure 0, makes 30; I say again, Can I have the following figure of the divisor 3 also 7 times in 30? *Ans.* Yes; and because 9 remains, I conclude, without further trial, that the divisor 234 may be had 7 times in the dividend 1709; so I put 7 in the quotient, multiply and subtract, and to the remainder 71 I bring down the following figure of the dividend 4 for a new dividend; and because this dividend consists of the same number of places as the divisor, I say, How often 2 in 7? *Ans.* 3, and 1 remains;

$$234)170948(7301.$$

$$\begin{array}{r} 1638 \\ \underline{714} \\ 702 \\ \underline{128 \text{ rem.}} \\ 20 \end{array}$$

$$234)2560(10s.$$

$$\begin{array}{r} 234 \\ \underline{220 \text{ rem.}} \\ 12 \end{array}$$

$$\begin{array}{r} 440 \\ \underline{220} \end{array}$$

$$234)2640(11d.$$

$$\begin{array}{r} 234 \\ \underline{300} \\ 234 \\ \underline{66} \\ 4 \end{array}$$

$$\begin{array}{r} 234)264(11\frac{30}{34}f. \\ \underline{234} \\ (30) \end{array}$$

mains; which 1, placed on the left of the following figure 1, makes 11; then I say again, Can I have 3 also 3 times in 11? *Ans.* Yes, and 2 remains; which 2, placed on the left of the following figure 4, makes 24; then I say again, Can I have the third figure of the divisor 4 also 3 times in 24? *Ans.* Yes; and consequently 234 can be had 3 times in 714; wherefore I put 3 in the quotient, multiply and subtract, and to the remainder 12 I bring down the next and last figure of the dividend 8 for a new dividual; but as the divisor cannot be had in this dividual, I put 0 in the quotient, and the dividual 128 becomes the last remainder.

Here, instead of annexing the fraction $\frac{128}{234}$ to the integral part of the quotient, I multiply the 128l. which remains to be divided among 234 men, by 20, the number of shillings in a pound, and the product 2560 is shillings, which I divide by 234, and the quotient gives 10s. to each man; the remainder 220s. I multiply by 12, the number of pence in a shilling, and the product 2640 is pence, which I divide by 234, and the quotient gives 11d. to each man; the remainder 66 pence I multiply by 4, the number of farthings in a penny, and the product 264 is farthings, which I divide by 234, and the quotient gives to each man 1 farthing, and $\frac{30}{234}$ of a farthing: so each man's share is L. 730, 10s. 11d. $1\frac{30}{234}$ f.

E X A M P L E V.

If 34168062 C. of goods be divided into 4875 equal lots, what will be the weight of each lot?

Here the divisor consists of four places; and because it is not contained in the four left-hand figures of the dividend, I point off 34168 as my first dividual, and say, How often 4 in 34? *Ans.* 8, and 2 remains; which 2 placed on the left of the following figure 1, makes 21: then I say, Can I have the following figure of the divisor 8 also 8 times in 21? *Ans.* No; consequently 4875 cannot be had 8 times in 34168; wherefore I try if I can have it 7 times, saying, 7 times 4 is 28, from 34, and 6 remains; which 6, placed on the left of the following figure 1, makes 61: then I say again, Can I have the following figure of the divisor 8 also 7 times in 61? *Ans.* Yes, and 5 remains; which 5, placed on the left of the following figure 6, makes 56: then I say, Can I have the third figure of the divisor 7 also 7 times in 56? *Ans.* Yes, and 7 remains; which 7, placed on the left of the following figure

4875)34168062(7008 C.

....
34125

43062

39000

4062 rem.

4

4875)16248(3 Q.

14625

1623 rem.

28

12984

3246

4875)45444(9 $\frac{1569}{4875}$ lb.

43875

(1569) rem.

figure 8, makes 78 : then I say, Can I have the fourth figure of the divisor 5 also 7 times in 78? *Ans.* Yes; consequently 4875 can be had 7 times in 34168; wherefore I put 7 in the quotient, multiply and subtract, and to the remainder 43 I bring down the next figure of the dividend 0 for a new dividual; and as I cannot have the divisor 4875 in this dividual 430, I put 0 in the quotient, and bring down the next figure of the dividend 6; but as the divisor 4875 is still greater than the dividual 4306, I put another 0 in the quotient, and bring down the next and last figure of the dividend 2; and as the dividual now consists of five places, and the divisor but of four, I say, How often 4 in 43? *Ans.* 9, and 7 remains; which 7, placed on the left of the following figure 0, makes 70; I say again, Can I have the following figure of the divisor 8 also 9 times in 70? *Ans.* No; consequently 4875 cannot be had 9 times in 43062; wherefore I try if I can have it 8 times, saying, 8 times 4 is 32, which, subtracted from 43, leaves a remainder above 9; therefore I conclude, without further trial, that the divisor can be had 8 times in the dividual; accordingly I put 8 in the quotient, multiply and subtract, and the last remainder is 4062.

Here, as in the former example, instead of annexing the fraction $\frac{4062}{4875}$ to the integral part of the quotient, I multiply the 4062 C. which remains, by 4, the number of quarters in a C. and the product 16248 is quarters; which I divide by 4875, and the quotient gives 3 Q. to each lot: the remainder, *viz.* 1623 Q. I multiply by 28, the number of pounds in a quarter, and the product 45444 is pounds; which I divide by 4875, and the quotient gives to each lot 9 pounds and $\frac{1569}{4875}$ of a pound: so the weight of each lot is 7008 C. 3 Q. $9\frac{1569}{4875}$ lb.

E X A M P L E VI.

If, as in the margin, a cipher, or ciphers, possess the right hand of the divisor, cut them off, and cut off, as many figures, *viz.* in this example, the figure 2 from the right hand of the dividend; then divide the remaining figures of the dividend, *viz.* 89678, by the remaining figures of the divisor, *viz.* 648, and you have the integral part of the quotient; but to the remainder 254 annex the figure cut off from the dividend, and you have 2542 for the numerator of your fraction, and the whole divisor 6480 is the denominator.

The reason will appear obvious by working a question in this manner, and also at full length, without cutting off the cipher or ciphers, and then comparing the two operations.

$$\begin{array}{r}
 648 \overline{) 089678} \mid 2 \left(138 \frac{2542}{6480} \right. \\
 \underline{2487} \\
 1944 \\
 \underline{5438} \\
 5184 \\
 \underline{} \\
 (2542)
 \end{array}$$

E X.

E X A M P L E VII.

If, as in the margin, the figures cut off from the right hand of the dividend, happen to be all ciphers; in this case, the last remainder, without regarding the ciphers cut off, is the numerator of your fraction, and the significant figures of the divisor the denominator. The reason is assigned in the doctrine of fractions.

$$48 \overline{) 009780} \overline{) 00} (203\frac{36}{48}$$

$$\begin{array}{r} 96 \\ \hline 180 \\ 144 \\ \hline (36) \end{array}$$

In like manner, if there be cut off from the dividend any number of significant figures, with a cipher or ciphers on their right hand; in this case, the last remainder, with the significant figures cut off, make the numerator of your fraction; and the significant figures of the divisor, with as many ciphers as the number of significant figures cut off from the dividend, make the denominator. Thus, if, in the above example, the figures cut off from the dividend had been 50, the numerator of your fraction would have been 365, and the denominator 480.

M O R E E X A M P L E S.

$$\begin{array}{r|l} 68)4768943(& 740)638007935(\\ 397)8620094(& 89000)8765432000(\\ 1679)9437856(& 2530000)325643874100(\end{array}$$

Contractions, and simple ways of working Division of Integers.

1. To divide any number by 10, 100, 1000, &c. you have only to point off for a remainder as many figures on the right hand of the dividend as the divisor has ciphers, and the other figures on the left of the point or separatrix are the quotient. Thus, 7489634 divided by 10, 100, 1000, &c. stands as follows.

$$\begin{array}{r} \text{Quot. rem.} \\ 10)748963.4 \\ 100)74896.34 \\ 1000)7489.634 \\ 10000)748.9634 \end{array}$$

2. If the figures of the divisor are all 9's, or all except the units figure, as 9, 99, 999, 98, 997, 9996, &c. work as follows.

Find a new divisor, by annexing to unity as many ciphers as there are figures in the given divisor, subtract the given from the new divisor, and the remainder or difference is the complement. Divide the given dividend by the new divisor, viz. point off so many

many figures on the right hand as there are ciphers in the said divisor; the figures thus pointed off are to be esteemed a remainder, and the other figures on the left hand are to be accounted a quotient; then multiply this quotient by the complement, placing the units of the product under the units of the former remainder; again, divide this product by the new divisor, by pointing off from the right hand the same number of figures as in the former remainder, and the figures to the left are to be esteemed another quotient; which quotient you are again to multiply by the complement, and divide as before. And in this manner proceed till the last quotient is nothing; then add as in addition of integers, observing the carriage from the left-hand column of the remainders; to the remainders add the product of the said carriage and complement, and the sum is the total remainder; and the sum of the several quotients is the total quotient required.

E X A M P L E I.

Divide 74698 by 98	
New divisor 100	100)746.98
Given divisor 98	14.92 = 746 × 2
	.28 = 14 × 2
Complement 2	
	Tot. quot. 762.18 + 4 = 22 tot. rem.
	Carriage 2 × 2 complement = 4.

E X P L I C A T I O N.

First, to unity I annex two ciphers, because the given divisor consists of two figures, and so the new divisor is 100; from which I subtract the given divisor 98, and there remains 2 for the complement.

Next, I divide the given dividend by the new divisor, *viz.* I point off 98, the two figures next the right hand, for the first remainder; and the figures on the left, namely 746, is the first quotient.

Then I multiply the said first quotient 746 by the complement 2; and by the new divisor I divide the product 1492, *viz.* I point off 92 for the second remainder, and 14 is the second quotient.

Again, I multiply the second quotient 14 by the complement 2, and the product 28, divided by 100, gives 28 for the third remainder, but nothing to the quotient.

Then I add the several remainders and quotients, and find the total quotient amounts to 762, and the remainders to 18.

Lastly, I multiply 2, the carriage from the left-hand column of the remainders, by the complement 2; and the product 4 I add to the remainders 18, and the sum 22 is the total remainder.

E X.

E X A M P L E II.

Divide 849675321 by 994.

New divisor 1000

Given divisor 994

Complement 6

1000)849675.321

5098.050 = 849675 × 6

30.588 = 5098 × 6

.180 = 30 × 6

Total quot. 854804.139 + 6 = 145 tot. rem.

Carriage 1 × 6 Complement = 6.

The sum of the remainders by itself, or with the product of the complement and carriage, may sometimes happen to be equal to or greater than the given divisor; in which case their difference is the true remainder; and you must add 1 to the sum of the quotients; as in the following

E X A M P L E III.

Divide 4769649 by 997.

New divisor 1000

Given divisor 997

Complement 3

1000)4769.649

14.307 = 4769 × 3

42 = 14 × 3

4783.998 sum of rem.

1.997 given divisor.

Tot. quot. 4784. 1 true rem.

Carriage 0 × 3 Comp. = 0

When the given divisor is all 9's, as 9, 99, 999, 9999, &c. the complement in this case being 1, you have only to transfer the several quotients toward the right hand; as in the following

E X A M P L E IV.

Divide 4379806425 by 9999.

New divisor 10000

Given divisor 9999

Complement 1

10000)437980.6425

43.7980

43

Quot. 438024.4448 + 1 = 4449 tot. rem.

Carriage 1 × 1 Comp. = 1, to be added to the sum of the remainders.

F

The

The reason of the preceding operations is obvious. For the quotient of any number divided by 100, is found by pointing off the two figures next the right hand. Thus, $47856 \div 100$, is 478.56, viz. 478 is the quotient, and 56 the remainder. But 478×99 wants 478 of 478×100 ; wherefore the number 478 on the left of the separatrix is to be considered not only as a partial quotient, but as the difference betwixt the foresaid two products: which difference, being again divided by 100, stands thus, 4.78; but 4×99 wants 4 of 4×100 ; wherefore this 4 must be added to the remainders. Again, the carriage from the remainders not only denotes the number of hundreds carried from them to the partial quotients, but also the excess above the 99's carried thither; wherefore this excess must be added to the remainder, in order to complete it.

Hence, if the divisor be 98, every difference, as well as the carriage, must be doubled; and if the divisor be 97, every difference, as well as the carriage, must be tripled, &c. that is, multiplied by the complement; and if the last remainder be equal to or exceed the given divisor, 1 must be carried from it to the quotient.

For the like reason, if the left-hand figure of the divisor be any digit under 9, and the other figures all 9's, as 79, 899, 6999, &c. you may work as follows.

Add unity to the given divisor, and the sum is the new divisor; by which divide the dividend and the several quotients successively, till nothing remain; then add the remainders and quotients, observing to carry from the remainders the quotient arising from the sum of their left-hand column, divided by the left-hand figure of the new divisor. Here observe, that the left-hand column of the remainders is that which has as many figures or places on its right hand as the new divisor has ciphers.

E X A M P L E I.

Divide 47983756 by 599.

New divisor 600, Complement 1

600)479837.56

79972.556

133.172 = $79972 \div 600$

.133 = $133 \div 600$

Quotient 80106.261 + 1 = 262 tot. rem.

Carriage 1 $\times$ 1 Com. = 1

Here the sum of the left-hand column of the remainders is 8; which divided by 6, the left-hand figure of the divisor, gives 1 of carriage, and 2 of a remainder; and the carriage 1×1 comp. gives 1 to be added to the sum of the remainders.

E X.

EXAMPLE II.

Divide 1936820 by 3999.

$$\begin{array}{r} 4000 \overline{) 1936.820} \\ \end{array}$$

$$\begin{array}{r} 484.820 \\ \end{array}$$

$$\begin{array}{r} .484 \\ \end{array}$$

Quotient 484.1304 tot. rem.

Carriage 0 X 1 Complement = 0

3. When the divisor is a single digit, or when it may be reduced to a digit by cutting off ciphers, the operation may be performed mentally; and in this case it is convenient to set the quotient under the dividend, so as its left-hand figure may stand under the right-hand figure of the first dividend; and the remainder may be subjoined by way of fraction to the quotient; as in the following examples.

Ex. 1.

$$\begin{array}{r} 2 \overline{) 568} \\ \end{array}$$

284 quot.

Ex. 2.

$$\begin{array}{r} 3 \overline{) 693} \\ \end{array}$$

231 quot.

Ex. 3.

$$\begin{array}{r} 8 \overline{) 336} \\ \end{array}$$

42 quot.

Ex. 4.

$$\begin{array}{r} 4 \overline{) 672} \\ \end{array}$$

168 quot.

Ex. 5.

$$\begin{array}{r} 2 \overline{) 1783} \\ \end{array}$$

891 $\frac{1}{2}$ quot.

Ex. 6.

$$\begin{array}{r} 5 \overline{) 4963} \\ \end{array}$$

992 $\frac{1}{5}$ quot.

Ex. 7.

$$\begin{array}{r} 2 \overline{) 0378} | 5 \\ \end{array}$$

189 $\frac{5}{20}$ quot.

Ex. 8.

$$\begin{array}{r} 4 \overline{) 053} | 7 \\ \end{array}$$

13 $\frac{7}{40}$ quot.

Ex. 9.

$$\begin{array}{r} 7 \overline{) 00269} | 45 \\ \end{array}$$

38 $\frac{45}{700}$ quot.

This method may also be profitably used, when the divisor is 11 or 12, or when it is 11 or 12 with a cipher or ciphers annexed.

Ex. 1.

$$\begin{array}{r} 11 \overline{) 561} \\ \end{array}$$

51 quot.

Ex. 2.

$$\begin{array}{r} 11 \overline{) 878} \\ \end{array}$$

79 $\frac{8}{11}$ quot.

Ex. 3.

$$\begin{array}{r} 12 \overline{) 288} \\ \end{array}$$

24 quot.

Ex. 4.

$$\begin{array}{r} 12 \overline{) 342} \\ \end{array}$$

28 $\frac{6}{12}$ quot.

Ex. 5.

$$\begin{array}{r} 11 \overline{) 0748} | 3 \\ \end{array}$$

68 $\frac{3}{11}$ quot.

Ex. 6.

$$\begin{array}{r} 12 \overline{) 0045} | 78 \\ \end{array}$$

3 $\frac{78}{1200}$ quot.

F 2

4. When

4. When the divisor is the product of two or more factors, or component parts, the given dividend may be divided by one of these factors, the quotient by another, the second quotient by a third, &c. till you have divided by every factor; the last quotient is the answer.

EXAMPLE I.

Divide 409248 by 84.

Because the divisor 84 may be considered as the product of $3 \times 4 \times 7$, I first divide 409248 by 3; and the quotient 136416 I divide by 4; and the second quotient 34104 I divide by 7; and the last quotient 4872 is the answer.

3)409248	Or, $12 \times 7 = 84$ and therefore
4)136416	12)409248
7) 34104	7) 34104
4872	4872

If there be a remainder in each or any of the divisions, the total remainder is found by subtracting the product of the given divisor and last quotient from the given dividend. Or rather let it be obtained thus: Multiply the last remainder into all the preceding divisors continually, adding the several remainders to the products of the divisors to which they belong; and the last product is the total remainder.

EXAMPLE II.

Divide 234786 by 168.

Here the factors, or component parts of the divisor, may be $4 \times 7 \times 6$; so I divide first by 4, and 2 remains; then I divide by 7, and 1 remains; lastly I divide by 6, and 3 remains.

4)234786	Rem.
7) 58696	2
6) 8385	1
1397	3
Total rem.	90
Complete quotient.	1397 $\frac{90}{168}$

Now, to find the total remainder, I multiply the last remainder 3 into the immediately preceding divisor 7; and to the product 21 I add 1, the remainder belonging to that divisor; and the sum 22 I multiply by the first divisor 4; and to the product 88 I add the first remainder 2; and the sum 90 is the total remainder. So the complete quotient is $1397 \frac{90}{168}$.

The reason of collecting the total remainder in this manner is obvious. For the last quotient multiplied back continually into the several divisors, (that is, into the component parts of the given divisor), produces the given dividend, lessened by or wanting the total remainder. And in like manner, the last remainder multiplied back into all the divisors prior to it (taking in the particular remainders

mainders that occur in the several divisions) will produce or make up the total remainder.

E X A M P L E III.

Divide 7427 by 24.

Here the component parts of the divisor are $3 \times 4 \times 2$; and as nothing remains in the last division, the total remainder is found by multiplying 3, the last remainder, into 3, the preceding divisor, and to 9, the product, adding 2, the remainder of the first division.

$$\begin{array}{r} 3)7427 \\ \hline \text{Rem.} \\ 4)2475 \quad 2 \\ \hline 2)618 \quad 3 \\ \hline \text{Quot. } 309 \\ \text{Total rem. } 11 \\ \text{Complete } \left. \begin{array}{l} \text{quotient.} \end{array} \right\} 309\frac{11}{24} \end{array}$$

E X A M P L E IV.

Divide 20612 by 56.

Here the component parts of the divisor are 7×8 ; and as 4 in the first division happens to be the last and only remainder, there is no prior divisor into which it can be multiplied; so this 4 is the total remainder.

$$\begin{array}{r} 7)20612 \\ \hline \text{Rem.} \\ 8)2944 \quad 4 \\ \hline 368 \\ \text{Tot. rem. } 4 \\ \text{Complete } \left. \begin{array}{l} \text{quotient.} \end{array} \right\} 368\frac{4}{56} \end{array}$$

E X A M P L E V.

Divide 7842 by 42.

Here the component parts of the divisor are 6×7 ; and as 5, the only remainder, is in the last division, the total remainder is found by multiplying it into the preceding divisor 6, and so the fraction is $\frac{30}{42}$. Or rather, when the only remainder happens in the last division, make it the numerator, and the last divisor the denominator; and so the fraction here will be $\frac{5}{7}$, which is equal in value to $\frac{30}{42}$; as will appear in the doctrine of vulgar fractions.

$$\begin{array}{r} 6)7842 \\ \hline 7)1307 \\ \hline \text{Rem.} \\ 186 \quad 5 \\ \text{Tot. rem. } 30 \\ \text{Complete } \left. \begin{array}{l} \text{quotient.} \end{array} \right\} 186\frac{30}{42} \\ \text{or, } 186\frac{5}{7} \end{array}$$

If the given divisor consist of any figure repeated, as 222, 333, 444, 555, &c. the component parts may be 111×2 , 111×3 , 111×4 , &c.

EXAMPLE VI.

Divide 78943 by 444.

Here the component parts of the divisor are 111×4 ; and the total remainder is made up as formerly, viz. $3 \times 111 = 333$, and $333 + 22 = 355$.

$$\begin{array}{r}
 4) \\
 111) 78943 (711 \\
 \underline{\dots} \quad \text{Rem.} \\
 777 \quad 177 \quad 3 \\
 \underline{\quad} \quad \times \quad \text{Total rem.} \quad 355 \\
 124 \quad \text{Complete} \\
 111 \quad \text{quotient.} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} 177 \frac{355}{444} \\
 \underline{\quad} \\
 133 \\
 111 \\
 \underline{\quad} \\
 \text{Rem.} \quad 22
 \end{array}$$

5. In division the operation may frequently be rendered more simple, if you divide both divisor and dividend by any number, or numbers, that will divide both without any remainder; and when by this method you can bring them no lower, divide the remaining dividend by the remaining divisor.

EXAMPLE I.

Divide 1692 by 468.

Here I divide both divisor and dividend twice by 2, and again twice by 3; and as I cannot proceed in this manner any further, without a remainder, I divide the remaining dividend 47 by the remaining divisor 13, and the quotient is $3 \frac{8}{13}$.

If, by this kind of procedure, the divisor at last dwindle to unity, the last dividend is the quotient.

EXAMPLE II.

Divide 7896 by 84.

Here I divide twice by 2, then by 3, and next by 7; and as the divisor is now reduced to unity, the dividend 94 becomes the quotient.

This method may likewise be used when the dividend, or divisor, or both, are continual products, expressed by the sign of multiplication betwixt the factors. Only observe, that in this case the like number of factors above and below must be divided by the common divisor; and if any factor occur both in the divisor and dividend, it may be dashed in both,

E X.

E X A M P L E III.

Divide 12×36 by 18.

In the first step I divide 18 and 12 by 6; in the next I divide 3 and 36 by 3; then I multiply 2 into 12, and the divisor being reduced to unity, the product 24 is the quotient.

$$\begin{array}{c|c|c|c} 12 \times 36 & 2 \times 36 & 2 \times 12 & 24 \\ \hline 18 & 3 & 1 & 1 \end{array}$$

E X A M P L E IV.

Divide $8 \times 72 \times 48$ by $4 \times 24 \times 72$.

Here I first dash the factor 72, both in the divisor and dividend; then I divide the two remaining factors, both above and below, by 4; and proceed to multiply 1 into 6, and 2 into 12, &c.

$$\begin{array}{c|c|c|c|c} 8 \times 72 \times 48 & 8 \times 48 & 2 \times 12 & 24 & 4 \\ \hline 4 \times 24 \times 72 & 4 \times 24 & 1 \times 6 & 6 & 1 \end{array}$$

E X A M P L E V.

Divide $5 \times 49 \times 3 \times 17$ by $17 \times 3 \times 5 \times 35$.

Here I dash the factors 17 $\times$ 3 $\times$ 5, both in the divisor and dividend; and then divide by 7, &c.

$$\begin{array}{c|c|c|c} 5 \times 49 \times 3 \times 17 & 49 & 7 & \\ \hline 17 \times 3 \times 5 \times 35 & 35 & 5 & \frac{2}{5} \end{array}$$

6. In computations or calculations that require the frequent use of the same divisor, the operation may be rendered more easy and expeditious, by making a table of the products of the divisor into all the nine digits; for then you have the quotient-figures, and their products, into the divisor by inspection.

E X A M P L E.

Divide 47896845 by 748.

$$748)47896845(64033$$

TABLE.

1	748
2	1496
3	2244
4	2992
5	3740
6	4488
7	5236
8	5984
9	6732

$$\begin{array}{r} 4488 \\ \hline 3016 \\ 2992 \\ \hline 2484 \\ 2244 \\ \hline 2405 \\ 2244 \\ \hline (161) \end{array}$$

Select methods of dividing Integers.

The method of dividing integers hitherto explained is generally used, as being the plainest and best; but yet a great many other methods are practised; the chief of which I shall here explain, by working a single example six different ways.

$$34)72685(2137$$

1. Here the products of the quotient-figures into the divisor are not set down, but subtracted mentally from the dividends.

$$\begin{array}{r} 46 \\ \hline 128 \\ \hline 265 \\ \hline (27) \\ \hline \end{array}$$

2. Here the quotient is placed above the dividend, and the remainders only are set down; which, with the subsequent figures of the dividend, make up the several dividends.

$$\begin{array}{r} 2137 \text{ quotient.} \\ 34)72685 \text{ dividend.} \\ \dots \\ 4 \\ 12 \\ 26 \\ (27) \\ \hline \end{array}$$

3. Here the quotient is placed below the dividend, and the several remainders are set above it.

$$\begin{array}{r} 12(2 \\ 426(7 \\ 34)72685 \text{ dividend.} \\ 2137 \text{ quotient.} \\ \hline \end{array}$$

4. Here the products of the quotient-figures into the divisor are set below the dividend, and the several remainders above it.

$$\begin{array}{r} 12(2 \\ 426(7 \\ 34)72685(2137 \\ 68 \\ 34 \\ 102 \\ 238 \\ \hline \end{array}$$

5. Here too the products of the quotient-figures into the divisor are placed under the dividend, and they, as well as the dividends, are dashed or cancelled as you subtract; which helps to prevent confusion or mistakes in the operation.

$$\begin{array}{r} 12(2 \\ 426(7 \\ 34)72685(2137 \\ \cancel{68} \cancel{428} \\ \cancel{308} \\ \cancel{12} \\ \hline \end{array}$$

6. Here

6. Here the divisor is repeated under the dividend; you multiply and subtract at once, and cancel as before.

$$\begin{array}{r} 12(2 \\ 426(7 \\ 7268(2137 \\ 34444 \\ 333 \end{array}$$

II. Division of the Parts of Integers.

Here there are three cases.

1. If the divisor be a digit, by it divide the integers of the dividend, reduce the remainder to the parts of the next inferior denomination, and add it, when thus reduced, to the said parts; then divide the sum, reducing and adding the remainder to the parts of the following denomination, &c.

Note. If the integral part of the dividend be less than the divisor, you must, in the first place, reduce it to the parts of the next denomination.

EXAMPLE I.

If L. 274 : 13 : 8 : 3 be equally divided among 8 men, what will each man's share be?

Here I first divide the integers
 L. 274 by 8, and the quotient is 8)274 13 8 3 dividend.
 L. 34, and L. 2 remains; which 34 6 8 2½ quotient.
 reduced to the next denomination makes 40 shillings; and these added to 13 shillings make 53 shillings; which divided by 8 gives 6 shillings to the quotient, and 5 shillings remains; which 5 shillings reduced make 60 d. and 60 d. added to 8 d. make 68 d.; which divided by 8 gives 8 d. to the quotient, and 4 d. remains, &c.

The operation may, if you please, be drawn out at large; as in the following

EXAMPLE II.

If C. 43 : 2 : 8 of tobacco be made up into 5 equal hhds, what will be the neat weight of each hhd?

Here I divide the C. 43 by 5, and the quotient is C. 8, and C. 3 remains; which reduced and added to the 2 Q. makes 14 Q. which I divide by 5, &c.

$$\begin{array}{r} \text{C. } q. \text{ lb. } \text{C. } q. \text{ lb.} \\ 5)43 \quad 2 \quad 8 \quad (8 \quad 2 \quad 24 \\ 40 \\ \hline 3 \text{ rem.} \\ 4 \\ \hline 14(\\ 10 \\ \hline 4 \text{ rem.} \\ 28 \\ \hline 120(\\ 10 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

2. If

2. If the divisor consist of two or more figures, and be a composite number, resolve it into its component parts, and divide the given dividend by one of these parts, the quotient by another, &c. and the last quotient is the answer.

E X A M P L E I.

If 54 pieces of cloth cost L. 526 : 12 : 5, what is that *per* piece?

Here the component parts are 6×9 ,
 so I divide by 6, and the quotient by 9; and the last remainder 1 multiplied into the preceding divisor 6, produces 6; to which I add 2, the remainder belonging to that division, and the sum 8 is the total remainder; so the fraction is $\frac{8}{34}$.

	L.	s.	d.	f.	rem.
6)	526	12	5		
9)	87	15	4	3	2
Ans.	9	15	0	2	$\frac{8}{34}$

E X A M P L E H.

If 1 C. or 112 lb. of nutmegs be valued at L. 76 : 18 : 8, what is that *per* lb.?

Here the component parts are 7×8
 $\times 2$, by which I divide continually, and make up the total remainder as before.

	L.	s.	d.	f.	rem.
7)	76	18	8		
8)	10	19	9	2	6
2)	1	7	5	2	6
Ans.	13	8	3	$\frac{48}{112}$	

But if the divisor be not a composite number, work as in the following

E X A M P L E III.

73 persons go equal shares in an adventure, which cost L. 3648, 15 s. 4 d. 2 f. : What is each person's share?

L.	s.	d.	f.	L.	s.	d.	f.
73)3648	15	4		2(49	19	7	$3\frac{1}{2}$
292	1420	576	276				
728	1435(	580(	278(				
657	73	511	219				
71 rem.	705	69 rem.	(59)				
20	657	4					
1420 s.	48 rem.	276 f.					
	12						
	96						
	48						
	576 d.						

3. If the divisor consist of integers and parts, reduce both divisor and dividend to the same denomination, and then proceed as in division of integers.

E X A M P L E I.

A nobleman distributes among some poor people, L. 20, 5 s.; each person got 7 s. 6 d.: What was the number of the poor?

Here I reduce both the divisor and the dividend to the same denomination, viz. pence, then I divide, and the quotient 54 is the number of poor people, or it is a number denoting how often 90 d. is contained in 4860 d.

s.	d.	L.	s.
7	6	20	5
12		20	
90 d.		405	
		12	
		810	
		405	

9|0)486|0(
Ans. 54 persons.

E X A M P L E II.

The content of a rectangular floor or pavement is 452 square feet, and 107 square inches; the breadth is 18 feet and 5 inches: What is the length?

Here I reduce the dividend to square inches, and the divisor to lineal inches; then I divide, and the quotient 295 is lineal inches, agreeable to the rules laid down in cross multiplication.

F.	in.	Sq.f.	Sq.in.
18	5	452	107
12		144	
36		1808	
18		1808	
221		452	
		2099	
		1989	
		1105	
		1105	

221)65195(295 = 24 7
442

More examples, in all the parts of division, may be had, by reversing the examples of multiplication.

III. The

III. *The Proof of Division.*

Division may be proved several ways, *viz.* by multiplication, by division, and by casting out the 9's.

1. By multiplication: Multiply the quotient by the divisor, or the divisor by the quotient; and the product, with the remainder added to it, will be equal to the dividend: Or take the products of the quotient-figures into the divisor, add them in the order they stand under the dividends; and their sum, with the remainder, will be equal to the dividend.

2. By division: Divide the difference of the dividend and remainder by the quotient, and your next quotient will be equal to your first divisor, without any remainder. But this method is tedious.

3. By casting out the 9's: Cast the 9's out of the divisor and quotient, place the excesses on the right and left sides of a cross; then multiply these two figures into one another, and cast the 9's out of their product; add the excess to the remainder; and, casting out the 9's, if need be, place the sum or excess at the top of the cross; then cast the 9's out of the dividend, and set the excess at the bottom. If the work be right, the figures at the top and bottom of the cross will agree, or be the same.

These methods of proof are a proper exercise to the learner in schools; but, in business, the only proof used is a careful revival of the operation.

IV. *Practical Questions.*

1. What number, multiplied by 74308, will produce 28534272 exactly? *Ans.* 384.

2. What number, divided by 87424, will quote 4783, and leave a remainder equal to a fourth part of the divisor? *Ans.* 418170848.

3. The sum of two numbers is 4940, the greater is 4572: What is the lesser, what their difference, what their product, and what the quotient of the greater divided by the lesser? *Ans.* 368. 4204. 1682496. $12\frac{1}{3}\frac{5}{8}$.

4. The remainder of a division is 649, the quotient 113, the divisor is the sum of both, and 24 more: What is the dividend? *Ans.* 89467.

5. There are two numbers, the greater is 24840, which divided by the lesser quotes 72: What is the lesser, what the difference of their squares, what the square of their sum, and what the cube of their difference? *Ans.* 345. 616906575. 634284225. 14697123087375.

6. If L. 768 be divided equally among 36 men, what will each man's share be? *Ans.* L. 21 : 6 : 8.

7. What is the value of 1 yard of cloth, when 84 yards of the same cost L. 36 : 13 : 3? *Ans.* 8 s. 8 d. 3 f.

8. A privateer takes a prize to the value of L. 2851, 4 s.; of which the captain gets $\frac{1}{8}$, each of six officers, $\frac{1}{12}$ of the remainder, and the private men, being 45 in number, get the rest equally divided among them: What is each man's share?

			L.	s.	d.
Ans.	{	Captain's share	—	—	—
		Each officer's share	—	—	—
		Each private man's share	—	—	—
			178	14	
			83	10	$7\frac{1}{2}$
			48	5	3

9. Suppose a person in ten years spends of his own L. 2346:18:6, and of contracted debts to the value of L. 132:7:8; what is his expence per year, per month, per week, per day?

			L.	s.	d.	f.
Ans.	{	Per year	—	—	—	—
		Per month	—	—	—	—
		Per week	—	—	—	—
		Per day	—	—	—	—
			247	18	7	$1\frac{6}{10}$
			20	13	2	$2\frac{16}{100}$
			4	15	4	$1\frac{96}{1000}$
			13	7		$\frac{84}{1000}$

10. The planet Mercury revolves round the sun in 88 days: How many revolutions will he perform in 17 years and 219 days? Ans. 73 revolutions.

11. How many bricks, 9 inches long and 4 inches broad, will floor a room that is 18 feet wide and 24 feet long? Ans. 1728 bricks.

12. A rectangular floor contains 919 square feet, 98 square inches, 108 square lines; and the length of it is 38 feet, 9 inches, 6 lines: What is the breadth? Ans. 23 feet 8 inches 6 lines.

13. The content of a parallelopiped is 146 solid yards, 1 solid foot, and 1024 solid inches; the length of it is 27 feet 4 inches; and breadth 17 feet 8 inches: What is the thickness or depth? Ans. 8 feet 2 inches.

C H A P. VI.

R E D U C T I O N.

REDUCTION teacheth how to bring a number of one name or denomination to another of the same value; and is either descending, ascending, or mixt.

I. Reduction descending brings a number of a higher denomination to a lower, when the lower is some aliquot part of the higher; as pounds to shillings, pence, or farthings; and is performed by multiplication.

II. Reduction ascending brings a number of a lower denomination to a higher, when the lower is some aliquot part of the higher; as shillings, pence, or farthings, to pounds; and is performed by division.

III. Mixt

III. Mixt reduction brings a number of one denomination to another, when the one is no aliquot part of the other; as pounds to guineas; and requires the use of both multiplication and division.

In treating of reduction I shall show its application to money, and to the most usual sorts of weights and measures; in doing of which I shall conjoin reduction descending and ascending, the one serving as a proof of the other; and shall afterwards treat of mixt reduction by itself.

In working reduction, of whatever kind, the following rule is to be observed, *viz.*

Multiply or divide as the tables of coin, weights, and measures, direct.

Reduction descending and ascending.

I. MONEY.

Quest. 1. In L. 472, how many shillings, pence, and farthings?

This reduction is descending; so I multiply the pounds by 20, because 20 shillings make 1 pound, and the product is shillings: then I multiply the shillings by 12, because 12 pence make 1 shilling, and the product is pence: lastly I multiply the pence by 4, because 4 farthings make 1 penny, and the product is farthings.

$$\begin{array}{r}
 472 \text{ pounds.} \\
 \underline{20} \\
 9440 \text{ shillings.} \\
 \underline{12} \\
 1888 \\
 944 \\
 \underline{113280 \text{ pence.}} \\
 \underline{4} \\
 453120 \text{ farthings.}
 \end{array}$$

Proof by Reduction ascending.

In 453120 farthings, how many pence, shillings, and pounds?

Here I divide the farthings by 4, because 4 farthings make 1 penny, and the quotient is pence: then I divide the pence by 12, because 12 pence make 1 shilling, and the quotient is shillings: lastly, I divide the shillings by 20, because 20 shillings make 1 pound, and the quotient is pounds.

$$\begin{array}{r}
 4)453120 \text{ farthings.} \\
 \underline{} \\
 12)113280 \text{ pence.} \\
 \underline{} \\
 2|0)944|0 \text{ shillings.} \\
 \underline{} \\
 472 \text{ pounds.}
 \end{array}$$

Note 1. To reduce pounds to pence at one operation, multiply by 240, the number of pence in 1 pound.

Note 2. To reduce pounds to farthings at one operation, multiply by 960, the number of farthings in 1 pound.

Note 3.

Note 3. To reduce shillings to farthings at one operation, multiply by 48, the number of farthings in 1 shilling.

I shall here reduce to farthings the L. 472 in Quest. 1. by these notes.

By note 1.	By note 2.	By note 3.
472 pounds.	472 pounds.	472 pounds.
240	960	20
1888	2832	9440 shillings.
944	4248	48
113280 pence.	453120 farthings.	7552
4		3776
453120 farthings.		453120 farthings.

Note 4. To reduce pence to pounds at one operation, divide by 240, the pence in 1 pound.

Note 5. To reduce farthings to pounds at one operation, divide by 960, the farthings in 1 pound.

Note 6. To reduce farthings to shillings at one operation, divide by 48, the farthings in 1 shilling.

Here follows the farthings of Quest. 1. reduced back to pounds by these notes.

By note 4.	By note 5.	By note 6.
4)453120 farthings.	96 0)45312 0(472 L.	2 0)
	384..	48)453120)944 0's.
24 0)11328 0 d. (472 L.	691	432... —
96..	672	472 L.
172	192	211
168	192	192
48	(0)	192
48		192
(0)		(0)

Quest. 2. In 4386 pounds, how many groats?

4386 pounds.

20

87720 shillings.

3 groats in 1 shilling.

Ans. 263160 groats.

Or

Or pounds may be reduced to groats at one operation, if you multiply by 60, the number of groats in 1 pound.

4386 pounds.
60

Ans. 263160 groats.

P R O O F.

In 263160 groats, how many pounds?

3)263160

Or thus: 6|0)26316|0

2|0)8772|0

Ans. L. 4386

Ans. L. 4386

Quest. 3. In 7643 pounds, how many three-pences?

7643 pounds.

20

152860 shillings.

4 three-pences in 1 shilling.

Ans. 611440 three-pences.

Or pounds may be reduced to three-pences at one operation, if you multiply by 80, the number of three-pences in 1 pound.

7643 pounds.
80

Ans. 611440 three-pences.

P R O O F.

In 611440 three-pences, how many pounds?

4)611440

Or thus: 8|0)61144|0

2|0)15286|0

Ans. L. 7643

Ans. L. 7643

Quest. 4. In 48 guineas, how many shillings, pence, and farthings?

48 guineas.

21 the shill. in 1 guinea.

Brought up, 1008 s.

12

48

96

12096d.

4

Carried up,

1008 shillings.

48384 f.

P R O O F.

P R O O F.

In 48384 farthings, how many pence, shillings, and guineas?

4)48384 farthings.

12)12096 pence.

21)1008 sh. (48 guineas.

84

168

168

(0)

Quest. 5. In 853 pounds, how many crowns, shillings, groats and pence?

Pounds 853

4 crowns in 1 pound.

Crowns 3412

5 shillings in 1 crown.

Shillings 17060

3 groats in 1 shilling.

Groats 51180

4 pence in 1 groat.

Pence 204720

P R O O F.

In 204720 pence, how many groats, shillings, crowns, and pounds?

4)204720 pence.

3)51180 groats.

5)17060 shillings.

4)3412 crowns.

853 pounds.

Quest. 6. In L. 458 : 16 : $7\frac{1}{4}$, how many shillings, pence, and farthings?

G

L

L. s. d.
 458 16 7 $\frac{1}{4}$
 20 I take in the 16 s.

Shillings 9176
 12 I take in the 7 d.

Pence 110119
 4 I take in the 3 f.

Farthings 440479

P R O O F.

In 440479 farthings, how many pence, shillings, and pounds?

Rem.
 4)440479 3 f.

The remainders in reduction ascending
 are of the same name or denomination
 with the dividends,

12)110119 7 d.

2)091716 16 s.

L. 458 16 7 $\frac{1}{4}$

MORE EXAMPLES.

Quest. 7. In L. 78342, how many groats and farthings?

Quest. 8. In 69453 guineas, how many shillings, threepences, pence, and farthings?

Quest. 9. In L. 45682, how many crowns, half-crowns, sixpences, and farthings?

Quest. 10. In L. 57843 : 19 : 10 $\frac{1}{2}$, how many shillings, pence, and halfpence?

2. A VOIR DUPOIS WEIGHT.

Quest. 1. In C. 47 : 1 : 20, how many ounces?

Method 1.

C. $\mathcal{Q}$. lb.

47 1 20

4

189 $\mathcal{Q}$.

28

1512

380

5312 lb.

16

31872

5312

84992 oz.

Method 2.

C. $\mathcal{Q}$. lb.

47 1 20

47

47

47

48 lb. in 1

20

5312 lb.

16

31872

5312

84992 oz.

Method 3.

C. $\mathcal{Q}$. lb.

47 1 20

564

48 lb. in 1

20

5312 lb.

16

31872

5312

84992 oz.

In

In method 1. in multiplying the C. by 4, I take in the 1 Q.; and in multiplying the Q. by 28, I take in the 20 lb.

In method 2. the operation is the same as multiplying the C. by 112. For 47 placed below 47, with units under units, is the same as multiplying 47 by 2; and what follows is the multiplication of 47 by 11.

In method 3. I multiply the C. by 12; and the product 564, being placed two figures to the right hand of 47, stands so, that being added, their sum will be equal to the product of 47 multiplied by 112. Or, the reason of the operation will still appear plainer by considering, that 47 C. is 4700 lb. and 47 times 12 lb. and therefore to 4700 lb. I add 12 times 47.

P R O O F.

In 84992 ounces, how many lb. Q. and C. ?

	28)	4)	C. Q. lb.
16)84992	(5312	(189	(47 1 20
80...	28..	16	
—	—	—	
49	251	29	
48	224	28	
—	—	—	
19	272	(1) Q.	
16	252		
—	—		
32	(20) lb.		
32			
—			
(0)			

M O R E E X A M P L E S.

Quest. 2. In C. 485 : 3 : 24 : 12 oz. how many Q. lb. oz. and dr. ?

Quest. 3. In 764 tuns 15 C. 2 Q. 16 lb. how many C. Q. and lb. ?

3. T R O Y W E I G H T.

Quest. 1. In 482 lb. 7 oz. 13 dw. 21 gr. how many ounces, penny-weights, and grains ?

lb. oz. dw. gr.
482 7 13 21
12
—
5791 ounces.
20
—
115833 penny-weights.
24
—
463333
231668
—
2780013 grains.
G 2

P R O O F.

P R O O F.

In 2780013 grains, how many penny-weights, ounces, and pounds?

24)2780013 (11583|3 13 penny-weights.

24.....

12)5791

7 ounces.

38

24

482 pounds.

140

120

200

192

81

72

lb. oz. dw. gr.
In all 482 7 13 21

93

72

Rem. (21) grains.

M O R E E X A M P L E S.

Quest. 2. In 7694 lb. 8 oz. how many grains?

Quest. 3. In 8546 lb, 10 oz. 0 dw. 18 gr. how many grains?

4. D R Y M E A S U R E.

Quest. 1. In 48 loads 12 bushels, how many bushels, pecks, and gallons?

Loads. Bush.

48

12

40

1932 bushels.

4

7728 pecks.

2

15456 gallons.

P R O O F.

In 15456 gallons, how many pecks, bushels, and loads?

M O R E

M O R E E X A M P L E S.

Quest. 2. In 74 bushels 2 pecks, how many pecks, gallons, and pottles?

Quest. 3. In 58 chalders 6 bolls 2 firlots, Scotch measure, how many bolls, firlots, and pecks?

5. W I N E - M E A S U R E.

Quest. 1. In 72 hogheads of wine, how many gallons and pints?

72 hogheads.

63

2160

432

4536 gallons.

8

36288 pints.

P R O O F.

In 36288 pints of wine, how many gallons and hogheads?

M O R E E X A M P L E S.

Quest. 2. In 45 tuns of wine, how many gallons?

Quest. 3. In 354 tuns 1 hhd. 42 gallons of wine, how many hogheads, gallons, quarts, and pints?

6. B E E R - M E A S U R E.

Quest. 1. In 54 butts of beer, how many hogheads and gallons?

54 butts.

2

108 hhds.

54

432

540

5832 gallons.

P R O O F.

In 5832 gallons of wine, how many hogheads and butts?

M O R E E X A M P L E S.

Quest. 2. In 96 tuns of beer, how many hogheads, gallons, and pints?

Quest. 3. In 25 hogheads of beer, Scotch measure, how many gallons, pints, and gills?

G 3

7. C L O T H.

7. CLOTH-MEASURE.

Quest. 1. In 34 yards 3 quarters, how many quarters and nails?

$$\begin{array}{r}
 \text{Yds.} \quad \text{qrs.} \\
 34 \quad 3 \\
 \underline{4} \\
 139 \text{ quarters.} \\
 \underline{4} \\
 556 \text{ nails.}
 \end{array}$$

P R O O F.

In 556 nails, how many quarters and yards?

M O R E E X A M P L E S.

Quest. 2. In 38 ells Flemish and 2 quarters, how many quarters and nails?

Quest. 3. In 45 ells English and 1 quarter, how many quarters and nails?

8. LONG MEASURE.

Quest. 1. From London to York is 151 miles, how many furlongs, poles, half-yards, inches, and barley-corns, will reach from the one city to the other?

$$\begin{array}{r}
 151 \text{ miles.} \\
 \underline{8} \\
 1208 \text{ furlongs.} \\
 \underline{40} \\
 48320 \text{ poles.} \\
 \underline{11} \\
 4832 \\
 \underline{4832} \\
 531520 \text{ half-yards.} \\
 \underline{18} \\
 425216 \\
 \underline{53152} \\
 9567360 \text{ inches.} \\
 \underline{3} \\
 28702080 \text{ barley-corns.}
 \end{array}$$

P R O O F.

P R O O F.

In 28702080 barley-corns, how many inches, half-yards, poles, furlongs, and miles?

M O R E E X A M P L E S.

Quest. 2. In 876 miles, how many chains, poles, half-yards, and inches?

Quest. 3. In 950 Scotch miles, how many furlongs, chains, feet, and inches?

9. L A N D - M E A S U R E.

Quest. 1. In 75 acres, how many roods and poles?

75 acres.

Or thus : 75 acres.

4

160

300 roods.

450

40

75

12000 poles.

12000 poles.

P R O O F.

In 12000 square poles, how many roods and acres?

M O R E E X A M P L E S.

Quest. 2. In 84 acres 3 roods and 25 poles, how many roods and poles?

Quest. 3. In 96 Scotch acres, how many roods, falls, and ells?

10. T I M E.

Quest. 1. In 28 years 24 weeks 4 days 16 hours 30 minutes, how many minutes?

Years.

weeks.

days.

hours.

minutes.

28

24

4

16

30

52

60

142

1480 weeks.

7

10364 days.

24

41462

20729

248752 hours.

60

14925150 minutes.

G 4

P R O O F.

P R O O F.

In 14925150 minutes, how many hours, days, weeks, and years?

The above reduction allows only 364 days to the year; but if you incline to be accurate, find the hours in 365 days 6 hours, a complete year, and then reduce as follows:

Days.	hours.	Weeks.	days.	hours.	min.
365	6	24	4	16	30
24		7			
1466		172 days.			
730		24			
8766	hours in one year.	694			
28		345			
70128		4144 hours.			
17532		245448			
245448	hours in 28 years.	249592	hours in all.		
		60			
14975550 minutes.					

P R O O F.

In 14975550 minutes, how many hours and years?

M O R E E X A M P L E S.

Quest. 2. How many hours and minutes from the creation to the birth of Christ, it being accounted 4004 years?

Quest. 3. How many hours and minutes from the birth of Christ to the end of the year 1785?

Mixt Reduction.

In working mixt reduction observe the following

R U L E.

By reduction descending bring the given name to some such third name as is an aliquot part both of the name given and of the name sought, and then by reduction ascending bring the third name to the name sought.

Mixt reduction, as well as reduction descending and ascending, extends to money, and to things weighed, or measured, as follows.

M O N E Y.

Quest. 1. In 764l. how many guineas?

Here the name given is pounds, the name sought is guineas, and the third name, to which the pounds are reduced, is shillings; for a shilling is an aliquot part both of a pound and of a guinea.

O P E R A T I O N.

764 pounds.
20
—
21)15280(727 guineas.
14700
—
58
42
—
160
147
—
(13) shillings.

P R O O F.

In 727 guineas 13 shillings,
how many pounds?

Guineas. shillings.

727 13
21
—
730
1455
—
210)15280
—
764 pounds.

Quest. 2. In 832 moidores, how many pounds Sterling?

Here I reduce the moidores to shillings, a shilling being an aliquot part both of a moidore and of a pound Sterling.

O P E R A T I O N.

832 moidores.
27 shill. in 1 moidore.
—
5824
1664
—
210)22464 shillings.
—
1123 L. Sterling.

P R O O F.

In 1123l. 4s. how many moidores?

L. s.

1123 4
20
—
27)22464(832 moidores.
21600
—
86
81
—
54
54
—
(0)

Quest. 3.

Quest. 3. In 968 l. how many merks?

Method 1.

968 l.

3

2)2904 half-merks.

1452 merks.

Method 2.

968 l.

240

3872

1936

16|0)23232|0(1452 merks.

16...

72

64

83

80

32

32

(0)

In method 1. I reduce the pounds to half-merks, a half-merk being an aliquot part both of a pound and of a merk.

In method 2. I reduce the pounds to pence, and then divide by 160, the number of pence in 1 merk; but method 1. is the easier and shorter way.

P R O O F.

In 1452 merks, how many pounds?

1452 merks.

2

3)2904 half-merks.

968 l.

Or thus: 1452 merks.

160

8712

1452

24|0)23232|0(968 l.

216...

163

144

192

192

(0)

Quest. 4. In 540 dollars, at 4s. 4d. per dollar, how many pounds Sterling?

s. d.	Method 1.	s. d.	Method 2.
4 4	540 dollars.	4 4	540 dollars.
12	52	3	13
52 d. in 1 dol.	108	Groats 13 in 1 dol.	162
	270		54
24 0)2808 0(117l.		6 0)702 0	
24			
		117 l.	
40			
24			
168			
168			
(0)			

In method 1. I reduce the dollars to pence, and then divide by 240, the number of pence in 1 pound.

In method 2. I reduce the dollars to groats, and then divide by 60, the number of groats in 1 pound.

P R O O F.

In 117 l. Sterling, how many dollars, at 4s. 4d. *per* dollar?

117 l.	Or thus: 117 l.
240	60
468	13)7020(540 dollars.
234	65
52)28080(540 dollars.	52
260	52
208	(0)
208	
(0)	

Quest. 5. In 4785 l. 13s. how many pieces of $13\frac{1}{2}$ d. *per* piece?

$13\frac{1}{2}$ d.

$13\frac{1}{2}d.$ L. 4785 13s.
 2 20
 —————
 27 half-pence. 95713 shillings.
 24 half-pence in 1 shilling.

382852
 191426
 —————

27) 2297112 (85078 pieces of $13\frac{1}{2}d.$
 216.....

137
 135
 —————

211
 189
 —————

222
 216
 —————

Rem. (6) half-pence, or 3d.

P R O O F.

In 85078 pieces of $13\frac{1}{2}d.$ each, and 3d. how many pounds?

Pieces. d.

85078 3
 27

595552
 170156
 —————

24) 2297112 (957113
 216.....

L. 4785 13s.

137
 120
 —————

171
 168
 —————

31
 24
 —————

72

72

(0)

Quest. 6. In L. 872, how many pieces of 6 d. of 5 d. and of 4 d. of each an equal number?

d.	L.
6	872
5	240
4	
	3488
Sum 15 pence.	1744
	15)209280(13952 pieces of 6 d. of 15..... 5 d. and 4 d.
	59
	45
	142
	135
	78
	75
	30
	30
	(0)

P R O O F .

In 13952 pieces of 6 d. of 5 d. of 4 d. of each an equal number, how many pounds?

13952 pieces.	24 0)20928 0(872 L.
15	192..
69760	172
13952	168
Total 209280 pence.	48
	48
	(0)

This sort of reduction could easily be extended much further, and might be used to reduce Sterling money to any sort of foreign money, or any sort of foreign money to Sterling: but as this is a wide field, it is more usual, and the better way, to assign a place for this by itself, under the title of *Exchange*. I shall therefore conclude mixt reduction of money by subjoining a few

M O R E

M O R E E X A M P L E S .

Quest. 7. A gentleman was robbed of 57 guineas 33 half-guineas and 48 moidores, how many pounds Sterling did he lose?

Ans. L. 141 : 19 : 6.

Quest. 8. In 685 merks 234 nobles and 142 quarter-guineas, how many pounds Sterling? *Ans.* L. 571 : 18 : 10.

Quest. 9. In L. 148, how many pieces of 13½ d. of 12 d. of 9 d. of 6 d. of 4 d. and of each an equal number? *Ans.* 798 pieces of each sort, and 9 d. over.

2. A V O I R D U P O I S W E I G H T .

Quest. 1. In 8 bags cotton, each containing C. 12 : 1 : 20, how many tuns?

$$\begin{array}{r}
 \begin{array}{ccc} \text{C.} & \text{q.} & \text{lb.} \\ 12 & 1 & 20 \\ & & 8 \end{array} \\
 \hline
 2 \overline{) 09} \begin{array}{ccc} \text{Tuns.} & \text{C.} & \text{q.} & \text{lb.} \\ 4 & 19 & 1 & 20 \end{array} \text{Ans.} \\
 \begin{array}{ccc} 8 & & \end{array}
 \end{array}$$

Rem. (19) C.

P R O O F .

If 4 tuns 19 C. 1 Q. 20 lb. of cotton be made up into 8 equal bags, how much will each bag contain?

$$\begin{array}{r}
 \begin{array}{ccc} \text{Tuns.} & \text{C.} & \text{q.} & \text{lb.} \\ 8 \overline{) 4} & 19 & 1 & 20 \end{array} \begin{array}{ccc} \text{C.} & \text{q.} & \text{lb.} \\ 12 & 1 & 20 \end{array} \text{Ans.} \\
 \begin{array}{ccc} 20 & & \end{array} \\
 \hline
 \begin{array}{ccc}) 99 & (& (5) \\ 8 & & 28 \end{array} \\
 \hline
 \begin{array}{ccc} 19 & &) 160 \\ 16 & & 16 \end{array} \\
 \hline
 \begin{array}{ccc} \text{rem.} & 3 & (0) \\ & 4 & \end{array} \\
 \hline
 \begin{array}{ccc}) 13 & (& \\ 8 & & \end{array} \\
 \hline
 (5)
 \end{array}$$

M O R E

M O R E E X A M P L E S.

Quest. 2. In 24 equal cheeses, weighing 3 Q. 15 lb. each, how many C.? *Ans.* C. 21 : 0 : 24.

Quest. 3. One buys of a grocer C. 4 : 1 : 14 of pepper, and orders it to be made up into parcels of 14 lb. of 12 lb. of 8 lb. of 6 lb. of 2 lb. and of each an equal number: Required the number of parcels of each sort? *Ans.* 11 parcels of each sort, and 28 pounds over.

3. T R O Y W E I G H T.

Quest. 1. In 96 pair of silver spurs, weighing 3 oz. 12 dw. per pair, how many pounds?

$$\begin{array}{r}
 \text{oz. dw.} \\
 3 \quad 12 \\
 20 \\
 \hline
 72 \text{ dw.} \\
 96 \\
 \hline
 432 \\
 648 \\
 \hline
 2 \overline{) 06912} \quad 12 \text{ dw.} \\
 \hline
 12 \overline{) 345} \quad 9 \text{ oz.} \\
 \hline
 28 \text{ lb.} \quad \text{Ans. } 28 \quad 9 \quad 12
 \end{array}$$

P R O O F.

How many pair of spurs, each pair weighing 3 oz. 12 dw. may be made out of 28 lb. 9 oz. 12 dw. of silver?

$$\begin{array}{r}
 \text{oz. dw.} \quad \text{lb. oz. dw.} \\
 3 \quad 12 \quad 28 \quad 9 \quad 12 \\
 20 \quad 12 \\
 \hline
 72 \text{ dw.} \quad 345 \\
 20 \\
 \hline
 72 \overline{) 6912} \quad (96 \text{ pair of spurs. } \text{Ans.} \\
 648 \\
 \hline
 432 \\
 432 \\
 \hline
 (0)
 \end{array}$$

M O R E

M O R E E X A M P L E S .

Quest. 2. In 7 dozen silver candlesticks, each weighing 10 oz. 14 dw. how many pounds? *Ans.* 74 lb. 10 oz. 16 dw.

Quest. 3. A merchant sent to a goldsmith 16 ingots of silver, each weighing 2 lb. 4 oz. and ordered it to be made into bowls of 2 lb. 8 oz. *per* bowl, and tankards of 1 lb. 6 oz. *per* piece, and salts of 10 oz. 10 dw. *per* salt, and spoons of 1 oz. 18 dw. *per* spoon, and of each an equal number, how many of each sort will there be? *Ans.* 7 vessels of each sort, and 224 dw. over.

4. W I N E - M E A S U R E .

Quest. 1. In 34 runlets of wine, each runlet containing 18 gallons, how many hogsheds?

O P E R A T I O N .

34 runlets.
18
—
272
34
—
63)612(9 hhds.
567
—
Rem. (45) gallons.
<i>Ans.</i> 9 hhds 45 gallons.

P R O O F .

In 9 hhds 45 gallons of wine,	
how many runlets?	
<i>Hhds.</i>	<i>gall.</i>
9	45
63	—
—	—
18)612(34 runlets. <i>Ans.</i>	
54	
—	
72	
72	
—	
(0)	

M O R E E X A M P L E S .

Quest. 2. In 752 runlets, how many tierce of wine? *Ans.* 322 tierce, and 12 gallons.

Quest. 3. In 840 hogsheds, how many puncheons of 80 gallons *per* puncheon? *Ans.* 661 puncheons, and 40 gallons.

5. C L O T H - M E A S U R E .

Quest. 1. In 96 yards, how many ells Flemish?

96 yards.
4
—
3)384
—
<i>Ans.</i> 128 ells Flemish.

P R O O F .

P R O O F.

In 128 ells Flemish, how many yards?

M O R E E X A M P L E S.

Quest. 2. In 450 yards, how many ells English? *Ans.* 360.

Quest. 3. In 785 ells Flemish, how many ells English? *Ans.* 471.

6. L O N G M E A S U R E.

Quest. 1. In 72 miles, how many furlongs, poles, half-yards, yards, and feet?

$$\begin{array}{r}
 72 \text{ miles.} \\
 8 \\
 \hline
 576 \text{ furlongs.} \\
 40 \\
 \hline
 23040 \text{ poles.} \\
 11 \\
 \hline
 2304 \\
 2304 \\
 \hline
 2)253440 \text{ half-yards.} \\
 \hline
 126720 \text{ yards.} \\
 3 \\
 \hline
 380160 \text{ feet.}
 \end{array}$$

P R O O F.

In 380160 feet, how many yards, half-yards, poles, furlongs, and miles?

M O R E E X A M P L E S.

Quest. 2. Supposing the circumference of the earth to be divided into 360 degrees, and each degree to contain 60 miles, I demand how many miles, furlongs, poles, half-yards, yards, feet, inches, and barley-corns, will reach round the globe of the earth? *Ans.* The barley-corns are 4105728000.

Quest. 3. In 150 English miles, how many Scotch miles? *Ans.* 133 miles, 6 furlongs, 10 falls, and 15 feet.

Mixt reduction of weights and measures, as well as that of money, might be carried a great way farther than what is done here; but the above specimens, it is hoped, will be found sufficient instruction to the learner.

H

Practical

Practical Questions.

1. In a purse containing 84 guineas, 56 half-guineas, and 48 moidores, how many pounds Sterling? *Ans.* L. 182, 8s.

2. A gentleman setting out on a journey carried along with him 8 Joannes's, being L. 3, 12s. each, 17 guineas, 13 half-guineas, 14 crowns, 10 half-crowns, 15 shillings, and 9 sixpences; and finds, on his return, by his pocket-book, that he had spent L. 49, 4s.: How many pounds Sterling did he carry out, and how many brought he home? *Ans.* He carried out L. 59, 4s. and brought home L. 10.

3. A merchant bought 30 hogheads sugar, viz. 12 hhds, containing C. 16 : 1 : 14 each, 12 hhds weighing C. 14 : 3 : 7 each, and 6 hhds containing C. 15 : 2 : 21 each: How many tuns of sugar did he purchase? *Ans.* 23 tuns 8 C. 1 Q. 14 lb.

4. In 96 firkins of beer, 9 gallons each, how many hhds of 54 gallons *per* hhd? *Ans.* 16 hhds.

5. A piece of ground, consisting of 26 acres is to be laid out in small divisions, of 3 roods and 10 poles each: How many small divisions will there be? *Ans.* 32.

6. Queen Elisabeth came to the throne of England the 17th of November 1558, and died the 24th of March 1603, in the 70th year of her age: What year was she born? and how many months, of 28 days each, did she reign, reckoning 365 days 6 hours to a year? *Ans.* She was born in the year 1533, and reigned 578 months 2 weeks and 1 day.

C H A P. VII.

THE RULE OF THREE.

THE Rule of Three, called also, on account of its excellence, the *Golden Rule*, from certain numbers given, finds another; and is divided into simple and compound, or into single and double.

SECTION I.

The simple or single Rule of Three.

The simple rule of three, from three numbers given, finds a fourth, to which the third bears the same proportion as the first does to the second.

The nature and properties of proportional numbers may be understood sufficiently for our purpose from the following observations.

In

In comparing any two numbers, with respect to the proportion which the one bears to the other, the first number, or that which bears proportion, is called the *antecedent*; and the other, to which it bears proportion, is called the *consequent*; and the quantity of the proportion or ratio is estimated by the quot arising from dividing the antecedent by the consequent. Thus, the ratio or proportion betwixt 6 and 3 is the quot arising from dividing the antecedent 6 by the consequent 3; namely, 2; and the ratio or proportion betwixt 1 and 2 is the quot arising from the division of the antecedent 1 by the consequent 2; namely $\frac{1}{2}$, or one half.

Four numbers are said to be proportional when the ratio of the first to the second is the same as that of the third to the fourth; and the proportional numbers are usually distinguished from one another, as in the following examples:

$$4 : 2 :: 16 : 8. \quad 6 : 9 :: 12 : 18.$$

Proportional numbers, or numbers in proportion, are usually denominated *terms*; of which the first and last are called *extremes*, and the intermediate ones get the name of *means*, or *middle terms*.

If four numbers are proportional, they will also be inversely proportional; that is, the first consequent will be to its own antecedent as the second consequent is to its antecedent; or the fourth term will be to the third as the second is to the first. Thus, if $6 : 3 :: 10 : 5$, then by inversion, $3 : 6 :: 5 : 10$, or $5 : 10 :: 3 : 6$. *Euclid* v. 4 cor. By either of these kinds of inversion may any question in the rule of three be proved.

If four numbers are proportional, they will also be alternately proportional; that is, the first antecedent will be to the second antecedent as the first consequent is to the second consequent; or the first term will be to the third term as the second term is to the fourth. Thus, if $8 : 4 :: 24 : 12$, then, by alternation, $8 : 24 :: 4 : 12$. *Euclid* v. 16.

But the celebrated property of four proportional numbers is, that the product of the extremes is equal to the product of the means. Thus, if $2 : 3 :: 6 : 9$, then $2 \times 9 = 3 \times 6 = 18$. *Euclid* vi. 16.

Hence we have an easy method of finding a fourth proportional to three numbers given, *viz.*

Multiply the middle number by the last, and divide the product by the first, the quot gives the fourth proportional.

EXAMPLE.

Given 6, 5, and 36, to find a fourth proportional; put x equal to the fourth proportional, then $6 : 5 :: 36 : x$, and $5 \times 36 = 180 = 6 \times x$; wherefore, dividing the product 180 by the factor 6, the quot gives the other factor x , namely 30, the fourth proportional sought.

H 2

Every

Every question in the rule of three may be divided into two parts, *viz.* a supposition and a demand; and of the three given numbers, two are always found in the supposition, and only one in the demand.

EXAMPLE.

If 4 yards cost 12 shillings, what will 6 yards cost at that rate?

In this question the supposition is, if 4 yards cost 12 shillings; and the two terms contained in it are 4 yards and 12 shillings; the demand lies in these words, What will 6 yards cost? and the only term found in it is 6 yards.

The supposition and demand being thus distinguished, proceed to state the question, or to put the terms in due order for operation, as the following rules direct.

R U L E I.

Place that term of the supposition, which is of the same kind with the number sought, in the middle. The two remaining terms are extremes, and always of the same kind.

R U L E II.

Consider, from the nature of the question, whether the answer must be greater or less than the middle term; and if the answer must be greater, the least extreme is the divisor; but if the answer must be less than the middle term, the greatest extreme is the divisor.

R U L E III.

Place the divisor on the left hand, and the other extreme on the right; then multiply the second and third terms, and divide their product by the first; and the quot gives the answer; which is always of the same name with the middle term.

When the divisor happens to be the extreme found in the supposition, the proportion is called *direct*; but when the divisor happens to be the extreme in the demand, the proportion is *inverse*.

The three rules delivered above are indeed so framed, as to preclude the distinction of direct and inverse, or render it needless, the left-hand term being always the divisor; but yet the direct questions being plainer in their own nature, and more easily comprehended by a learner, I shall, in the first place, exemplify the rules by a set of questions of the direct kind, and shall afterwards adduce a collection of such as are inverse,

I. *The simple Rule of Three direct.*

Quest. 1. If 4 yards cost 12 shillings, what will 6 yards cost at that rate?

The supposition and demand of this question have already been distinguished,

distinguished, and the two terms in the former are 4 yards 12 shillings, and the only term in the latter is 6 yards.

The number sought is the price of 6 yards, and the term in the supposition of the same kind is the price of 4 yards, viz. 12 shillings, which I place in the middle, as directed in Rule I. and the two remaining terms are extremes, and of the same kind, viz. both lengths.

It is easy to perceive that the answer must be greater than the middle term; for 6 yards will cost more than 4 yards; therefore the least extreme, viz. 4 yards, is the divisor, according to Rule II.

$$\begin{array}{rcl}
 \text{yds.} & \text{s.} & \text{yds.} \\
 \text{If } 4 & : 12 & :: 6 \\
 & 6 & \\
 & \text{—} & \\
 & 4)72 & (18 \text{ shillings } \textit{Ans.} \\
 & 4 & \\
 & \text{—} & \\
 & 32 & \\
 & 32 & \\
 & \text{—} & \\
 & (0) & \text{£ } 18 \text{ } 0
 \end{array}$$

Wherefore I place the divisor 4 yards on the left hand, and the other extreme 6 yards on the right; and multiplying the second and third terms, I divide their product by the first term, and the quot 18 is the answer, and of the same name with the middle term, viz. shillings, according to Rule III.

And because the divisor is the extreme found in the supposition, the proportion is direct.

Quest. 2. If 7 C. of pepper cost 21 l. how much will 5 C. cost at that rate?

The supposition in this question is, that 7 C. of pepper costs 21 l. and the two terms in it are 7 C. and 21 l.; the demand is, How much will 5 C. cost? and the term in it is 5 C.

The number sought is the price of 5 C. and the term in the supposition of the same kind is the price of 7 C. viz. 21 l. which I place in the middle. The two remaining terms are extremes, and of the same kind, viz. quantities of pepper.

It is obvious, that the answer must be less than the middle term; for 5 C. will cost less than 7 C.; and therefore the greatest extreme, viz. 7 C. is the divisor.

$$\begin{array}{rcl}
 \text{C.} & \text{L.} & \text{C.} \\
 \text{If } 7 & : 21 & :: 5 \\
 & 5 & \\
 & \text{—} & \\
 & 7)105 & (15 \text{ l. } \textit{Ans.} \\
 & 7 & \\
 & \text{—} & \\
 & 35 & \\
 & 35 & \\
 & \text{—} & \\
 & (0) &
 \end{array}$$

Accordingly I place the divisor 7 C. on the left hand, and the other extreme 5 C. on the right; and having multiplied the second and third terms, I divide their product by the first term, and the quot 15 is the answer, of the same name with the middle term, viz. L. Sterling.

And because the divisor happens to be the extreme in the supposition, the proportion is direct.

Quest. 3. If 13 yards of velvet cost L. 21, what will 27 yards cost at that rate?

<i>T.</i>	<i>L.</i>	<i>T.</i>	
If 13 :	21 ::	27	* Rem. 4 s.
	27		12
	—		—
	147		13)48(3 d.
	42		39
	—		—
	13)567(43 L.		Rem. 9 d.
	52		4
	—		—
	47		13)36(2 f.
	39		26
	—		—
	Rem. 8 L.		Rem. 10 f.
	20		
	—		
	13)160(12 s.		
	13		
	—		
	30	<i>Ans.</i> 43	<i>L. s. d. f.</i>
	26	12	3 2 ¹⁰ / ₁₃
	—		

* Rem. 4 s.

Such remainders are always of the same name with the preceding part of the quot. Thus, the first remainder 8, and the first part of the quot 43, are both pounds; and the second remainder 4, and the second part of the quot 12, are both shillings; and the third remainder 9, and the third part of the quot 3, are both pence; and the fourth remainder 10, and the fourth part of the quot 2, are both farthings.

As we have no money under farthings, the last remainder cannot be reduced any lower; so there remains 10 farthings to be divided by 13; that is, there is wanting to complete the quot the thirteenth part of 10 farthings, or the thirteenth part of every remaining farthing; that is, ten thirteen parts of one farthing; so I set the remainder 10 above and the divisor 13 below a line drawn between them, in the form of a fraction, of which the remainder is the numerator and the divisor the denominator.

Quest. 4. If 14 lb. of tobacco cost 27 shillings, what will 478 lb. cost at that rate?

If

lb. s. lb.
If 14 : 27 :: 478

27
—
3346
956
— 2|0)
14)12906(92|1
12600
— L. 46 2

Here the middle term being shillings, the first part of the quot is also shillings; which I divide by 20, and so reduce it to pounds.

There remains at last 2 farthings, to be divided by 14; so I subjoin these by way of fraction to the preceding part of the quot; as in the former example.

L. s. d. f.
Ans. 46 1 10 1 $\frac{3}{4}$.

30
28
—
26
14
—

Rem. 12s.

12

14)144(10d.
14

Rem. 4d.

4

14)16(1f.
14

Rem. 2f.

Quest. 5. If 15 ounces of silver be worth L. 3, 15 s. what are 86 ounces worth at that rate?

If the middle term be complex, or mixt, and so consist of two or more parts, reduce it to the lowest, and the answer will come out of the same name with that of the lowest part. Thus,

The middle term here is complex, consisting of two parts, viz. pounds and shillings; so I reduce it to shillings, and the quot comes out in shillings; which I reduce to pounds.

oz. L. s. oz.
If 15 : 3—15 :: 86

20

—

75 shill.

86

—

450

600

— 2|0
15)6450(43|0 shillings,
6000

— L. 21 10

45

45

—

(0)

L. s.
Ans. 21 10

The

The parts of a complex term may be distinguished from one another by a stroke; as in this example.

Quest. 6. If 18 C. sugar cost L. 54, what will 7 C. 3 Q. 14 lb. cost at that rate?

C. L. C. Q. lb.
If 18 : 54 :: 7—3—14

18 7
18 7
18 798
—
2016 lb. 882 lb.

54

3528

4410

2016) 47628 (23 L.

4032

7308

6048

Rem. 1260 L.

20

2016) 25200 (12 s.

2016

5040

4032

Rem. 1008 s.

12

2016) 12096 (6 d.

12096

(0)

L. s. d.

Ans. 23 12 6

Quest. 7. If C. 3 : 1 : 14 of raisins cost L. 10 : 2 : 6, what will 6 C. 3 Q. cost at that rate?

1f

C. q. lb.	L. s. d.	C. q.
If 3—1—14 :	10—2—6 ::	6—3
3	20	6
3	—	6
342	202	684
—	12	—
378 lb.	—	756 lb.
	243 ^o d.	
	756	

Here all the terms are complex, and the two extremes are both reduced to lb. that being the lowest of the parts.

The middle term, for the like reason, is reduced to pence, and accordingly the quot or answer comes out in pence.

1458		
1215		
1701		
—	12)	210)
378)1837080	(4860	(4015
1512...	48..	—
—	—	L. 20 5
3250	60	
3024	60	
—	—	
2268	(0)	
2268		
—	Ans. L. 20 5.	
(0)		

Quest. 8. A goldsmith bought a wedge of gold, which weighed 14 lb. 3 oz. 8 dw. for the sum of L. 514, 4 s. : I demand what it stood him *per ounce* ?

lb. oz. dw.	L. s.	oz.
If 14—3—8 :	514—4 ::	1
12	20	20
—	—	—
171 oz.	10284 shill.	20 dw.
20	20	
—	—	210)
dw. 3428	3428)205680	(610 shill.
	20568	
	—	L. 3.
	(0)	

Ans. L. 3 per ounce.

Here the left-hand extreme being complex, both it and the other extreme are reduced to penny-weights, the lowest of the parts.

The middle term, being likewise complex, is reduced to shillings; and so the answer comes out in shillings; which I reduce to pounds.

Quest. 9.

Quest. 9. A grocer bought 4 hogheads of sugar, each weighing neat C. 6 : 2 : 14, which cost him L. 2 : 8 : 6 per C. : Required the value of the 4 hogheads at that rate?

C.	q.	lb.	lb.	L.	s.	d.	lb.
6	2	14	If 112	2	8	6	2968
		4			20		582
26	2			48			5936
26				12			23744
26							14840
2656				582 d.			12) 1727376
2968 lb. in 4 hhds.							(15423 d. 3
							112) 1727376
							112) 1727376
							2) 1012815
							607
							560
							473
							448
							257
							224
							336
							336
							(0)

When part of a term only is given, and the term to be completed by multiplication, it is generally convenient to complete the term, or even to reduce it, before you state the question.

Quest. 10. If 4s. 6d. is the price of 1 yard, how many yards will L. 16, 4s. buy?

s.	d.	yd.	L.	s.
If 4	6	1	16	4
2			20	
9			324	2
			9) 648	(72 y.
			63	
			18	
			18	
			(0)	

When the middle term happens to be 1, or unity, in this case, as 1 neither multiplies nor divides, you have only to divide the third term by the first, and the quot is the answer.

Ans 72 yards.

In this example both extremes are complex, and reduced to sixpences.

Quest. 11.

Quest. 11. If the yearly rent of a house be L. 73, how much is that per day?

Days. L. day.
If 365 : 73 :: 1

When the third term happens to be 1, you have only to divide the middle term by the first, and the quot is the answer.

20
365)1460(4 s.
1460
—
(0)

Ans. 4 s. per day.

But if in this case the middle term be less than the first, it must be reduced to some lower denomination, viz. till the middle term thus reduced may be divided by the first.

Quest. 12. A merchant bought 14 pieces of broad cloth, each piece containing 28 yards, at 13 s. 6½ d. per yard: How much did the 14 pieces cost?

	yd.	s.	d.	Yds.
14 pieces.	28	13	6½	392
	—	12		325
	112	162		1960
	28	2		784
	—			1176
392 yds. in 14 p.	325			210
				24)127400(530 8 s.
				120000
				— L. 265 8
				74
				72
				—
				200
				192
				—

Ans. L. 265 8 4

Rem. (8) halfpence.

When the first term happens to be 1, the product of the second and third is the answer.

The middle term here being complex, is reduced to halfpence; and accordingly the product, or answer, comes out in halfpence, which I reduce to pounds.

Quest. 13. A draper buys 50 pieces of kersey, each piece containing 34 ells Flemish, to pay at the rate of 8 s. 4 d. per ell English: How much did the 50 pieces cost?

If

<i>qrs.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>	
If 5	: 8—4	::	5100	
	12		100	
100			5)510000	34
				50
			12)102000	1700 ells Flemish.
				3
			2 0)850 0	5100 quarters.
<i>Ans.</i> 425 l.				

Quest. 14. What is the yearly interest of L. 78 : 16 : 6 at 5 per cent.?

<i>Prin.</i>	<i>int.</i>	<i>Prin.</i>	
<i>L.</i>	<i>L.</i>	<i>L. s. d.</i>	
If 100	: 5	::	78—16—6
			5
			1 00)3 94—2—6(3 18 9 3 ⁶ / ₁₀ <i>Ans.</i>
			20
			)18 82(
			12
			)9 90(
			4
			3 60 = 3 ⁶ / ₁₀ f.

MORE EXAMPLES.

Quest. 15. If 17 yards of cloth cost L. 19 : 2 : 6, what will 35 yards cost at that rate? *Ans.* L. 39 : 7 : 6.

Quest. 16. If 24 lb. of raisins cost 6s. 6d. what will 18 fraills cost, each frail weighing 3 Q. 18 lb.? *Ans.* L. 24 : 17 : 3.

Quest. 17. If 40 pieces of cloth cost L. 750, 15s. what is the price of 1 piece? *Ans.* L. 18 : 15 : 4¹/₂.

Quest. 18. If 1 ell of cloth cost 8s. 4d. how many ells will L. 10 : 16 : 8 buy? *Ans.* 26 ells.

Quest. 19. A gentleman expends daily L. 5, 15s. : How much is that in a year? *Ans.* L. 2098, 15s.

Quest. 20. A merchant buys cloth to the value of L. 537, 12s. at 5s. 4d. per yard : How many yards did he buy? *Ans.* 2016 yards.

Quest. 21. If I sell 14 yards for L. 10, 10s. how many ells Flemish shall I sell for L. 283 : 17 : 6, at that rate? *Ans.* 504²/₇ ells Flemish.

Quest. 22.

Quest. 22. What is the price of 14 ingots of silver, each ingot weighing 7 lb. 5 oz. 10 dw. at 5s. *per* ounce? *Ans.* L. 313, 5s.

Quest. 23. A grocer mingles 17 C. 3 Q. 14 lb. of sugar, at L. 4, 4s. *per* C. with 8 C. 3 Q. 21 lb. other sugar, at 6d. *per* lb.: What is 1 C. of this mixture worth? *Ans.* L. 3 : 14 : 8.

Quest. 24. A draper buys 8 packs of cloth, each pack containing 4 parcels, and each parcel 10 pieces, and each piece 26 yards, for which he paid at the rate of L. 4, 16s. for 6 yards: How much did the whole cost? *Ans.* L. 66½.

Quest. 25. A merchant bought 242 yards of broad cloth, which cost him in all L. 201 : 12 : 8; for 86 yards of which he paid at the rate of 13s. 4d. *per* yard: How much did he give *per* yard for the rest? *Ans.* 18s. 6d. *per* yard.

Quest. 26. An oilman bought 3 tuns of oil at the rate of L. 50, 11s. 4d. *per* tun; and so it happened that it leaked out 85 gallons; but he is minded to sell it again, so as to be no loser: How must he sell it *per* gallon? *Ans.* At 4s. 6 $\frac{1}{8}$ d. *per* gallon.

Quest. 27. A shopkeeper bought 4 bales of cloth, each bale containing 6 pieces, and each piece 27 yards, at L. 16, 4s. *per* piece: What was the price of the whole? and what the rate *per* yard? *Ans.* The whole cost L. 388, 16s. and the rate *per* yard was 12s.

Quest. 28. A bankrupt, who owed L. 1490 : 5 : 10, compounds with his creditors for 12s. 6d. *per* L.: How much did he pay in all? *Ans.* L. 931 : 8 : 7 $\frac{1}{4}$.

Quest. 29. If the rent of a small estate be L. 250, what will this afford to spend every calendar month, in order to save, or lay up, L. 80 at the year's end? *Ans.* L. 14 : 3 : 4.

Quest. 30. The rents of a whole parish amount to L. 3500, and a rate is granted of L. 131, 5s.: What is that *per* pound? *Ans.* 9d.

Quest. 31. Received 42 C. 2 Q. cotton, at L. 3, 15s. *per* C. and 12 lb. cloves, at 9s. 1d. *per* lb.; for which I have given in part 1000 yards linen, at 2s. 9d. *per* yard: What balance do I owe? *Ans.* L. 27 : 6 : 6.

Quest. 32. A chapman bought a parcel of serge and shalloon, which together cost him L. 61 : 9 : 2; the quantity of serge he bought was 236 yards, at 3s. 4d. *per* yard; and for every 2 yards of serge he had 3 yards of shalloon: How much shalloon had he; and what did it cost him *per* yard? *Ans.* 354 yards of shalloon, at 15 d. *per* yard.

II. The simple Rule of Three inverse.

Quest. 1. If 8 men can do a piece of work in 12 days, in how many days will 16 men do the same?

In this question the supposition is, If 8 men do a piece of work in 12 days, and the two terms contained in it are 8 men and 12 days: The demand lies in these words, In how many days will 16 men do the same? and the only term contained in it is 16 men.

The

The number sought here is the days in which 16 men will do the work, and the term in the supposition of the same kind is 12 days; wherefore I place 12 days as the middle term, according to Rule I. the two remaining terms are extremes, and of the same kind, *viz.* both of them men.

It is obvious that the answer must be *men. days. men.*
 less than the middle term; for 16 men $16 : 12 :: 8$
 will do the work in fewer days than 8 8
 men; and therefore, by Rule II. the $—$
 greatest extreme, *viz.* 16, is the divisor; $16)96(6 \text{ days. } \textit{Ans.}$
 which I place on the left hand, and the 96
 other extreme on the right, as directed $—$
 in Rule III. Then multiplying the se- (0)
 cond and third, and dividing their pro-
 duct by the first, the quot comes out in days; that is, of the same
 name with the middle term.

And because the extreme found in the demand happens to be the divisor, the proportion is inverse.

Quest. 2. If 30 yards of cloth that is 5 quarters broad, be required to hang a bed, how many yards of 3 quarters broad will serve the same purpose?

The supposition here is, that 30 yards of cloth 5 quarters broad are sufficient to hang a bed, and the two terms in it are 30 yards of length, and 5 quarters of breadth: The demand is, How many yards of cloth that is 3 quarters broad will hang the same bed? and the only term in it is 3 quarters of breadth.

The thing sought is the length or number of yards of cloth 3 quarters broad requisite to hang the bed, and the term in the supposition of the same kind, is 30 yards, which therefore I place in the middle; the two remaining terms are extremes, and of the same kind, *viz.* both breadths.

It is easy to perceive that the *breadth. length. breadth.*
 answer must be greater than the *grs. yds. grs.*
 middle term; for more yards of $3 : 30 :: 5$
 cloth 3 quarters broad will be 5
 necessary to hang a bed than of $—$
 cloth 5 quarters broad, and the $3)150(50 \text{ yards. } \textit{Ans.}$
 narrower the cloth is the more 15
 length will be required; where- $—$
 fore the least extreme is the divi- (0)
 for, which I place on the left
 hand, and set the greater extreme on the right.

And because the extreme found in the demand is the divisor, the proportion is inverse.

Quest. 3. If the penny-loaf weigh 8 ounces, when flour is sold at 2s. or 24d. *per* peck, what ought to be the weight of the penny-loaf when flour is sold at 18d. *per* peck?

It

$$\begin{array}{rcl} d. & oz. & d. \\ 18 & : 8 :: & 24 \\ & & 8 \end{array}$$

$$\begin{array}{r} 18 \overline{) 192} (10 \text{ oz.} \\ 18 \cdot \end{array}$$

$$\text{Rem. } 12 \text{ oz.}$$

$$20$$

$$\begin{array}{r} 18 \overline{) 240} (13 \text{ dw.} \\ 18 \end{array}$$

$$60$$

$$54$$

$$\text{Rem. } 6 \text{ dw.}$$

$$24$$

$$18 \overline{) 144} (8 \text{ gr.}$$

$$144$$

$$(0)$$

Ans. 10 oz. 13 dw. 8 gr.

Quest. 4. If 1200lb. weight be carried 36 miles for a sum of money, how many miles ought 1800lb. weight to be carried for the same sum?

$$\begin{array}{rcl} lb. & miles, & lb. \\ 1800 & : 36 :: & 1200 \\ & & 1200 \end{array}$$

$$18 \overline{) 00432} \overline{) 00} (24 \text{ miles. } \text{Ans.}$$

$$36 \cdot$$

$$72$$

$$72$$

$$(0)$$

Here the answer must be less than the middle term; because the heavier the load is, the fewer miles ought it to be carried for the same money; and so the greater extreme is the divisor, and placed on the left hand.

Quest. 5. If L. 100 in 12 months gain a sum of interest, what principal will gain the same sum of interest in 8 months?

$$\begin{array}{rcl} m. & L. & m. \\ 8 & : 100 :: & 12 \end{array}$$

$$100$$

$$8 \overline{) 1200}$$

$$\text{Ans. } 150 \text{ L.}$$

Quest. 6.

Here the answer must be greater than the middle term; because the fewer the months are, the more principal will be required to gain the same sum of interest, and therefore the least of the extremes is the divisor.

Quest. 6. If 40 poles in length, and 4 in breadth, make an acre, what must be the length to make an acre when the breadth is 15 poles?

breadth. length. breadth.

15 : 40 :: 4

4

15)160(10 poles.

15

Rem. 10 poles.

33 half-feet in 1 pole.

— 2)

15)330(22 half-feet.

30 —

— 11 feet.

30

30

(0)

Here the answer must be less than the middle term; because the broader a piece of ground is, the less length is required to make the acre; and so the greatest of the extremes is the divisor.

poles. feet.
Ans. 10 11

Quest. 7. How much plush of 3 quarters wide will line a cloak that hath in it 4 yards of 7 quarters wide?

Q. yd. Q.

3 : 4 :: 7

7

3)28(9 $\frac{1}{3}$ yards. *Ans.*

27

(1)

Here the answer must be greater than the middle term; for the plush being narrower than the cloth of which the cloak is made, will require more length.

Quest. 8. If 36 yards be a rood of mason-work, at 3 feet high, how many yards will make a rood at 9 feet high?

Feet. yards. feet.

9 : 36 :: 3

3

9)108

Ans. 12 yards.

MORE EXAMPLES.

Quest. 9. If a messenger makes a journey in 24 days when the day is 12 hours long, in how many days may he make out the same journey when the day is 16 hours long? *Ans.* in 18 days.

Quest. 10. If 360 men be in garrison, and have provisions only for

for 6 months; how many men must be turned out, that the same stock of provisions may last 9 months? *Ans.* 240 men are to be retained, and the rest, *viz.* 120, must be turned out.

Quest. 11. If I have 1200 lb. weight carried 36 miles for a sum of money, how many pound weight shall I have carried 24 miles for the same sum? *Ans.* 1800 lb.

Quest. 12. If I borrow of a friend L. 64 for 8 months, what sum ought I to lend him for 12 months in order to requite the favour? *Ans.* L. 42 : 13 : 4.

Quest. 13. A fabric is reared in 8 months by 120 workmen; and another fabric, on the same plan, and of the same dimensions, is to be reared in 2 months: How many workmen must be employed? *Ans.* 480.

Quest. 14. A regiment of soldiers, consisting of 1000 men, are to have new coats, each coat containing 3 yards 2 quarters of cloth that is 6 quarters wide, and they are to be lined with shalloon that is yard wide: How many yards of cloth will the coats take? and how many yards of shalloon will line them? *Ans.* 3500 yards of cloth, and 5250 yards of shalloon.

Quest. 15. How many yards of paper that is 3 quarters wide will be sufficient to line a room that is 24 yards round and 4 yards high? *Ans.* 128 yards.

Quest. 16. If I lend my friend L. 650 for 22 months, how long ought he to lend me L. 953 : 6 : 8 to requite my kindness? *Ans.* 15 months.

Quest. 17. If 50 horses are maintained a year in corn for a certain sum, when oats are sold at 9s. *per* boll, how many horses may be maintained a year in corn for the same sum, when oats are at 10s. *per* boll? *Ans.* 45 horses.

Quest. 18. If the penny-loaf weigh 10 oz. when wheat is sold at 12s. *per* bushel, what ought to be the price of wheat when the penny-loaf weighs 8 oz.? *Ans.* 15s. *per* bushel.

Quest. 19. If 12 inches in length and 12 inches in breadth make a square foot, what length of a plank that is 9 inches broad will be equal to a square foot? *Ans.* 16 inches.

Quest. 20. If 40 poles in length and 4 poles in breadth make an acre, what must be the breadth to make an acre when the length is only 32 poles? *Ans.* 5 poles.

S E C T. II.

The Compound Rule of Three.

THE Compound Rule of Three, from five given numbers finds a sixth, or from seven given numbers finds an eighth, or from nine given numbers finds a tenth, or from eleven finds a twelfth, &c.

This rule easily and naturally admits of subdivisions, which,

I

from

from the number of the terms given, may be denominated the rule of Five, the rule of Seven, the rule of Nine, the rule of Eleven, &c.

Questions in the compound rule of three are also resolved into two parts, *viz.* a supposition and a demand.

If five terms be given, three of these are always found in the supposition, and two in the demand; if seven terms be given, four of these are in the supposition, and three in the demand; if nine terms are given, five of these are in the supposition, and four in the demand; if eleven terms be given, six of these are in the supposition, and five in the demand, &c.

The supposition and demand being distinguished, proceed to state the question; that is, to put the terms in due order for operation, as the following rules direct.

R U L E I.

Place that term of the supposition which is of the same kind with the number sought, in the middle. The remaining terms are extremes, which must be classed into similar pairs, by making each pair consist of one term taken from the supposition, and another of the same kind taken from the demand.

R U L E II.

Out of each similar pair, joined with the middle term, form a simple question; and in each simple question, so formed, find the divisor; *viz.* consider from the nature of the question, whether the answer must be greater or less than the middle term; and if the answer must be greater, the least extreme is the divisor; but if the answer must be less than the middle term, the greatest extreme is the divisor.

R U L E III.

Place all the divisors on the left hand, and the other extremes on the right; then multiply the divisors, or extremes on the left, continually, for a divisor, and multiply the extremes on the right hand and the middle term, continually, for a dividend; and lastly, divide the dividend by the divisor; and the quot is the answer, of the same name with the middle term.

The answer to questions in the compound rule of three may also be had by working the simple questions separately, or by themselves, in the following manner, *viz.*

The middle term, with any one pair of similar extremes, make the first simple question, and the answer to this question must be made the middle term to the next similar pair of extremes; and the answer to this second question must, in like manner, be made the middle term to the following similar pair of extremes, &c.; and the answer to the last simple question is the number sought.

But the joint operation prescribed in Rule III. is the shorter as well

well as the easier method; for in working some of the simple questions, there may happen to be a remainder, and consequently the middle term of the next simple question will have some fractional part; which inconveniency is avoided by working jointly.

In every simple question, when the divisor is an extreme found in the supposition, the proportion is direct: but when the divisor is an extreme found in the demand, the proportion is inverse.

The three rules delivered above are indeed so calculated, as to make no difference betwixt direct and inverse, or so as to render that distinction needless, the left-hand extremes being all divisors; but yet, as questions consisting entirely of direct proportions are the plainest and easiest, it will be proper, in the first place, to exemplify the rules by questions of the direct kind, and afterwards introduce such as are inverse.

And as questions in the rule of five are by far more numerous, and occur much oftener, than questions in the rule of seven, nine, or eleven; I shall, first of all, adduce a set of questions in the rule of five, wherein both proportions are direct; then propose another set, wherein one or both proportions are inverse; and, lastly, give a few examples of the rules of seven, nine, and eleven.

I. *The Rule of five direct.*

Quest. 1. If 14 horses eat 56 bushels of corn in 16 days, how many bushels will 20 horses eat in 24 days?

The supposition in this question is, If 14 horses eat 56 bushels in 16 days; and the three terms contained in it are, 14 horses, 56 bushels, and 16 days: The demand is, How many bushels will 20 horses eat in 24 days? and the two terms contained in it are 20 horses, and 24 days.

The number sought is bushels, and the term in the supposition of the same kind is 56 bushels; wherefore, according to Rule I. I place 56 bushels in the middle. The remaining four terms are extremes, which I class into similar pairs, by making each pair consist of one term taken from the supposition, and another of the same kind taken from the demand. Thus, 14 horses and 20 horses make one pair; again, 16 days and 24 days make another pair.

Out of the several similar pairs, joined with the middle term, I form so many simple questions, according to Rule II. *viz.* I say,

1. If 14 horses eat 56 bushels in a certain number of days, how many bushels will 20 horses eat in the same time?

2. If 16 days eat up, or consume, 56 or any other number of bushels, how many bushels will 24 days consume?

In the first simple question it is obvious, that the answer will be greater than the middle term; for 20 horses will eat more bushels than 14 horses will do in the same time; and so the least extreme, *viz.* 14, is the divisor; and because 14 is an extreme found in the supposition, the proportion is direct.

In the second simple question it is also plain, that the answer will be greater than the middle term; for 24 days will consume more bushels than 16 days; and consequently the least extreme, viz. 16, is the divisor; and because 16 is an extreme found in the supposition, the proportion is direct.

According to Rule III. I place the divisors on the left hand, and the other extremes on the right, and both of them under one another; so that the two upper ones make a pair, or be of one kind, and the two lower ones make another pair, or be of one kind; and no matter which of the pairs be uppermost: then I multiply the divisors, or the extremes on the left hand, for a divisor; and again I multiply the extremes on the right, and the middle term, continually, for a dividend; and dividing the dividend by the divisor, the quot, or answer, comes out of the same name with the middle term, viz. 120 bushels.

Joint operation.

<i>Horses.</i>	<i>bushels.</i>	<i>Horses.</i>
If 14 :	56 ::	20
da. 16		24 da.
—		—
84		480
14		56
—		—
224		288
		240
		—
	224)26880(120	
	224	
	—	
	448	
	448	
	—	
	(0)	

Ans. 120 bushels.

The two simple questions into which the compound question is resolved, are stated, and wrought separately, as follows.

<i>H.</i>	<i>B.</i>	<i>H.</i>
If 14 :	56 ::	20
	20	
—		—
14)1120(80 B.		
112		
—		
(0)		

Ans. 120 bushels, as before.

<i>Days.</i>	<i>B.</i>	<i>Days.</i>
If 16 :	80 ::	24
	80	
—		—
16)1920(120 B.		
160		
—		
32		
32		
—		
(0)		

Quest. 2. If 40 acres of grass be cut down by 8 men in 7 days, how many acres shall be cut down by 24 men in 28 days?

Joint

Joint operation.

<i>Men.</i>	<i>acres.</i>	<i>Men.</i>
If 8	: 40	:: 24
da. 7		28 da.
<u>56</u>		<u>192</u>
		48
		<u>672</u>
		40

56)26880(480 Acres. *Ans.*224

448

448

(0)

The same resolved into two simple questions, and wrought separately.

<i>M.</i>	<i>a.</i>	<i>M.</i>	<i>Days.</i>	<i>a.</i>	<i>Days.</i>
If 8	: 40	:: 24	If 7	: 120	:: 28
	<u>40</u>			<u>120</u>	
8)960			7)3360		

120 acres.

Ans. 480 acres.

Quest. 3. If L. 100 in 12 months gain L. 5 interest, what will L. 75 gain in 9 months?

Joint operation.

<i>Pr.</i>	<i>int.</i>	<i>Pr.</i>
<i>L.</i>	<i>L.</i>	<i>L.</i>
If 100	: 5	:: 75
months 12		9 months.
<u>1200</u>		<u>675</u>
		5

L. s. d.12|00)33|75(2 16 3. *Ans.*24

975 L. rem.

20

)195|00(

12

75

72

3 s. rem.

12

)36(

36

(0)

1 3

The

The same question resolved into two simple ones, and wrought separately.

$$\begin{array}{rcl} \text{Pr.} & \text{int.} & \text{Pr.} \\ \text{L.} & \text{L.} & \text{L.} \\ \text{If } 100 : 5 :: 75 \end{array}$$

$$\begin{array}{r} \text{L. s.} \\ 1|00) 3|75 (3-15 \\ \underline{20} \\ 15|00 \end{array}$$

$$\begin{array}{rcl} \text{Mo.} & \text{L.} & \text{s.} & \text{Mo.} \\ \text{If } 12 : 3-15 :: 9 \end{array}$$

$$\begin{array}{r} \text{L. s. d.} \\ 12)675 (5|6 (2 \text{ } 16 \text{ } 3 \\ \underline{60} \quad \underline{4} \\ 75 \quad 16\text{s.} \\ 72 \\ \underline{\quad} \end{array}$$

$$\begin{array}{r} \text{Rem. } 3\text{s.} \\ 12 \\ \underline{\quad} \\)36(3\text{d.} \\ \underline{36} \\ (0) \end{array}$$

Quest. 4. If a carrier receive 42 shillings for the carriage of 3 C. weight 150 miles, how much ought he to receive for the carriage of 7 C. 3 Q. 14 lb. 50 miles?

Before the question is stated, it will be convenient to reduce one of the pairs as follows.

$$\begin{array}{rcl} \text{C.} & & \text{C. Q. lb.} \\ 3 & & 7-3-14 \\ 3 & & 7 \\ 3 & & 7 \\ 3 & & 7 \\ \underline{3} & & \underline{7} \\ 336\text{lb.} & & 798 \end{array}$$

882lb.

Joint

Joint operation.

If 336 : 42 :: 882
miles 150 50 miles.

1680	44100
336	42
50400	882

1764

504|00)18522|00(36—9. Ans.
1512.

3402
3024

Rem. 378 shill.

12

4536(9d.

4536

(0)

The same question resolved into two simple ones, and wrought separately.

lb. s. lb.
If 336 : 42 :: 882

42

1764

3528

s. d.

336)37044(110—3

336..

344

336

Rem. 84 shill.

12

1008(3d.

1008

(0)

m. s. d. m.
If 150 : 110—3 :: 50

12

1323

50

12)

15|0)6615|0(441

60..

36—9

61

60

15

15

(0)

s. d.
Ans. 36—9

MORE EXAMPLES.

Quest. 5. If a regiment of 936 soldiers eat up 351 quarters of wheat in 168 days, how many quarters of wheat will 11,232 soldiers eat in 56 days at that rate? *Ans.* 1404 quarters.

Quest. 6. If 48 bushels of corn yield 576 bushels in one year, how much will 240 bushels yield in 6 years at that rate? *Ans.* 17,280 bushels.

Quest. 7. If 40 shillings be the wages of 8 men for 5 days, what will be the wages of 32 men for 24 days? *Ans.* 768 shillings, or L. 38, 8s.

Quest. 8. If 8 cannons in 1 day spend 48 barrels of powder, how many barrels will 24 cannons spend in 12 days at that rate? *Ans.* 1728 barrels.

Quest. 9. An usurer receives L. 3 : 7 : 6 as the interest of L. 75 for 9 months : How much is that *per cent. per annum*? *Ans.* 6 *per cent.*

Quest. 10. If 18 roods of ditching be done by 3 men in 6 days, how many roods will be done by 8 men in 4 days at the same rate of working? *Ans.* 32 roods.

Quest. 11. If 4 students spend L. 19 in 3 months, how much will 8 students spend in 9 months? *Ans.* L. 114.

Quest. 12. If 600 seamen in 1 week eat 1500 lb. of beef, how much will serve 120 seamen 12 weeks? *Ans.* 3600 lb.

II. The Rule of Five inverse.

The questions that fall under this rule have commonly one of the proportions inverse, and the other direct, and sometimes the upper, and sometimes the lower, is the inverse proportion; and in some few questions both proportions are inverse. Now, though the three rules delivered above make no difference betwixt direct and inverse; yet, to bring the learner to some measure of acquaintance with this useful distinction, I shall, in stating the following questions, expose the same to view, by affixing an asterisk to the extremes of every inverse proportion.

Quest. 1. If 14 horses eat 56 bushels of corn in 16 days, in how many days will 20 horses eat 120 bushels at that rate?

In this question the supposition is, that 14 horses eat 56 bushels in 16 days; and the demand is, In how many days 20 horses will eat 120 bushels?

The number sought is days, and the term in the supposition of the same kind is 16 days; and accordingly I place 16 days in the middle. The remaining four terms are extremes; which I class into similar pairs, by making each pair consist of one term taken from the supposition, and another of the same kind taken from the demand.

demand. Thus, 14 horses and 20 horses make one pair; again, 56 bushels and 120 bushels make another pair.

Out of the similar pairs, joined with the middle term, I form so many simple questions; namely,

1. If 14 horses eat a certain number of bushels in 16 days, in how many days will 20 horses eat the same quantity?

2. If 56 bushels are eat up in 16 days, in how many days will 120 bushels be eat up by the same eaters?

In the first simple question it is plain, that the answer must be less than the middle term; for 20 horses will eat the same number of bushels in fewer days than 14 horses; and so the greatest extreme, *viz.* 20, is the divisor; and because 20 is an extreme found in the demand, the proportion is inverse.

In the second simple question it is also obvious, that the answer must be greater than the middle term; for 120 bushels will require more days to be eat up in, than 56 bushels; and therefore the least extreme, *viz.* 56, is the divisor; and because 56 is an extreme found in the supposition, the proportion is direct.

I now proceed to state the question, by placing the divisors on the left hand, and the other extremes on the right; then I multiply and divide, as directed in R. III. and the answer comes out of the same name with the middle term, *viz.* 24 days.

Joint operation.		
Hors.	day.	Hors.
* 20	: 16	:: 14 *
bush. 56		120 bush.
1120		1680
		16
		1008
		168
		days.
	112 0	2688 0(24. Ans.
		224
		448
		448
		(0)

The two simple questions into which the compound question is resolved, are stated and wrought separately, as follows:

Hors.

<i>Horf.</i>	<i>day.</i>	<i>Horf.</i>	<i>Busb.</i>	<i>D.</i>	<i>b.</i>	<i>m.</i>	<i>Busb.</i>
* 20	:	16	::	14	*		
		14					56 : 11-4-48 :: 120
		<u>64</u>					24
		16					<u>48</u>
							22
2 0	22	4	(11 days.				268
	24						60
	9 6	4	hours.				<u>16128</u>
	16						120
	60						6 0)
	96 0	48	min.	56	1935360	(3456 0	
				168			
						24)576	(24 days,
						255	48
						224	<u>96</u>
						313	96
						280	<u>(0)</u>
						336	
						336	
						<u>(0)</u>	

Quest. 2. If 40 acres of grafs be cut down by 8 men in 7 days, in how many days will 480 acres be cut down by 24 men?

Joint operation.

	<i>Acres.</i>	<i>days.</i>	<i>Acres.</i>
	40	:	7 :: 480
Men *	24		8 * Men.
	<u>960</u>		
			<u>3840</u>
			7
			<u>96 0</u> 2688 0
			(28 days. <i>Ans.</i>
			192
			<u>768</u>
			768
			<u>(0)</u>

The same question resolved into two simple questions, and wrought separately,

Acres.

Acres. days. Acres.

40 : 7 :: 480

7

4|0)336|0(84 days.

32

16

16

(0)

Men. days. Men.

* 24 : 84 :: 8 *

8

24)672(28 days. Ans.

48

192

192

(0)

Quest. 3. If 100l. principal, in 12 months, gain 5l. interest, what principal will gain L. 2 : 16 : 3 interest in 9 months?

Joint operation.

L.

L. s. d.

5

2—16—3

240

20

Prin.

Mon. L. Mon.

1200 d.

56

* 9 : 100 :: 12 *

12

Int. d. 1200

675 d. Int.

675 d.

10800

8100

100

108|00)8100|00(75 l. Ans.

756

540

540

(0)

The same question resolved into two simple questions, and wrought separately.

Mo. L. Mo.

* 9 : 100 :: 12 *

12

9)1200(133 l.

rem. 3

20

9)60(6 s.

54

6

12

9)72(8 d.

72

(0)

D.

L. s. d.

D.

1200 : 133—6—8 :: 675

20

2666

12

32000

675

1350

2025

12)

12|00)216000|00(18000

12

2|0)150|0

96

96

(000)

75 l. Ans.

Quest.

Quest. 4. If 12 inches of length, 12 of breadth, and 12 of thickness, make a solid foot, what length of a plank that is 6 inches broad and 4 inches thick will make a solid foot?

Joint operation,

$$\begin{array}{rcl} \text{Br. leng.} & \text{Br.} & \\ * 6 : 12 :: 12 * & & \\ \text{Th.} * 4 & 12 * \text{Th.} & \end{array}$$

24

144

12

24)1728(72 inches. *Ans.*
168.

48

48

(0)

The same question resolved into two simple questions, both inverse, and wrought separately.

Br. leng. Br.*

* 6 : 12 :: 12 *

12

6)144

24 inches.

Th. leng. Th.

* 4 : 24 :: 12 *

12

4)288

Ans. 72 inch. in length.

MORE EXAMPLES.

Quest. 5. If a carrier receive 42 shillings for the carriage of 3 C. weight 150 miles, how many miles ought he to carry C. 7 : 3 : 14 for 36s. 9d? *Ans.* 50 miles.

Quest. 6. If a regiment of 936 soldiers eat up 351 quarters of wheat in 168 days, how many soldiers will eat up 1404 quarters in 56 days at that rate? *Ans.* 11,232 soldiers.

Quest. 7. If 8 men in 5 days earn 40 shillings of wages, in how many days will 32 men earn 38l. 8s.? *Ans.* In 24 days.

Quest. 8. If 8 cannons in 1 day spend 48 barrells of powder, in how many days will 24 cannons spend 1728 barrells at that rate? *Ans.* In 12 days.

Quest. 9. If 75l. principal, in 9 months, gain L. 3 : 7 : 6 interest, in how many months will 100l. principal gain 6l. interest? *Ans.* In 12 months.

Quest. 10. If the penny-loaf weigh 12 ounces when wheat is sold at 3. 4d. per bushel, what ought to be the weight of a loaf worth 9d. when wheat is sold at 10s. per bushel? *Ans.* 36 ounces.

Quest.

Quest. 11. If one travel 300 miles in 10 days, when the day is 12 hours long, in how many days may he travel 600 miles, when the day is 16 hours long? *Ans.* In 15 days.

Quest. 12. If 48 pioneers, in 12 days, cast a trench 24 yards long, how many pioneers will cast a trench 168 yards long in 16 days? *Ans.* 252 pioneers.

III. The Rule of Seven, Nine, Eleven, &c.

Quest. 1. If 15 men eat 156 d. worth of bread in 6 days, when wheat is sold at 12s. per bushel, in how many days will 30 men eat 520 d. worth of bread, when wheat is at 10s. per bushel?

This question belongs to the rule of seven, the number sought is days, and the term of the same kind in the supposition is 6 days, which I place in the middle. The remaining six terms are extremes, which I class into similar pairs, by taking one term of each pair out of the supposition, and another of the same kind out of the demand.

Out of the similar pairs, joined with the middle term, I form so many simple questions, in each of which I find the divisor by Rule II; then I place the divisors on the left hand, and the other extremes on the right, as directed in Rule III. and multiply and divide, as follows:

$$\begin{array}{r}
 \text{Joint operation.} \\
 \text{Men. days. Men.} \\
 \begin{array}{l}
 \text{s.} \quad * 30 : 6 :: 15 * \quad \text{s.} \\
 * 10 : d. 156 \quad \quad \quad 520 d. : 12 * \\
 \hline
 4680 \quad \quad \quad 30 \\
 10 \quad \quad \quad 75 \\
 \hline
 46800 \quad \quad \quad 7800 \\
 \quad \quad \quad 12 \\
 \hline
 \quad \quad \quad 93600 \\
 \quad \quad \quad 6 \\
 \hline
 468 | 00) 5616 | 00 (12 \text{ days. } \text{Ans.} \\
 468 \cdot \\
 \hline
 936 \\
 936 \\
 \hline
 (0)
 \end{array}
 \end{array}$$

This compound question is resolved into three simple ones, as follows:

$$30 : 6 :: 15 : 3$$

$$156 : 3 :: 520 : 10$$

$$10 : 10 :: 12 : 12 \text{ days. } \text{Ans.}$$

Quest. 2. If 18 men build a wall 40 feet long 3 feet thick and 16 feet high in 12 days, how many men will build a wall 360 feet long 8 feet thick and 10 feet high in 60 days?

This question belongs to the rule of nine, and is stated and wrought in the same manner with the preceding question in the rule of seven, as follows:

Joint operation.

Length. men. Length.

d. h.

40 : 18 :: 360

h. d.

* 60 : 16 : Th. 3

8 Th. : 10 : 12 *

120

2880

16

10

72

28800

12

12

1920

345600

60

18

115200

27648

3456

1152|00)62208|00(54 men. *Ans.*

5760

4608

4608

(0)

This compound question is resolved into four simple ones, as follows:

$$40 : 18 :: 360 : 162$$

$$3 : 162 :: 8 : 432$$

$$16 : 432 :: 10 : 270$$

$$60 : 270 :: 12 : 54 \text{ men. } \text{Ans.}$$

Quest. 3. If 12 men cast a ditch 30 feet long, 6 deep, and 3 broad, in 15 days, when the day is 12 hours long; in how many days will 60 men cast a ditch 300 feet long, 8 deep, and 6 broad, when the day is but 8 hours long?

This question belongs to the rule of eleven, and is stated and wrought

wrought in the same manner with the preceding questions in the rules of seven and nine, as follows:

Joint operation.

Men. days. Men.

$$\begin{array}{ccccccc} \text{h.} & \text{b.} & \text{d.} & * & 60 & : & 15 & :: & 12 & * & & \text{d.} & \text{b.} & \text{h.} \\ * & 8 & : & 3 & : & 6 & : & 1. & 30 & & 300 & \text{l.} & : & 8 & : & 6 & : & 12 & * \end{array}$$

And $60 \times 30 \times 6 \times 3 \times 8 = 259200$ the divisor.

And $12 \times 300 \times 8 \times 6 \times 12 \times 15 = 31104000$ the dividend.

$2592 \overline{) 00} 311040 \overline{) 00}$ (120 days. *Ans.*

$2592 \cdot \cdot$

$\underline{5184}$

$\underline{5184}$

(0)

The work may be contracted by omitting the equal extremes, 6 and 8, found both on the left hand and the right, thus:

$60 \times 30 \times 3 = 5400$ the divisor.

And $12 \times 300 \times 12 \times 15 = 648000$ the dividend.

$54 \overline{) 00} 6480 \overline{) 00}$ (120 days. *Ans.*

$54 \cdot \cdot$

$\underline{108}$

$\underline{108}$

(0)

This compound question is resolved into five simple ones, as under:

$$60 : 15 :: 12 : 3$$

$$30 : 3 :: 300 : 30$$

$$6 : 30 :: 8 : 40$$

$$3 : 40 :: 6 : 80$$

$$8 : 80 :: 12 : 120 \text{ days. } \textit{Ans.}$$

MORE EXAMPLES.

Quest. 4. If 18 roods of ditching be wrought by 3 men in 16 days, when the day is 15 hours long; how many roods will be wrought by 8 men in 4 days, when the day is 9 hours long? *Ans.* $7\frac{1}{2}$ roods.

Quest. 5. If 12 men build a wall 30 feet long, 6 feet high, and 3 feet thick, in 15 days; in how many days will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick? *Ans.* In 80 days.

Quest.

Quest. 6. If 10 men, by working 6 hours a-day, can, in 4 days, dig a cellar that is 24 feet long, 16 wide, and 12 deep; how many men, by working 8 hours a-day, will, in 2 days, dig a cellar that is 32 feet long, 24 wide, and 8 deep? *Ans.* 20 men.

Contractions in the Rule of Three.

1. If the first term be an aliquot part of the second or third, divide the said second or third term by the first, and the product of the quot and the other term is the answer. Thus, if $9 : 27 :: 42$, then divide 27 by 9, and the quot $3 \times 42 = 126$, the answer. Again, if $8 : 5 :: 64$, divide 64 by 8, and the quot $8 \times 5 = 40$, the answer.

E X A M P L E.

A, B, C, and D, have L. 100 to be divided among them, in such manner, that for every L. 3 A receives, B must have 5, C 7, and D 10; that is, their shares are to be as the numbers 3, 5, 7, 10: Required their shares?

Add the proportional numbers, by saying, $3 + 5 + 7 + 10 = 25$; then say,

$$\text{If } 25 : 100 :: \begin{cases} 3 \times 4 = 12 \text{ A's share.} \\ 5 \times 4 = 20 \text{ B's share.} \\ 7 \times 4 = 28 \text{ C's share.} \\ 10 \times 4 = 40 \text{ D's share.} \end{cases}$$

100 Proof.

Because 25 measures 100, I multiply the quot 4 into all the extremes on the right hand, and the products are the answers or shares.

2. If the first term be a multiple of the second or third, divide the first term by the said second or third; and the remaining term, divided by the quot, gives the answer.

E X A M P L E I.

If 60 yards cost L. 20, what will 45 yards cost?

Here the first term 60 is a multiple of the second term 20, and $\frac{60}{20} = 3$, and $\frac{45}{3} = 15$ L. *Ans.*

E X A M P L E II.

If 18 yards cost L. 12, what will 6 yards cost?

Here the first term 18 is a multiple of the third term 6, and $\frac{18}{6} = 3$, and $\frac{12}{3} = 4$ L. *Ans.*

3. In compound questions the operation may frequently be rendered more simple, by placing the component parts of the divisor and dividend in a fractional form, rejecting such parts or factors of the

the numerator and denominator as happen to be equal, or cutting off an equal number of ciphers, or dividing by some common divisor.

EXAMPLE I.

If 35 ells of Vienna make 24 at Lyons, and 3 ells at Lyons make 5 at Antwerp, and 100 ells at Antwerp make 125 at Frankfort, how many ells at Frankfort make 42 at Vienna?

This question belongs to the rule of seven; and because the number sought is ells at Frankfort, therefore the given ells at Frankfort, *vis.* 125, is the middle term: the remaining terms are extremes; which I class into similar pairs, and state as follows.

	<i>Ant.</i>	<i>Frank.</i>	<i>Ant.</i>	
Vienna	100	:	125	:
35 : Lyons	3	:	24	:
	300		120	
	35		42	
	10500		5040	
			125	
			2520	
			1008	
			504	
			105 00	
			6300 00	
			630	
			(0)	

The simple questions are,

$$\begin{aligned}
 100 : 125 &:: 5 : 6\frac{1}{4} \\
 3 : 6\frac{1}{4} &:: 24 : 50 \\
 35 : 50 &:: 42 : 60 \text{ ells at Frankfort. } \textit{Ans.}
 \end{aligned}$$

But the question becomes more simple, by being stated and wrought in the fractional form, as follows:

$$\begin{aligned}
 &\frac{42 \times 24 \times 5 \times 125}{35 \times 3 \times 100} = \frac{42 \times 24 \times 5 \times 125}{3 \times 35 \times 100} = \\
 &\frac{42 \times 8 \times 1 \times 25}{1 \times 7 \times 20} = \frac{8400}{140} = \frac{840}{14} = 60 \text{ ells at Frankfort. } \textit{Ans.}
 \end{aligned}$$

EXAMPLE II.

If 100 lb. of Venice weigh 70 lb. of Lyons, and 120 lb. of Lyons

ons weigh 100 lb. of Roan, and 80 lb. of Roan weigh 100 lb. of Tolouse, and 100 lb. of Tolouse weigh 74 lb. of Geneva, how many pounds of Geneva will 100 lb. of Venice weigh?

This question belongs to the rule of nine; and because pounds of Geneva is the number sought, the given pounds of Geneva, *viz.* 74, must be the middle term: the remaining terms are extremes; which may be classed into similar pairs, and stated as follows:

	<i>Tol. Gen.</i>	<i>Tol.</i>	
Ven. Ly.	100 : 74 ::	100	Ly. Ven.
100 : 120 : Roan	80	100	Roan : 70 : 100
	8000	10000	
	120	70	
	960000	700000	
	100	100	
	96000000	70000000	
		74	
	96 000000	5180 000000	(53 $\frac{92}{96}$ lb. of Gen. <i>Ans.</i>
		480	
		380	
		288	
		(92)	

But the question becomes more simple, and is wrought with greater ease and advantage, by being stated in the fractional form, as follows:

$$\frac{100 \times 70 \times 100 \times 100 \times 74}{100 \times 120 \times 80 \times 100} = \frac{70 \times 100 \times 74}{120 \times 80} =$$

$$\frac{7 \times 10 \times 74}{12 \times 8} = \frac{5180}{96} = 53\frac{92}{96} \text{ lb. of Geneva. } \textit{Ans.}$$

I shall now conclude, by observing, that every compound question, whether in the rule of five, seven, nine, or eleven, &c. properly and strictly speaking, consists but of three given terms. For the first term, or divisor, is to be considered as one compound term made up, or produced, by the continual multiplication of the extremes on the left hand, as so many component parts. In like manner, the third term is to be considered as one compound term, made up by the continual multiplication of the extremes on the right, as component parts. Suppose the question to be,

If

If L. 100 in 12 months gain L. 5 interest, what will L. 75 gain in 9 months?

Here it is obvious, that it is neither the L. 100 principal, nor the 12 months of time, taken separately, that gains the L. 5 interest, but both contribute their share; that is, they conspire, as joint causes, to produce one effect; and therefore their product, *viz.* the first term, is to be considered as the cause producing the effect; that is, the first term, *viz.* 100×12 , causeth, produceth, or gains, L. 5 of interest. And in like manner, the product of the extremes on the right hand, or the third term, *viz.* 75×9 , is to be esteemed the cause that produceth a similar effect; that is, gains a like sum of interest, namely, the fourth term, or answer. In reference to this way of considering the first and third terms, the question might be stated as under.

$$\text{If } 100 \times 12 : 5 :: 75 \times 9$$

C H A P. VIII.

VULGAR FRACTIONS.

A FRACTION is a part or parts of an unit, or of any integer or whole; and is expressed by two numbers, one above and the other below a line drawn between them; as, $\frac{1}{4}$.

The number under the line shews into how many parts the unit or integer is divided; and is called the *denominator*, because it gives name to the fraction: the number above the line shews or tells how many of these parts the fraction contains; and is therefore called the *numerator*.

In the fraction $\frac{3}{4}$ l. a pound Sterling is the unit, integer, or whole; and the denominator 4 shews that the pound is broken or divided into four equal parts, *viz.* 4 crowns; and the numerator 3 shews that the fraction contains three of these parts, that is, three crowns; and so the value of this fraction is fifteen shillings.

C O R O L L A R Y I.

Hence it follows, 1. When the numerator of a fraction is less than the denominator, the value of such a fraction is less than unity, or the integer. 2. When the numerator is equal to the denominator, the value of the fraction is exactly an unit or integer. 3. When the numerator is greater than the denominator, the value of the fraction is more than an unit; and so often as the denominator is contained in the numerator, so many units or wholes are contained in the fraction. If, therefore, the numerator of a fraction be divided by the denominator, the quot will be a number of units or integers, and the remainder so many parts.

The numerator of a fraction is to be considered as a dividend,

and the denominator as a divisor; and the fraction itself may be taken to denote the quotient.

COROLLARY II.

From this view of a fraction, it is evident, that if the numerator and denominator of a fraction be either both multiplied or both divided by the same number, the products or quotients will retain the same proportion to one another; and consequently the new fraction thence arising will be of the same value with the given one. Thus the numerator and denominator of the fraction $\frac{3}{4}$ multiplied by 2 produces $\frac{6}{8}$, and divided by 4 quotes $\frac{1}{2}$, both which fractions are of the same value with $\frac{3}{4}$.

Fractions having 10, 100, 1000, or 1 with any number of ciphers annexed to it, for a denominator, are called *decimal fractions*; and fractions having any other denominator, are called *vulgar fractions*.

But the doctrine of vulgar fractions, or the rules therein laid down for reducing, adding, subtracting, multiplying, and dividing fractions, are equally applicable to fractions of every kind; to the decimal as well as the vulgar; and therefore, in what follows, the decimal and vulgar fractions are promiscuously used in the exemplification of the rules.

Decimal fractions, indeed, may be added, subtracted, &c. and have their operations conducted, not only by the rules assigned in the doctrine of vulgar fractions, which are of a general or universal nature, and extend alike to fractions of every sort; but also by a more easy and simple method, peculiar to themselves, which vulgar fractions do not admit of; and this may be called *the doctrine of decimals*; the explication whereof is reserved to some subsequent part of this treatise. In the mean time, we proceed to describe the other classes or kinds into which fractions in general are divided.

1. A proper fraction is that whose numerator is less than its denominator, and consequently is in value less than unity; as $\frac{3}{4}$.

2. An improper fraction is that whose numerator is equal to or greater than its denominator; and consequently is in value equal to or greater than an unit; as $\frac{4}{3}$, $\frac{7}{4}$.

3. A simple fraction is that which has but one numerator, and one denominator: and may be either proper or improper; as $\frac{5}{6}$ or $\frac{7}{4}$.

4. A compound fraction is made up of two or more simple fractions, coupled together with the particle *of*, and is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{3}{4}$, or $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.

5. A mixt number consists of an integer, and a fraction joined with it; as $7\frac{3}{4}$.

Because in most cases fractions can neither be added nor subtracted, till they be reduced, we begin with reduction.

Reduction

Reduction of Vulgar Fractions.

P R O B. I.

To reduce an improper fraction to an integer, or mixt number.

R U L E.

Divide the numerator by the denominator, the quot gives integers; and the remainder, if there be any, placed over the divisor or denominator, gives the fraction to be annexed.

E X A M P L E S.

1. $\frac{525}{8} = 85$ integers, there being no remainder.
2. $\frac{437}{8} = 54\frac{5}{8}$, the remainder being 5.
3. $\frac{1382}{14} = 98\frac{10}{14}$, the remainder being 10.
4. $\frac{23576}{136} = 173\frac{48}{136}$, the remainder being 48.

The reason of the rule is evident from Corollary I. For the denominator of every fraction is one whole; and if the numerator be divided by the denominator, the quot will be units, integers, or wholes, and the remainder so many parts.

P R O B. II.

To reduce a mixt number to an improper fraction.

R U L E.

Multiply the integer by the denominator; to the product add the numerator: the sum is the numerator of the improper fraction; and the denominator is the same as before.

E X A M P L E S.

$$1. 54\frac{5}{8} = \frac{437}{8}; \text{ for } 54 \times 8 = 432$$

$$+ 5$$

Numerator 437

$$2. 98\frac{10}{14} = \frac{1382}{14}; \text{ for } 98 \times 14 = 1372$$

$$+ 10$$

Numerator 1382

$$3. 173\frac{48}{136} = \frac{23576}{136}, \text{ for } 173 \times 136 = 23528$$

$$+ 48$$

Numerator 23576

K 3

The

The reason of this rule needs not be assigned, both the rule and the examples being the reverse of the foregoing.

P R O B. III.

To reduce a whole number to a fraction of a given denominator.

R U L E.

Multiply the whole number by the given denominator; and place the product by way of numerator over the given denominator.

E X A M P L E S.

1. Reduce 9 to a fraction whose denominator is 5.

$$9 \times 5 = 45; \text{ so the fraction is } \frac{45}{5}.$$

2. Reduce 36 to a fraction whose denominator is 4.

$$36 \times 4 = 144; \text{ so the fraction is } \frac{144}{4}.$$

3. Reduce 8 to a fraction whose denominator is 1.

$$8 \times 1 = 8; \text{ so the fraction is } \frac{8}{1}.$$

It is easy to perceive, that the fractions of this problem will all be improper; and from Ex. 3. we may learn, that any whole number may be reduced to the form of a fraction, by making unity the denominator.

The reason of the rule appears by reversing the operation; for if the numerator be divided by the denominator, it will quot the integer, or whole number.

P R O B. IV.

To reduce a compound fraction to a simple one.

R U L E.

Multiply the numerators continually for the numerator of the simple fraction; and multiply the denominators continually for its denominator.

E X A M P L E S.

Ex. 1.

$$\frac{2}{3} \text{ of } \frac{4}{5} = \frac{8}{15}.$$

Ex. 2.

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{24}.$$

An operation may sometimes be shortened by connecting the numerators and denominators with the sign of multiplication, and then rejecting such of the factors above and below as happen to be the

same. Thus, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5} = \frac{2 \times 3 \times 4}{3 \times 4 \times 5}$, and rejecting 3 and 4, both above and below, the fraction is quickly reduced to $\frac{2}{5}$.

The

The reason of the rule will appear by reviewing Ex. 1.; in which $\frac{4}{5}$, by Corollary II. is equal to $\frac{4 \times 3}{5 \times 3} = \frac{12}{15}$; and one third of $\frac{12}{15}$ is $\frac{4}{15}$; consequently two thirds is $\frac{8}{15} = \frac{2 \times 4}{3 \times 5}$.

COROLLARY.

From this problem may be reduced a method of reducing a fraction of a lesser denomination to a fraction of a greater denomination; namely,

Form a compound fraction, by comparing the given fraction with the superior denominations; and then reduce the compound fraction to a simple one.

EXAMPLES.

1. What fraction of a pound Sterling is $\frac{3}{4}$ of a penny?
 $\frac{3}{4}$ d. is $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ L. $= \frac{3}{960}$ L.
2. What fraction of a C. is $\frac{7}{8}$ of a pound?
 $\frac{7}{8}$ lb. is $\frac{7}{8}$ of $\frac{1}{8}$ of $\frac{1}{4}$ C. $= \frac{7}{896}$ C.
3. What fraction of a pound Troy is $\frac{4}{5}$ of a penny-weight?
 $\frac{4}{5}$ dw. is $\frac{4}{5}$ of $\frac{1}{16}$ of $\frac{1}{12}$ lb. $= \frac{4}{1200}$ lb. Troy.

PROB. V.

To reduce a fraction of a greater denomination to a fraction of a lesser denomination.

RULE.

Multiply the numerator of the given fraction, as in reduction of integers descending; and the product is the numerator, to be placed over the denominator of the given fraction.

EXAMPLES.

1. What fraction of a shilling is $\frac{3}{4}$ of a pound?
 Here, as in reduction descending, I multiply the numerator 3 by 20, because 20 shillings make a pound; as under,

$$\begin{array}{r} \text{L.} \\ 3 \times 20 \\ \hline 4 \end{array} = \frac{60}{4} \text{ shilling.}$$

K 4

2. What

2. What fraction of a penny is
- $\frac{4}{5}$
- L.?

L.

$$\frac{4 \times 20 \times 12}{5} = 240 \text{ d.}$$

3. What fraction of a farthing is
- $\frac{2}{3}$
- L.?

L.

$$\frac{2 \times 20 \times 12 \times 4}{3} = 1280 \text{ f.}$$

4. What fraction of a pound is
- $\frac{7}{8}$
- C.?

C.

$$\frac{7 \times 4 \times 28}{8} = 784 \text{ lb.}$$

5. What fraction of a grain is
- $\frac{1}{12}$
- lb. Troy?

lb.

$$\frac{5 \times 12 \times 20 \times 24}{12} = 28800 \text{ gr.}$$

6. What fraction of an inch is
- $\frac{1}{2}$
- yard?

Yard.

Yard.

$$\frac{1 \times 3 \times 12}{2}$$

$$= 18 \text{ inch.}$$

$$\frac{1 \times 36}{2}$$

$$= 18 \text{ inch.}$$

The reason of this rule will appear by observing, that every fraction may be considered in two views. Thus, $\frac{3}{4}$ may either be considered as expressing three fourths of one unit, or as denoting the fourth part of three units. Now, if the unit be a pound Sterling, the fraction, in the latter view, will denote the fourth part of three pounds; and by reducing the numerator L. 3 to shillings, we have $\frac{60}{4}$ s.; and again reducing 60 shillings to pence, we have $\frac{720}{4}$ d. equal to $\frac{60}{4}$ s. or to $\frac{3}{4}$ L.

P R O B. VI.

To find the value of a fraction.

R U L E.

Reduce the numerator to the next inferior denomination; divide by the denominator; and the quot, if nothing remain, is the value complete.

If there be any remainder, it is the numerator of a fraction whose denominator is the divisor. This fraction may either be annexed to the quotient, or reduced to value, if there be any lower denomination.

E X.

EXAMPLES.

1. What is the value of $\frac{3}{4}$ L.?

Here I consider $\frac{3}{4}$ L. as expressing the fourth part of three pounds Sterling; so I reduce L. 3, the numerator, to shillings, and divide by the denominator 4; and as nothing remains, the quot, viz. 15 shillings, is the value complete.

$$\begin{array}{r} \text{L. } s. \\ \text{Ans. } \frac{3}{4} = 15 \end{array}$$

$$\begin{array}{r} 3 \\ 20 \\ \hline 4)60(15s. \\ 4 \\ \hline 20 \\ 20 \\ \hline (0) \\ \hline \end{array}$$

2. What is the value of $\frac{2}{8}$ L.?

Here there is a remainder of 4, which makes $\frac{4}{8}$ of a shilling; this I reduce to value, viz. I multiply 4 by 12, the pence in a shilling, and divide the product by the denominator 16; and the quot gives 3d.

$$\begin{array}{r} \text{L. } s. \text{ d.} \\ \text{Ans. } \frac{2}{8} = 11 \quad 3 \end{array}$$

$$\begin{array}{r} 9 \\ 20 \\ \hline 16)180(11s. \\ 16 \\ \hline 20 \\ 16 \\ \hline 4 \text{ rem.} \\ 12 \\ \hline 16)48(3d. \\ 48 \\ \hline (0) \\ \hline \end{array}$$

3. What is the value of $\frac{16}{21}$ C.?

Here the last remainder 7 makes $\frac{7}{21}$ of a pound? which I annex to the quotient.

$$\begin{array}{r} \text{C. } s. \text{ lb.} \\ \text{Ans. } \frac{16}{21} = 3 \quad 1\frac{7}{21} \end{array}$$

$$\begin{array}{r} 16 \\ 4 \\ \hline 21)64(3Q. \\ 63 \\ \hline 1 \\ 28 \\ \hline 21)28(1\frac{7}{21}\text{lb.} \\ 21 \\ \hline (7) \\ \hline \end{array}$$

MORE

MORE EXAMPLES.

4. What is the value of
- $\frac{37}{9}$
- L. Sterling?

Ans. 18s. 7d. $1\frac{2}{3}$ f.

5. What is the value of
- $\frac{33}{8}$
- of a tun?

Ans. 11 C. 1 Q. $2\frac{3}{4}$ lb.

6. What is the value of
- $\frac{6}{10}$
- lb. Troy?

Ans. 4 oz. 10 dw.

7. What is the value of
- $\frac{41}{6}$
- of a year?

Ans. 299 days, 7 hours, 12 minutes.

The reason of this rule is the same with that in the preceding problem. It is by the practice of this problem that remainders in the rule of three are reduced to value.

P R O B. VII.

To reduce a fraction to its lowest terms.

R U L E.

Divide both numerator and denominator by their greatest common divisor; the two quots make the new fraction.

The greatest common divisor of the numerator and denominator of a fraction is found by the following rule.

Divide the greater of these two numbers by the lesser; and again divide the divisor by the remainder; and so on, continually, till 0 remains. The last divisor is their greatest common divisor.

E X A M P L E S.

1. Reduce
- $\frac{784}{952}$
- to its lowest terms.

First I find the greatest common divisor of the numerator and denominator, as follows:

$$\begin{array}{r}
 784)952(1 \\
 \underline{784} \\
 168)784(4 \\
 \underline{672} \\
 112)168(1 \\
 \underline{112} \\
 \text{Greatest common divisor} \quad 56)112(2 \\
 \underline{112} \\
 (0)
 \end{array}$$

I now proceed to reduce the given fraction to its lowest terms, by dividing both numerator and denominator by 56, the greatest common divisor.

$$56)784(14 \text{ new numerator.}$$

$$\begin{array}{r} 56 \\ \hline 224 \\ 224 \\ \hline (0) \end{array}$$

$$56)952(17 \text{ new denominator.}$$

$$\begin{array}{r} 56 \\ \hline 392 \\ 392 \\ \hline (0) \end{array}$$

$$\text{So } \frac{784}{952} = \frac{14}{17}.$$

2. Reduce $\frac{846}{468}$ to its lowest terms.

First find the greatest common divisor, as under.

$$468)846(1$$

$$\begin{array}{r} 468 \\ \hline \end{array}$$

$$378)468(1$$

$$\begin{array}{r} 378 \\ \hline \end{array}$$

$$90)378(4$$

$$\begin{array}{r} 360 \\ \hline \end{array}$$

Greatest common divisor

$$18)90(5$$

$$\begin{array}{r} 90 \\ \hline \end{array}$$

$$(0)$$

Now reduce the given fraction to its lowest terms.

$$18)846(47 \text{ new numerator.}$$

$$\begin{array}{r} 72 \\ \hline \end{array}$$

$$126$$

$$126$$

$$\hline$$

$$(0)$$

$$18)468(26 \text{ new denominator.}$$

$$\begin{array}{r} 36 \\ \hline \end{array}$$

$$108$$

$$108$$

$$\hline$$

$$(0)$$

$$\text{So } \frac{846}{468} = \frac{47}{26} = 1\frac{21}{26}$$

3. Reduce

3. Reduce $\frac{147}{294}$ to its lowest terms.

$$\begin{array}{r} 147 \overline{) 323} 2 \\ 294 \end{array}$$

$$\begin{array}{r} 29 \overline{) 147} 5 \\ 145 \end{array}$$

$$\begin{array}{r} 2 \overline{) 29} 14 \\ 2 \end{array}$$

$$\begin{array}{r} 9 \\ 8 \end{array}$$

$$\begin{array}{r} 1 \\ 2 \end{array}$$

Greatest common divisor $\begin{array}{r} 1 \overline{) 2} 2 \\ 2 \end{array}$

$$\begin{array}{r} 0 \end{array}$$

$$(0)$$

The greatest common divisor being unity, the given fraction is already in its lowest terms.

MORE EXAMPLES.

4. What is $\frac{308}{884}$ in its lowest terms? *Ans.* $\frac{53}{171}$.
 5. What is $\frac{3548}{912}$ in its lowest terms? *Ans.* $\frac{7}{8}$.
 6. What is $\frac{3728}{976}$ in its lowest terms? *Ans.* $\frac{11}{3}$.

This way of abbreviating or reducing a fraction to its lowest terms, by finding the greatest common divisor, being tedious, you may use the following method.

Divide the numerator and denominator of the given fraction by any number that will divide both without any remainder, and you have your fraction in lower terms. In like manner, reduce this new fraction to lower terms still, and so on continually, till no common divisor, except unity, is found, and you have your fraction in its lowest terms.

In order to discover what number will divide both numerator and denominator without a remainder, observe the following practical rules or directions.

1. If the units, or right-hand figure, of both numerator and denominator, are even numbers, you may always halve them, or divide by 2,

$$\text{Ex. I. } \begin{array}{c} 2) \\ 784 \\ 952 \end{array} \mid \begin{array}{c} 2) \\ 392 \\ 476 \end{array} \mid \begin{array}{c} 2) \\ 196 \\ 238 \end{array} \mid \begin{array}{c} 7) \\ 98 \\ 166 \end{array} \mid \frac{14}{17}$$

In the above example I proceed by halving, till the new fraction is

is $\frac{8}{9}$, where 9, the unit of the denominator, is an odd number, so 2 cannot measure both; consequently neither can 4, 6, or 8, the multiples of 2. By trial I find 3 cannot measure both, and so neither can 6 or 9, the multiples of 3. Then I try 7, and find it measures both. So the given fraction is reduced to $\frac{14}{17}$, and is in its lowest terms, unity being the only number that measures both numerator and denominator.

The divisors in the above example may be considered as component parts of the greatest common divisor, and accordingly the product arising from their continual multiplication, viz. $2 \times 2 \times 2 \times 7 = 56$, is the greatest common divisor.

$$\text{Ex. 2. } \frac{102}{576} \mid \frac{96}{288} \mid \frac{48}{144} \mid \frac{24}{72} \mid \frac{12}{36} \mid \frac{6}{18} \mid \frac{3}{9} \mid \frac{1}{3}$$

2. If the right-hand figure of both numerator and denominator be 5, or if the right-hand figure of the one be 5, and of the other 0, you may in either case divide by 5.

$$\text{Ex. 1. } \frac{375}{1125} \mid \frac{75}{225} \mid \frac{15}{45} \mid \frac{3}{9} \mid \frac{1}{3}$$

$$\text{Ex. 2. } \frac{525}{800} \mid \frac{105}{160} \mid \frac{21}{32} \mid \frac{7}{8}$$

$$\text{Ex. 3. } \frac{140}{245} \mid \frac{28}{49} \mid \frac{4}{7}$$

3. If there are ciphers on the right hand of both numerator and denominator, cut off an equal number of ciphers from both; for to cut off one cipher is the same thing as to divide by 10, and to cut off two is the same as to divide by 100, &c.

$$\text{Ex. 1. } \frac{180}{240} \mid \frac{18}{24} \mid \frac{3}{4}$$

$$\text{Ex. 2. } \frac{1000}{31500} \mid \frac{10}{315} \mid \frac{2}{63} \mid \frac{2}{9} \mid \frac{2}{9}$$

$$\text{Ex. 3. } \frac{3}{5} \mid \frac{000}{000} = \frac{3}{5}$$

The reason of this rule has been already assigned in Corollary II. viz. When the numerator and denominator of a fraction are both divided by the same number, the new fraction formed by the quotients is of the same value with the given one.

The rule for finding the greatest common divisor may be thus demonstrated: If any number measure both the remainder of a division, and also the divisor, it will likewise measure the dividend; for the dividend consists of two parts, whereof one is a multiple of the

the divisor, produced by multiplying the quot into the divisor; and the other part is the remainder itself: now, since the supposed number measures the remainder, and also measures the divisor, and all its multiples, it will measure both parts of the dividend; that is, it will measure the dividend.

Thus, in Ex. 1. since every number measures itself, the remainder 56, in the third division, will measure itself; but it also measures the divisor 112; and therefore will also measure the dividend 168; that is, it measures the remainder and the divisor of the second division; and consequently measures the dividend 784; that is, it measures both the remainder and the divisor of the first division; and therefore will also measure the dividend 952, and at the same time is the highest or greatest number that will do so.

P R O B. VIII.

To reduce fractions of different denominators to a common denominator.

R U L E.

Multiply the denominators continually for the common denominator; and multiply each numerator into all the denominators, except its own, for the several numerators.

E X A M P L E S.

1. Reduce $\frac{3}{4}$ and $\frac{4}{5}$ to a common denominator.

$4 \times 5 = 20$, the common denominator.

$3 \times 5 = 15$, the first numerator.

$4 \times 4 = 16$, the second numerator.

So the new fractions are $\frac{15}{20}$ and $\frac{16}{20}$.

2. Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$, to a common denominator.

$3 \times 6 \times 8 = 144$, the common denominator.

$2 \times 6 \times 8 = 96$, the first numerator.

$5 \times 3 \times 8 = 120$, the second numerator.

$7 \times 3 \times 6 = 126$, the third numerator.

The new fractions are $\frac{96}{144}$, $\frac{120}{144}$, and $\frac{126}{144}$.

3. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{6}{7}$, $\frac{9}{10}$, to a common denominator.

$2 \times 5 \times 7 \times 10 = 700$, the common denominator.

$1 \times 5 \times 7 \times 10 = 350$, the first numerator.

$3 \times 2 \times 7 \times 10 = 420$, the second numerator.

$6 \times 2 \times 5 \times 10 = 600$, the third numerator.

$9 \times 2 \times 5 \times 7 = 630$, the fourth numerator.

New fractions, $\frac{350}{700}$, $\frac{420}{700}$, $\frac{600}{700}$, $\frac{630}{700}$.

M O R E

MORE EXAMPLES.

$$\begin{aligned} 4. \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8} &= \frac{720}{960}, \frac{768}{960}, \frac{800}{960}, \frac{840}{960}. \\ 5. \frac{1}{3}, \frac{4}{5}, \frac{5}{7}, \frac{2}{3} &= \frac{315}{945}, \frac{420}{945}, \frac{675}{945}, \frac{378}{945}. \\ 6. \frac{1}{4}, \frac{5}{7}, \frac{9}{10}, \frac{1}{2} &= \frac{140}{560}, \frac{400}{560}, \frac{504}{560}, \frac{280}{560}. \end{aligned}$$

When the denominator of one fraction happens to be an aliquot part of the denominator of another fraction, the former may be reduced to the same denominator with the latter, by multiplying both its numerator and denominator by the number which denotes how often the lesser denominator is contained in the greater.

$$\text{Thus, } \frac{2}{3} + \frac{5}{12} = \frac{8}{12} + \frac{5}{12}.$$

Here 3 is contained in 12 four times; so I multiply both 2 and 3 by 4, and I have $\frac{8}{12} = \frac{2}{3}$.

$$\text{Again, } \frac{1}{2} + \frac{3}{4} + \frac{5}{8} = \frac{4}{8} + \frac{6}{8} + \frac{5}{8}.$$

Sometimes, too, the fraction that has the greater denominator may, in like manner, be reduced to the same denominator, with that which has the lesser, by division.

$$\text{Thus, } \frac{3}{9} + \frac{2}{3} = \frac{1}{3} + \frac{2}{3}.$$

$$\text{And } \frac{16}{48} + \frac{9}{36} + \frac{7}{12} = \frac{4}{12} + \frac{3}{12} + \frac{7}{12}.$$

The reason of the above rule for reducing fractions to a common denominator is evident from Corollary II.; for both numerator and denominator of every fraction are multiplied by the same number, or by the same numbers.

After fractions are reduced to a common denominator, they may frequently be reduced to lower terms, by dividing all the numerators, and also the common denominator, by any divisor that leaves no remainder, or by cutting off an equal number of ciphers from both.

The lower fractions are reduced, the more simple and easy will any operation be; if, therefore, it be required to reduce fractions to the lowest or least common denominator possible, it may be done as follows.

The least multiple of all the given denominators is the least common denominator.

Divide this common denominator severally by the denominators of the given fractions; and multiply each numerator respectively by the quot arising from its own denominator; and the products will be the numerators.

The least multiple of two or more numbers is found thus.

Place the given numbers in a line; then divide two of them, or as many of them as you can, by 2, or 3, or any small divisor that leaves no remainder; place the quots, and the numbers not divided,

ded, in a line below; again, divide the numbers in this line, either by the former, or by some other divisor, placing the quots, and the numbers not divided, in a line below; proceed by dividing, in the same manner, till the last quot of every number be unity; then multiply the divisors into one another continually, and their product is the least multiple required.

EXAMPLE I.

Required the least multiple of 24 and 36?

Divisors 6 24. 36.

2	4.	6.
2	2.	3.
3	1.	3.
		1.

$6 \times 2 \times 2 \times 3 = 72$, the least multiple.

EXAMPLE II.

Required the least multiple of 24, 36, 54?

Divisors 6 24. 36. 54.

3	4.	6.	9.
2	4.	2.	3.
2	2.	2.	3.
3	1.		3.
			1.

$6 \times 3 \times 2 \times 2 \times 3 = 216$, the least multiple.

EXAMPLE III.

Required the least multiple of 2, 4, 6, 10, 12, 15?

Divisors 2 2. 4. 6. 10. 12. 15.

2	1.	2.	3.	5.	6.	15.
3		1.	3.	5.	3.	15.
5			1.	5.	1.	5.
				1.		1.

$2 \times 2 \times 3 \times 5 = 60$, the least multiple.

The reason of the operation is obvious. For the continual product of the divisors used in dividing any of the given numbers is equal to the said given number; and if this product be multiplied by the remaining divisor or divisors, the product will be a multiple of the given number. Thus, in Ex. 1. the divisors used in dividing 24 are 6, 2, 2, and $6 \times 2 \times 2 = 24$, the given number; and if we multiply this by 3, the remaining divisor, we shall have $6 \times 2 \times 2 \times 3 = 72$, a multiple of 24. In like manner, the divisors used in dividing 36 are 6, 2, 3, and $6 \times 2 \times 3 = 36$, the given number; and if we multiply this by 2, the remaining divisor, we shall have, as before, $6 \times 2 \times 2 \times 3 = 72$, a multiple of 36, which, at the same time, is the least common multiple of 24 and 36.

If any of the smaller given numbers be an aliquot part of some of the higher given numbers, you may, if you please, neglect the aliquot parts. Thus, in Ex. 3. you may neglect, 2, 4, and 6, as being aliquot parts of 12; and by dividing 10, 12, 15, you will have 60 for the least common multiple, as before.

I now proceed to the exemplification of the above rule.

Ex. 1. Reduce $\frac{1}{2}$ and $\frac{7}{8}$ to their least common denominator.

Divisors 6 | 12. 18. $6 \times 2 \times 3 = 36$, the least common denominator.

2	2.	3.
3	1.	3.
		1.

12)36($3 \times 5 = 15$ the first numerator.

18)36($2 \times 7 = 14$ the second numerator.

The new fractions are $\frac{15}{36}$ and $\frac{14}{36}$.

Ex. 2. Reduce $\frac{1}{8}$, $\frac{7}{12}$, $\frac{4}{9}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{3}$, to their least common denominator.

Divisors 2 | 8. 12. 9. 3. 6. 4. $2 \times 2 \times 2 \times 3 \times 3 = 72$, the common denominator.

2	4.	6.	9.	3.	3.	2.
2	2.	3.	9.	3.	3.	1.
3	1.	3.	9.	3.	3.	
3		1.	3.	1.	1.	
			1.			

8)72($9 \times 5 = 45$ first numerator.

12)72($6 \times 7 = 42$ second numerator.

9)72($8 \times 4 = 32$ third numerator.

3)72($24 \times 2 = 48$ fourth numerator.

6)72($12 \times 5 = 60$ fifth numerator.

4)72($18 \times 1 = 18$ sixth numerator.

New fractions $\frac{45}{72}$, $\frac{42}{72}$, $\frac{32}{72}$, $\frac{48}{72}$, $\frac{60}{72}$, $\frac{18}{72}$.

L

The

The method of finding the new numerators may be thus accounted for. The value of a fraction is not altered by multiplying both its numerator and denominator by the same number; but when the common denominator is divided by the denominator of the given fraction, the quot gives the number into which the denominator of the given fraction is multiplied; and consequently its numerator must be multiplied by the same. Thus, in the last example, the common denominator, 72, divided by 8, the denominator of the given fraction $\frac{5}{8}$, quotes 9; and so the new fraction is

formed thus, $\frac{5 \times 9}{8 \times 9} = \frac{45}{72}$; in like manner, the new fraction $\frac{7}{8}$ is

formed thus, $\frac{7 \times 6}{12 \times 6} = \frac{42}{72}$. &c.

Addition of Vulgar Fractions.

Rule I. If the given fractions have all the same denominator, add the numerators, and place the sum over the denominator.

Ex. 1. What is the sum of $\frac{2}{7} + \frac{3}{7}$? *Ans.* $\frac{5}{7}$.

Ex. 2. What is the sum of $\frac{3}{15} + \frac{6}{15}$? *Ans.* $\frac{9}{15} = \frac{3}{5}$, by Prob. VII.

Ex. 3. What is the sum of $\frac{3}{8} + \frac{5}{8} + \frac{7}{8}$? *Ans.* $\frac{15}{8} = 1\frac{7}{8}$, by Prob. I.

Rule II. If the given fractions have different denominators, reduce them to a common denominator, by Prob. VIII. then add the numerators, and place the sum over the common denominator.

Ex. 1. What is the sum of $\frac{2}{5} + \frac{3}{8}$?
 $\frac{2}{5} + \frac{3}{8} = \frac{16}{40} + \frac{15}{40}$, by Prob. VIII. and $\frac{16}{40} + \frac{15}{40} = \frac{31}{40}$.

Ex. 2. What is the sum of $\frac{1}{3} + \frac{3}{4} + \frac{5}{6}$?

$\frac{1}{3} + \frac{3}{4} + \frac{5}{6} = \frac{4}{12} + \frac{9}{12} + \frac{10}{12}$, by Prob. VIII.

and $\frac{4}{12} + \frac{9}{12} + \frac{10}{12} = \frac{23}{12}$.

and $\frac{23}{12} = 1\frac{11}{12}$, by Prob. I. = $1\frac{11}{12}$, by Prob. VII.

The operation becomes more simple, if the given fractions be reduced to the least common denominator. The former example wrought in this manner follows.

What is the sum of $\frac{1}{3} + \frac{3}{4} + \frac{5}{6}$?

$\frac{1}{3} + \frac{3}{4} + \frac{5}{6} = \frac{4}{12} + \frac{9}{12} + \frac{10}{12} = \frac{23}{12} = 1\frac{11}{12}$, by Prob. I.

Rule III. If mixt numbers be given, or if mixt numbers and fractions be given, reduce the mixt numbers to improper fractions, by Prob.

Prob. II.; then reduce the fractions to a common denominator, by Prob. VIII. and add the numerators.

Ex. 1. What is the sum of $7\frac{3}{4} + 5\frac{2}{3}$?

$$7\frac{3}{4} + 5\frac{2}{3} = \frac{31}{4} + \frac{17}{3}, \text{ by Prob. II.}$$

$$\text{and } \frac{31}{4} + \frac{17}{3} = \frac{93}{12} + \frac{68}{12}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{93}{12} + \frac{68}{12} = \frac{161}{12} = 13\frac{5}{12}, \text{ by Prob. I.}$$

Ex. 2. What is the sum of $6\frac{2}{3} + \frac{4}{5}$?

$$6\frac{2}{3} = \frac{20}{3}, \text{ by Prob. II.}$$

$$\text{and } \frac{20}{3} + \frac{4}{5} = \frac{100}{15} + \frac{12}{15}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{100}{15} + \frac{12}{15} = \frac{112}{15} = 7\frac{7}{15}, \text{ by Prob. I.}$$

When mixt numbers, or mixt numbers and fractions, are given, you may, with greater expedition, work by the following rule, *viz.* reduce only the fractions to a common denominator, and add the sum of the fractions to the integers. The above two examples wrought in this manner follow.

Ex. 1. What is the sum of $7\frac{3}{4} + 5\frac{2}{3}$?

$$\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}.$$

$$\text{and } 7 + 5 + 1\frac{5}{12} = 13\frac{5}{12}.$$

Ex. 2. What is the sum of $6\frac{2}{3} + \frac{4}{5}$?

$$\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}.$$

$$\text{and } 6 + 1\frac{7}{15} = 7\frac{7}{15}.$$

Rule IV. If any, or all of the given fractions, be compound, first reduce the compound fractions to simple ones, by Prob. IV.; then reduce the simple fractions to a common denominator, by Prob. VIII. and add the numerators.

Ex. 1. What is the sum of $\frac{2}{3}$ of $\frac{4}{5} + \frac{3}{4}$?

$$\frac{2}{3} \text{ of } \frac{4}{5} = \frac{8}{15}, \text{ by Prob. IV.}$$

$$\text{and } \frac{8}{15} + \frac{3}{4} = \frac{32}{60} + \frac{45}{60} = \frac{77}{60}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{32}{60} + \frac{45}{60} = \frac{77}{60} = 1\frac{17}{60}, \text{ by Prob. I.}$$

Ex. 2. What is the sum of $\frac{1}{2}$ of $\frac{2}{3} + \frac{3}{4}$ of $\frac{1}{4}$?

$$\frac{1}{2} \text{ of } \frac{2}{3} + \frac{3}{4} \text{ of } \frac{1}{4} = \frac{2}{6} + \frac{3}{16}, \text{ by Prob. IV.}$$

$$\text{and } \frac{2}{6} + \frac{3}{16} = \frac{40}{120} + \frac{18}{120}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{40}{120} + \frac{18}{120} = \frac{58}{120} = \frac{29}{60}, \text{ by Prob. VII.}$$

Rule V. If the given fractions be of different denominations, first reduce them to the same denomination, by Cor. of Prob. IV. or by Prob. V.; then reduce the fractions, now of one denomination, to

a common denominator, by Prob. VIII. and add the numerators; or reduce each of the given fractions separately to value, by Prob. VI. and then add their values.

EXAMPLE.

What is the sum of $\frac{3}{4}$ s. and $\frac{7}{8}$ l.?

METHOD I.

$\frac{3}{4}$ s. = $\frac{3}{4}$ of $\frac{1}{20}$ l. = $\frac{3}{80}$ l. by Cor. Prob. IV.

and $\frac{7}{8}$ + $\frac{7}{8}$ = $\frac{7}{8}$ + $\frac{7}{8}$ = $\frac{14}{8}$ l. by Prob. VIII.

and $\frac{3}{80}$ + $\frac{14}{8}$ = $\frac{146}{80}$ l. = 18s. 3d. by Prob. VI.

METHOD II.

$\frac{7}{8}$ l. = $\frac{7 \times 20}{8}$ s. = $\frac{140}{8}$ s. by Prob. V.

and $\frac{3}{4}$ + $\frac{140}{8}$ = $\frac{6}{8}$ + $\frac{140}{8}$, by Prob. VIII.

and $\frac{6}{8}$ + $\frac{140}{8}$ = $\frac{146}{8}$ s. = 18s. 3d. by Prob. I. and VI.

METHOD III.

$\frac{3}{4}$ s. = $\frac{3 \times 12}{4}$ d. = $\frac{36}{4}$ d. = 9d. } by Prob. VI.

$\frac{7}{8}$ l. = $\frac{7 \times 20}{8}$ s. = $\frac{140}{8}$ s. = 17s. 6d. }
18s. 3d.

MORE EXAMPLES.

1. What is the sum of $\frac{1}{2}$ + $\frac{1}{4}$ + $\frac{1}{8}$? *Ans.* $1\frac{7}{8}$.
2. What is the sum of $\frac{7}{8}$ + $\frac{11}{16}$ + $\frac{17}{32}$? *Ans.* $2\frac{11}{32}$.
3. What is the sum of $13\frac{3}{4}$ + $24\frac{5}{8}$? *Ans.* $38\frac{11}{8}$, or $38\frac{7}{4}$.
4. What is the sum of $\frac{3}{4}$ of $\frac{1}{2}$ + $\frac{1}{8}$ of $\frac{6}{7}$ + $\frac{5}{8}$? *Ans.* $1\frac{117}{80}$.
5. What is the sum of $\frac{1}{2}$ l. + $\frac{3}{4}$ s. + $\frac{1}{8}$ d.? *Ans.* $\frac{11}{16}$ l. in value 15s. 9d. 3f.

The reason of the above rules appears from Axiom XI. because none but similar or like things can be added. Thus, third parts cannot be added to fourth parts, for the sum would neither be third parts nor fourth parts; third parts can only be added to third parts, and fourth parts only to fourth parts; and consequently

quently fractions to be added must have all the same denominator. In like manner, the fraction of a pound cannot be added to the fraction of a shilling, or of a penny, being unlike things; and therefore fractions to be added must be all of the same denomination.

Subtraction of Vulgar Fractions.

Rule I. If the given fractions have the same denominator, subtract the lesser numerator from the greater, and place the remainder over the denominator.

Ex. 1. From $\frac{5}{7}$ subtract $\frac{3}{7}$.

$$\frac{5}{7} - \frac{3}{7} = \frac{2}{7}.$$

Ex. 2. From $\frac{10}{13}$ subtract $\frac{4}{13}$.

$$\frac{10}{13} - \frac{4}{13} = \frac{6}{13}.$$

Rule II. If the given fractions have different denominators, reduce them to a common denominator, by Prob. VIII.; then subtract the lesser numerator from the greater, and place the remainder over the common denominator.

Ex. 1. From $\frac{3}{4}$ subtract $\frac{2}{5}$.

$$\frac{3}{4}, \frac{2}{5} = \frac{9}{20}, \frac{8}{20}, \text{ by Prob. VIII. and } \frac{9}{20} - \frac{8}{20} = \frac{1}{20}.$$

Ex. 2. From $\frac{9}{10}$ subtract $\frac{7}{8}$.

$$\frac{9}{10}, \frac{7}{8} = \frac{72}{80}, \frac{70}{80}, \text{ by Prob. VIII. and } \frac{72}{80} - \frac{70}{80} = \frac{2}{80} = \frac{1}{40}, \text{ by Prob. VII.}$$

Rule III. If it be required to subtract one mixt number from another, or to subtract a fraction from a mixt number, reduce the mixt numbers to improper fractions, by Prob. II.; then reduce the fractions to a common denominator, by Prob. VIII. and subtract the one numerator from the other.

Ex. 1. From $7\frac{3}{4}$ subtract $5\frac{1}{2}$.

$$7\frac{3}{4}, 5\frac{1}{2} = \frac{29}{4}, \frac{11}{2}, \text{ by Prob. II.}$$

$$\text{and } \frac{29}{4}, \frac{11}{2} = \frac{29}{4}, \frac{22}{4}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{29}{4} - \frac{22}{4} = \frac{7}{4} = 1\frac{3}{4}, \text{ by Prob. I. and VII.}$$

Ex. 2. From $8\frac{2}{3}$ subtract $\frac{4}{5}$.

$$8\frac{2}{3} = \frac{26}{3}, \text{ by Prob. II.}$$

$$\text{and } \frac{26}{3}, \frac{4}{5} = \frac{130}{15}, \frac{12}{15}, \text{ by Prob. VIII.}$$

$$\text{and } \frac{130}{15} - \frac{12}{15} = \frac{118}{15} = 7\frac{8}{15}, \text{ by Prob. I.}$$

In subtracting one mixt number from another, or in subtracting a fraction from a mixt number, you may, with greater ease and expedition, proceed thus, *viz.* Reduce only the fractions to a com-

mon denominator, and then work by one or other of the two rules following.

1. If the numerator of the fraction to be subtracted be less than the numerator of the fraction from which you are to subtract, then subtract the lesser numerator from the greater, place the remainder over the common denominator, and to this fraction prefix the difference of the integers of the two mixt numbers; or the integral part of the mixt number when a fraction is subtracted.

Ex. 1. From $7\frac{3}{4}$ subtract $5\frac{1}{8}$.

$\frac{3}{4}, \frac{1}{8} = \frac{6}{8}, \frac{4}{8}$, by Prob. VIII.

and $\frac{6}{8} - \frac{4}{8} = \frac{2}{8} = \frac{1}{4}$, by Prob. VII. and $7 - 5 = 2$.

So the difference of the two mixt numbers is $2\frac{1}{4}$.

Ex. 2. From $9\frac{3}{4}$ subtract $\frac{2}{3}$.

$\frac{3}{4}, \frac{2}{3} = \frac{9}{12}, \frac{8}{12}$, by Prob. VIII.

and $\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$; and to this fraction I prefix the integral part of the mixt number, and the difference, or remainder, is $9\frac{1}{12}$.

Note. In subtracting a fraction from an unit, you have only to subtract the numerator from the denominator; and the remainder, placed over the denominator, is the difference, or answer. Thus, $1 - \frac{2}{3} = \frac{1}{3}$: for $1 = \frac{3}{3}$, and $\frac{3}{3} - \frac{2}{3} = \frac{1}{3}$.

2. If the numerator of the fraction to be subtracted be greater than the numerator of the fraction from which you are to subtract; in this case subtract the greater fraction from an unit borrowed; that is, subtract the greater numerator from the common denominator, add the remainder to the numerator of the lesser fraction, and place the sum over the common denominator, for the fractional part of the answer; or, which is the same in effect, subtract the greater numerator from the sum of the numerator and denominator of the lesser fraction, and place the remainder over the common denominator for the fractional part of the answer: then, for the unit thus borrowed, add 1 to the integral part of the lesser mixt number; subtract this sum from the integral part of the greater mixt number; and prefix the difference to the fractional part of the answer. But in subtracting a fraction from a mixt number, for the unit borrowed, take 1 from the integral part of the mixt number, and prefix the remainder to the fractional part of the answer.

Ex. 1. From $8\frac{1}{2}$ subtract $5\frac{2}{3}$.

$\frac{1}{2}, \frac{2}{3} = \frac{3}{6}, \frac{4}{6}$, by Prob. VIII.

Here having reduced the fractions to a common denominator, I say, $\frac{4}{6}$ from $\frac{3}{6}$ I cannot; wherefore I subtract $\frac{4}{6}$ from an unit borrowed, *viz.* I subtract the numerator 4 from the common denominator

minator 6; and add the remainder 2 to the numerator of the lesser fraction 3; and the sum 5, placed over the common denominator, gives $\frac{5}{6}$ for the fractional part of the answer. Or, which is the same in effect, I subtract 4 from $3 + 6 = 9$, and there remains $\frac{5}{6}$ for the fractional part of the answer, as before. Then, for the unit borrowed, I say, 1 borrowed and 5 make 6; which, subtracted from 8, leaves two of a remainder; and this prefixed to the fractional part, gives $2\frac{5}{6}$ for the difference, remainder, or answer.

Ex. 2. From $9\frac{2}{3}$ subtract $\frac{4}{5}$.

$\frac{2}{3}, \frac{4}{5} = \frac{10}{15}, \frac{12}{15}$, by Prob. VIII.

Here having reduced the fractions to a common denominator, I say, $\frac{12}{15}$ from $\frac{10}{15}$ I cannot; but, borrowing an unit, I subtract the numerator 12 from the common denominator 15, and 3 remains; which 3, added to the lesser numerator 10, gives $\frac{13}{15}$ from the fractional part of the answer. Or, I subtract 12 from $10 + 15 = 25$, and 13 remains; which 13, placed over the common denominator, gives $\frac{13}{15}$ for the fractional part, as before. Then, for the unit borrowed, I take 1 from the integral part of the mixt number; and the remainder, prefixed to the fractional part, gives $8\frac{13}{15}$ for the difference, or answer.

Rule IV. If it be required to subtract a mixt number, or a fraction, from an integer, first subtract the fraction from an unit borrowed: that is, subtract the numerator from the denominator, and place the remainder, as a numerator, over the denominator, for the fractional part of the answer: then, for the unit borrowed, add 1 to the integral part of the mixt number; subtract the sum from the given integer; and prefix the remainder to the fractional part of the answer. But when a fraction is subtracted from an integer, for the unit borrowed, take 1 from the given integer, and prefix the remainder to the fractional part of the answer.

Ex. 1. From 14 subtract $7\frac{3}{5}$.

Here I say, $5 - 3 = 2$; so $\frac{2}{5}$ is the fractional part of the answer: then I say, 1 borrowed and 7 make 8, and 8 subtracted from 14 leaves 6; which I prefix to the fractional part: so the difference or answer is $6\frac{2}{5}$.

Ex. 2. From 12 subtract $\frac{3}{4}$.

Here I say, $7 - 3 = 4$; so $\frac{4}{4}$ is the fractional part: then I say, 1 borrowed from 12, and 11 remains: so $11\frac{4}{4}$ is the difference, or answer.

Note. When an integer is given to be subtracted from a mixt number, you have only to subtract the given integer from the in-

tegral part of the mixt number; and to the remainder annex the fractional part. Thus, $9\frac{3}{4} - 5 = 4\frac{3}{4}$.

Rule V. If one or both of the given fractions be compound, first reduce the compound fractions to simple ones, by Prob. IV.; then reduce the simple fractions to a common denominator, by Prob. VIII.; and subtract the one numerator from the other.

Ex. 1. From $\frac{4}{7}$ subtract $\frac{3}{7}$ of $\frac{1}{2}$.

$\frac{3}{7}$ of $\frac{1}{2} = \frac{3}{14}$, by Prob. IV.

and $\frac{4}{7}, \frac{6}{14} = \frac{48}{80}, \frac{30}{80}$, by Prob. VIII.

and $\frac{48}{80} - \frac{30}{80} = \frac{18}{80} = \frac{9}{40}$, by Prob. VII.

Ex. 2. From $\frac{2}{3}$ of $\frac{1}{2}$ subtract $\frac{3}{7}$ of $\frac{1}{2}$.

$\frac{2}{3}$ of $\frac{1}{2}, \frac{1}{3}$ of $\frac{1}{2} = \frac{2}{6}, \frac{1}{6}$, by Prob. IV.

and $\frac{2}{6}, \frac{1}{6} = \frac{40}{60}, \frac{10}{60}$, by Prob. VIII.

and $\frac{40}{60} - \frac{10}{60} = \frac{30}{60} = \frac{1}{2}$, by Prob. VII.

Rule VI. When the given fractions are of different denominations, first reduce them to the same denomination, by Cor. of Prob. IV. or by Prob. V.; then reduce the fractions, now of one denomination, to a common denominator, by Prob. VIII.; and subtract the one numerator from the other. Or, reduce each of the given fractions, separately, to value, by Prob. VI.; and subtract the one value from the other.

EXAMPLE

From $\frac{3}{4}$ l. subtract $\frac{3}{4}$ s.

METHOD I.

$\frac{3}{4}$ s. = $\frac{3}{4}$ of $\frac{1}{20}$ l. = $\frac{3}{80}$ l. by Cor. Prob. IV.

and $\frac{3}{4}, \frac{2}{5} = \frac{180}{400}, \frac{80}{400}$, by Prob. VIII.

and $\frac{180}{400} - \frac{80}{400} = \frac{100}{400}$ l. = 14s. 4d. by Prob. VI.

METHOD II.

$\frac{3}{4}$ l. = $\frac{3 \times 20}{4}$ s. = $\frac{60}{4}$ s. by Prob. V.

and $\frac{60}{4}, \frac{2}{5} = \frac{180}{12}, \frac{8}{12}$, by Prob. VIII.

and $\frac{180}{12} - \frac{8}{12} = \frac{172}{12}$ s. = 14s. 4d. by Prob. I. and VI.

METHOD III.

$$\left. \begin{array}{l} \frac{3}{4}l. = \frac{3 \times 20}{4} s. = \frac{60}{4} s. = 15 \quad 0 \\ \frac{2}{3}s. = \frac{2 \times 12}{3} d. = \frac{24}{3} d. = 8 \end{array} \right\} \text{by Prob. VI.}$$

$$\begin{array}{r} 15 \quad 0 \\ \underline{8} \\ 14 \quad 4 \end{array}$$

MORE EXAMPLES.

- | | |
|---|--|
| 1. From $\frac{3}{4}l.$ subtract $\frac{5}{12}$. | Rem. $\frac{1}{12} = \frac{1}{12}$. |
| 2. From $\frac{8}{9}$ subtract $\frac{2}{3}$. | Rem. $\frac{2}{9}$. |
| 3. From $7\frac{1}{8}$ subtract $5\frac{3}{4}$. | Rem. $2\frac{1}{8}$. |
| 4. From 1 subtract $\frac{4}{7}$. | Rem. $\frac{3}{7}$. |
| 5. From $7\frac{1}{4}$ subtract $5\frac{1}{8}$. | Rem. $1\frac{1}{8}$. |
| 6. From $12\frac{7}{8}$ subtract $\frac{5}{8}$. | Rem. $12\frac{2}{8}$. |
| 7. From $13\frac{3}{8}$ subtract $\frac{5}{8}$. | Rem. $12\frac{6}{8}$. |
| 8. From 50 subtract $9\frac{7}{8}$. | Rem. $40\frac{1}{8}$. |
| 9. From 7 subtract $\frac{2}{3}$. | Rem. $6\frac{1}{3}$. |
| 10. From $15\frac{1}{4}$ subtract 7. | Rem. $8\frac{1}{4}$. |
| 11. From $\frac{8}{9}$ subtract $\frac{2}{3}$ of $\frac{5}{8}$. | Rem. $\frac{1}{9}$. |
| 12. From $\frac{2}{3}$ of $\frac{5}{8}$ subtract $\frac{1}{3}$ of $\frac{1}{4}$. | Rem. $\frac{1}{12}$. |
| 13. From $\frac{1}{3}l.$ subtract $\frac{7}{8}d.$ | Rem. $\frac{1}{24}l. = 6s. 7\frac{1}{2}d.$ |

The reason of the rules in subtraction is the same as in addition. For like things only can be subtracted from one another; and therefore in subtraction the fractions must have all the same denominator, and be of the same denomination.

Multiplication of Vulgar Fractions.

In multiplication of fractions there is no occasion to reduce the given fractions to a common denominator, as in addition and subtraction: only if a mixt number be given, reduce it to an improper fraction; if an integer be given, reduce it to an improper fraction, by putting an unit for its denominator; if a compound fraction be given, you may either reduce it to a simple one; or, instead of the particle *of*, insert the sign of multiplication: then work by the following

R U L E.

Multiply the numerators for the numerator of the product, and multiply the denominators for its denominator.

E X.

EXAMPLES.

1. $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

2. $\frac{3}{4} \times 5\frac{2}{3} = \frac{3}{4} \times \frac{17}{3} = \frac{51}{4} = 12\frac{3}{4} = 12\frac{3}{4}$.

3. $\frac{5}{7} \times 8 = \frac{5}{7} \times \frac{8}{1} = \frac{40}{7} = 5\frac{5}{7}$.

4. $\frac{2}{3}$ of $\frac{3}{4} \times \frac{5}{8} = \frac{6}{12} \times \frac{5}{8} = \frac{30}{96} = \frac{5}{16}$.

Or, $\frac{2}{3} \times \frac{3}{4} \times \frac{5}{8} = \frac{30}{96} = \frac{5}{16}$.

NOTES.

1. If any number be multiplied by a proper fraction, the product will be less than the multiplicand; for multiplication is the taking of the multiplicand as often as the multiplier contains unity; and consequently, if the multiplier be greater than unity, the product will be greater than the multiplicand; if the multiplier be unity, the product will be equal to the multiplicand; and if the multiplier be less than unity, the product will, in the same proportion, be less than the multiplicand. Thus, supposing the multiplier to be $\frac{1}{2}$ or $\frac{1}{3}$, the product, in this case, will be equal to one half or to one third of the multiplicand.

2. Mixt numbers may be multiplied without reducing them to improper fractions, by working as in the margin; where I first multiply the integral parts, viz. 54 by 24; then I multiply the integral parts cross-ways into their altern fractions, viz. 54 by $\frac{1}{2}$, and the product 27 I set down; in like manner, I multiply 24 by $\frac{1}{4}$, and the product 6 I likewise set down; then I add; and to the sum I annex $\frac{1}{8}$, the product of the two fractions.

$$\begin{array}{r} 54\frac{1}{2} \\ 24\frac{1}{2} \\ \hline 216 \\ 108 \\ 27 \\ 6 \\ \hline 1329\frac{1}{8} \end{array}$$

3. In multiplying a fraction by an integer, you have only to multiply the numerator by the integer, the putting 1 for the denominator being only matter of form. And to multiply a fraction by its denominator is to take away the denominator, the product being an integer, the same with, or equal to the numerator. Thus, $\frac{7}{8} \times 8 = 7$. For $\frac{7}{8} \times \frac{8}{1} = \frac{56}{8} = 7$.

4. If the numerators and denominators of two equal fractions be multiplied cross-ways, the products will be equal. Thus, if $\frac{3}{4} = \frac{9}{12}$, then will $3 \times 12 = 9 \times 4$; for multiplying both by 9, we have

$$3 = \frac{9 \times 4}{12}; \text{ and multiplying these by 12, we have } 3 \times 12 = 9 \times 4.$$

Hence, if four numbers be proportional, the product of the extremes will be equal to the product of the means: for if $3 : 9 :: 4 : 12$, then $\frac{3}{9} = \frac{4}{12}$; and it has been proved that $3 \times 12 = 9 \times 4$. Therefore if, of four proportional numbers, any three be given, the fourth may easily be found, viz. when one of the extremes is sought, divide the product

product of the means by the given extreme ; and when one of the means is sought, divide the product of the extremes by the given mean.

5. In multiplying fractions, equal factors above and below may be dashed or dropt. Thus, $\frac{1}{2}$ of $\frac{2}{3} \times \frac{3}{4}$ of $\frac{4}{5} = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$; and dropping the factors 2, 3, 4, both above and below, the product is $\frac{1}{5}$. In like manner, to facilitate an operation, a factor above and another below may be divided by the same number : Thus,

$\frac{6}{7} \times \frac{5}{12} = \frac{1}{7} \times \frac{5}{2} = \frac{5}{14}$. Or we may exchange one numerator for another : Thus, $\frac{6}{7} \times \frac{5}{12} = \frac{6}{12} \times \frac{5}{7} = \frac{1}{2} \times \frac{5}{7} = \frac{5}{14}$.

6. To take any part of a given number, is to multiply the said number by the fraction. Thus, $\frac{5}{8}$ of 320 is found thus, $\frac{5}{8} \times \frac{320}{1} = \frac{5}{8} \times \frac{320}{8} = \frac{5}{1} \times \frac{40}{1} = \frac{200}{1} = 200$. In like manner, $\frac{2}{3}$ of $45\frac{3}{8}$, is $\frac{2}{3} \times 45\frac{3}{8} = \frac{2}{3} \times \frac{363}{8} = \frac{2}{8} \times \frac{363}{3} = \frac{1}{4} \times \frac{121}{1} = \frac{121}{4} = 30\frac{1}{4}$. Hence, to reduce a compound fraction to a simple one, is to multiply the parts of it into one another.

7. If a multiplicand of two or more denominations be given to be multiplied by a fraction, reduce the higher part or parts of the multiplicand to the lowest species, and then multiply. Thus, to multiply 8l. 10 $\frac{1}{4}$ s. by $\frac{2}{3}$, I say, 8l. = 8 $\times$ 20s. = 160s. and 160 + 10 $\frac{1}{4}$ = 170 $\frac{1}{4}$ s. = $\frac{681}{4}$, and $\frac{2}{3} \times \frac{681}{4} = \frac{1362}{12} = 113\frac{10}{12}$ s. = L. 5, 13s. 10d. Or, without reducing, you may multiply the given multiplicand by the numerator of the fraction, and divide the product by the denominator.

MORE EXAMPLES.

- | | |
|---|---------------------------------------|
| 1. Multiply $\frac{2}{3}$ by $\frac{9}{11}$. | Product $\frac{4}{11}$. |
| 2. Multiply $7\frac{8}{9}$ by $\frac{4}{5}$. | Product $6\frac{14}{5}$. |
| 3. Multiply $8\frac{2}{3}$ by $9\frac{1}{4}$. | Product $84\frac{1}{2}$. |
| 4. Multiply $6\frac{4}{7}$ by 8. | Product $54\frac{2}{7}$. |
| 5. Multiply $9\frac{1}{2}$ by $\frac{1}{2}$ of $\frac{3}{4}$. | Product $3\frac{9}{8}$. |
| 6. Multiply 12 by $\frac{3}{4}$ of $\frac{4}{5}$. | Product $7\frac{1}{5}$. |
| 7. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$. | Product $\frac{1}{6}$. |
| 8. Multiply $\frac{1}{8}$ by 18. | Product 13. |
| 9. Multiply L. 3 : 12 : 6 $\frac{1}{2}$ by $\frac{3}{4}$. | Product L. 2 : 14 : 4 $\frac{7}{8}$. |

The reason of the rule may be shewn thus : $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$: for $\frac{4}{5} = \frac{12}{15}$, and $\frac{1}{3}$ of $\frac{12}{15}$ is $\frac{4}{15}$; and consequently $\frac{2}{3}$ of $\frac{12}{15}$ is $\frac{8}{15}$.

The truth of the rule may also be proved thus : Assume two fractions equal to two integers, such as, $\frac{8}{4}$ and $\frac{6}{2}$, equal to 2 and 3, and the product of the fractions will be equal to the product of the integers; for $\frac{8}{4} \times \frac{6}{2} = \frac{48}{8} = 6$, and $2 \times 3 = 6$.

Division

Division of Vulgar Fractions.

In division of fractions, if a mixt number be given, reduce it to an improper fraction; if an integer be given, put an unit for its denominator; if a compound fraction be given, reduce it to a simple one, and then work by the following

R U L E.

Multiply cross-ways, viz. the numerator of the divisor into the denominator of the dividend, for the denominator of the quot; and the denominator of the divisor into the numerator of the dividend, for the numerator of the quot.

E X A M P L E S.

1. $\frac{2}{3}) \frac{4}{5} (\frac{12}{15} = 1\frac{2}{15} = 1\frac{1}{7\frac{1}{2}}.$
2. $\frac{1}{4}) 4\frac{1}{2} (= \frac{9}{2}) \frac{17}{4} (\frac{68}{16} = 5\frac{8}{16} = 5\frac{1}{2}.$
3. $4\frac{2}{3}) \frac{7}{8} (= \frac{14}{3}) \frac{7}{8} (\frac{21}{16} = 1\frac{5}{16}.$
4. $3\frac{1}{4}) 6\frac{5}{8} (= \frac{13}{4}) \frac{53}{8} (\frac{212}{32} = 2\frac{4}{32} = 2\frac{1}{8}.$
5. $\frac{4}{5}) 8 (= \frac{4}{5}) \frac{8}{1} (\frac{40}{40} = 10.$
6. $7) \frac{3}{4} (= 7) \frac{3}{4} (\frac{3}{28}.$
7. $2\frac{1}{2}) 15 (= \frac{5}{2}) \frac{15}{1} (\frac{15}{5} = 6.$
8. $5) 10\frac{1}{2} (= \frac{21}{2}) \frac{21}{2} (\frac{21}{10} = 2\frac{1}{10}.$
9. $\frac{7}{8}) \frac{1}{4} \text{ of } \frac{5}{6} (= \frac{7}{8}) \frac{15}{24} (\frac{15}{168} = \frac{5}{56}.$
10. $\frac{1}{2} \text{ of } \frac{3}{4}) 4\frac{1}{2} (= \frac{9}{8}) \frac{9}{2} (\frac{72}{72} = 12.$
11. $\frac{2}{3} \text{ of } \frac{4}{5}) 8 (= \frac{8}{15}) \frac{8}{1} (\frac{120}{15} = 15.$
12. $\frac{1}{4} \text{ of } \frac{2}{3} \text{ of } \frac{1}{2}) \frac{4}{5} \text{ of } \frac{5}{6} (= \frac{6}{14}) \frac{30}{14} (\frac{480}{140} = 2\frac{2}{7}.$

N O T E S.

1. Instead of working division of fractions as taught above, you may invert the divisor, and then multiply it into the dividend. Thus, in Example 1. instead of $\frac{2}{3}) \frac{4}{5} (\frac{12}{15}$, you may say, $\frac{3}{2} \times \frac{4}{5} = \frac{12}{10} = 1\frac{2}{10} = 1\frac{1}{5}.$

2. If any number be divided by a proper fraction, the quot will be greater than the dividend: for in division the quot shews how often the divisor is contained in the dividend; and consequently if the divisor be greater than unity, the quot will be less than the dividend; if the divisor be unity, the quot will be equal to the dividend; and if the divisor be less than unity, the quot will, in the same proportion, be greater than the dividend. Thus, supposing the divisor to be $\frac{1}{2}$, or $\frac{1}{3}$, the quot in this case will be double or triple of the dividend.

3. To divide a fraction by an integer, is only to multiply the integer into the denominator of the fraction, the numerator being continued. Thus, $7\frac{1}{2} \div \frac{1}{2}$. See Ex. 6.

4. A mixt number may sometimes be divided by an integer, with more ease, in the following manner. Divide the integral part of the mixt number by the given integer; and if there be no remainder, divide likewise the fraction of the mixt number by the given integer, and annex the quot to the integral quot formerly found. Thus, in working Ex. 8. by this method, I say, $5)10(2$, and $5)\frac{1}{2}(\frac{1}{10}$; and so the complete quot is $2\frac{1}{10}$, as before. But if, in dividing the integral part, there happen to be a remainder, prefix this remainder to the fraction for a new mixt number; which reduce to an improper fraction: then divide the improper fraction by the given integer; and annex the quot to the integral quot formerly found. Thus, if it be required to divide $15\frac{1}{2}$ by 8, I say, $8)15(1$, and 7 remains; which 7, prefixed to the fraction, gives $7\frac{1}{2}$ for a new mixt number; and this, reduced to an improper fraction, is $\frac{15}{2}$, and $8)\frac{15}{2}(\frac{3}{4}$; so the complete quot is $1\frac{3}{4}$.

5. If the factors of the numerator and denominator of the quot, instead of being actually multiplied, be only connected with the sign of multiplication, it will be easy to drop such factors, above

and below, as happen to be the same, thus: $\frac{3}{4}$ of $\frac{3}{8}$ $\left(\frac{4 \times 4 \times 3}{3 \times 5 \times 8} \right)$

$\frac{4 \times 4}{5 \times 8} = \frac{16}{40} = \frac{2}{5}$. Or a factor above and below may be divided

by the same number, thus: $\frac{6 \times 7 \times 7}{5 \times 12 \times 5 \times 2} = \frac{7}{10}$. Or the factors of the numerator of the quot may be exchanged, thus: $\frac{2}{3} \times \frac{5}{9}$

$\left(\frac{3 \times 5 \times 5 \times 3}{2 \times 9 \times 2 \times 9} \right) = \frac{25}{54}$

6. To divide an integer by a fraction, is to divide the product of the denominator and integer by the numerator, thus: $\frac{4}{5}) 8$

$\left(\frac{5 \times 8}{4} = 5 \times 2 = 10 \right)$. See Ex. 5.

7. If the divisor and dividend have the same denominator, you have only to divide the numerator of the dividend by the numerator of the divisor, thus: $\frac{4}{5}) \frac{3}{5} (= \frac{3}{5})$; for $\frac{5}{8}) \frac{3}{8} (= \frac{3}{8})$

$\left(\frac{8 \times 3}{5 \times 8} = \frac{3}{5} \right)$

8. If a dividend of two or more denominations be given to be divided by a fraction, reduce the higher part or parts of the dividend to the lowest species, and then divide. Thus, to divide $6l. 9\frac{3}{4}s.$ by

by $\frac{2}{3}$, I say, $6l. = 6 \times 20s. = 120$; and $120 + 9\frac{3}{4} = 129\frac{3}{4}s. = \frac{519}{4}$; and $\frac{2}{3})\frac{519}{4}(\frac{1557}{8} = 194\frac{5}{8}s. = 9l. 14s. 7\frac{1}{2}d.$

Or, Divide the given dividend by the numerator of the fraction, and multiply the quot by the denominator.

E X A M P L E.

Divide L. 276 : 16 : 8 among 4 men, A, B, C, D, so that A, B, C, may have equal shares, and D only two thirds of one of their shares.

$$1 + 1 + 1 + \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{11}{3} \text{ of } \frac{1}{4} = \frac{11}{12}.$$

L.	s.	d.	L.	s.	d.	L.	s.	d.
11)276	16	8	(25	3	4	$4 \times 3 = 75$	10	
						$\times 3 = 75$	10	A.
						$\times 3 = 75$	10	B.
						$\times 2 = 50$	6	8 D.

Proof 276 16 8

M O R E E X A M P L E S.

- | | |
|--|---|
| 1. By $\frac{3}{4}$ divide $\frac{5}{8}$. | Quot $\frac{25}{12} = 1\frac{1}{12}$. |
| 2. By $3\frac{3}{4}$ divide $2\frac{2}{3}$. | Quot $\frac{16}{15}$. |
| 3. By 9 divide $19\frac{1}{2}$. | Quot $2\frac{1}{6}$. |
| 4. By $5\frac{2}{3}$ divide 18. | Quot $3\frac{1}{3}$. |
| 5. By $7\frac{1}{2}$ divide $\frac{4}{3}$ of $\frac{5}{8}$. | Quot $\frac{4}{15}$. |
| 6. By $\frac{3}{4}$ divide 12l. 6s. 8d. | Quot $\frac{11840}{3}d. = 16l. 8s. 10\frac{2}{3}d.$ |

The reason of the rule will appear by considering, that the method here used is nothing else but the reducing the divisor and dividend to a common denominator, and then dividing the one numerator by the other. Thus, $\frac{3}{4})\frac{5}{8}(\frac{5}{6}$, for reducing the divisor and dividend to a common denominator, we have $\frac{9}{12})\frac{10}{12}(\frac{5}{6}$.

The truth of the rule may also be proved by assuming two fractions equal to two integers, such as, $\frac{6}{3}$ and $\frac{16}{4}$, equal to 2 and 4, and the quot of the fractions will be equal to the quot of the integers. Thus, $\frac{6}{3})\frac{16}{4}(\frac{48}{3} = 2$, and $2)4(2$.

Practical Questions.

Quest. 1. The lesser of two numbers is $36\frac{8}{9}$; their difference is $11\frac{2}{3}$: What is the greatest? Ans. $48\frac{1}{3}$.

Quest. 2. What number added to $4\frac{7}{8}$, will give $12\frac{4}{5}$? Ans. $7\frac{11}{40}$.

Quest. 3. A has $\frac{7}{10}$ of a ship, and B has $\frac{2}{5}$ of the same vessel: Which of them has the greatest share? and what the difference? Ans. A has the greatest share; and the difference of their shares is $\frac{1}{10}$.

Quest.

Quest. 4. Three purses contain $130\frac{2}{3}$ l.; in one purse is $55\frac{1}{3}$ l. in another $42\frac{4}{7}$ l.: How much is in the third purse? *Ans.* $32\frac{1}{7}$ l.

Quest. 5. What is $\frac{2}{3}$ parts of $130\frac{2}{3}$? *Ans.* $81\frac{2}{3}$.

Quest. 6. What number multiplied by $\frac{1}{2}$ will produce $25\frac{2}{3}$? *Ans.* $42\frac{2}{3}$.

The Simple Rule of Three in Vulgar Fractions.

The question is stated as formerly taught in the rule of three. The extremes must be of one denomination. Reduce mixt numbers and integers to improper fractions, compound fractions to simple ones, and then work by the following rule, *viz.*

Multiply the second and third terms, and divide the product by the first term: that is, multiply the numerator of the first term into the denominators of the second and third, for the denominator of the answer; and multiply the denominator of the first term into the numerators of the second and third, for the numerator of the answer.

I. Direct Questions.

Quest. 1. If $\frac{3}{4}$ yard cost $\frac{5}{8}$ l. what will $\frac{9}{10}$ yard cost?

Ans. *Rs.* *L.* *Rs.*

If $\frac{3}{4} : \frac{5}{8} :: \frac{9}{10} : ?$

$$\text{Ans. } \frac{4 \times 5 \times 9}{3 \times 8 \times 10} = \frac{5 \times 9}{3 \times 2 \times 10} = \frac{9}{3 \times 2 \times 2} = \frac{3}{2 \times 2} = \frac{3 \times 20}{4} = \frac{60}{4} = 15 \text{ s.}$$

Quest. 2. If $\frac{5}{8}$ yard cost $\frac{2}{3}$ l. what will $\frac{11}{12}$ yard cost?

Ans. *Rs.* *L.* *Rs.*

If $\frac{5}{8} : \frac{2}{3} :: \frac{11}{12} : ?$

$$\text{Ans. } \frac{6 \times 2 \times 11}{5 \times 3 \times 12} = \frac{2 \times 11}{5 \times 3 \times 2} = \frac{11}{5 \times 3} = \frac{11 \times 20}{15} = \frac{220}{15} = 14 \text{ s. } 8 \text{ d.}$$

Quest. 3. If 3 yards cost $2\frac{4}{5}$ l. what will $14\frac{3}{4}$ yards cost?

Ans. *Rs.* *L.* *Rs.*

If $3 : 2\frac{4}{5} :: 14\frac{3}{4} : ?$

$\frac{3}{1} : \frac{14\frac{3}{4}}{1} :: \frac{2\frac{4}{5}}{1} : ?$

$$\text{Ans. } \frac{1 \times 14 \times 59}{3 \times 5 \times 4} = \frac{7 \times 59}{3 \times 5 \times 2} = \frac{413}{30} = 13 \text{ l. } 15 \text{ s. } 4 \text{ d.}$$

MORE

MORE EXAMPLES.

Quest. 4. If $2\frac{1}{4}$ lb. tobacco cost $3\frac{1}{2}$ s. what will $242\frac{1}{4}$ lb. cost?

Ans. L. 15 : 8 : $3\frac{1}{2}$ r.

Quest. 5. If $\frac{1}{4}$ of $\frac{1}{8}$ C. sugar cost $\frac{1}{2}$ l. what will 82 C. cost?

Ans. 262 l. 8 s.

Quest. 6. A mercer bought $3\frac{1}{2}$ pieces of silk, each piece containing $24\frac{1}{2}$ ells, at 6 s. $\frac{1}{4}$ d. per ell: Required the value of the $3\frac{1}{2}$ pieces at that rate? *Ans.* L. 26 : 3 : $4\frac{1}{4}$.

Quest. 7. If $\frac{1}{2}$ oz. silver cost 2 s. what will be the price of $11\frac{1}{2}$ lb. at that rate? *Ans.* 35 l.

Quest. 8. If $1\frac{1}{2}$ lb. of gold is worth 61 $\frac{1}{2}$ l. Sterling, what is a grain worth at that rate? *Ans.* $1\frac{1}{2}$ d.

Quest. 9. If $\frac{1}{4}$ yards of silk cost $\frac{1}{4}$ of $\frac{1}{8}$ l. what is the price of $15\frac{1}{2}$ ells Flemish? *Ans.* L. 9 : 15 : 10.

Quest. 10. If $\frac{2}{3}$ of $\frac{1}{4}$ lb. of cloves cost 6 s. $2\frac{1}{2}$ d. what cost the C. at that rate? *Ans.* L. 69 : 6 : 8.

II. Inverse Questions.

Quest. 1. If $\frac{1}{4}$ yard of cloth that is 2 yards wide, will make a garment, how much of any other cloth that is $\frac{3}{4}$ yard wide will make the same garment?

Bread.	len.	Bread.
$\frac{1}{4}$	2	$\frac{3}{4}$
5	3	2
3	4	1

Ans. $\frac{5 \times 3 \times 2}{3 \times 4 \times 1} = \frac{5 \times 2}{4} = \frac{10}{4} = 2\frac{1}{2}$ yards.

Quest. 2. If I lend my friend 48 l. for $\frac{1}{4}$ of a year, how much ought he to lend me for $\frac{3}{4}$ of a year?

Yea.	L.	Yea.
$\frac{1}{4}$	48	$\frac{3}{4}$
12	48	3
5	1	4

Ans. $\frac{12 \times 48 \times 3}{5 \times 1 \times 4} = \frac{12 \times 12 \times 3}{5} = \frac{432}{5} = 86\text{ l. } 8\text{ s.}$

MORE EXAMPLES.

Quest. 3. If $\frac{3}{4}$ yard of cloth that is $2\frac{1}{2}$ yards wide, will make a coat, what is the breadth of the cloth whereof $1\frac{1}{4}$ yard will make the same coat? *Ans.* $\frac{1}{2}$ yd, or 3 qrs $2\frac{1}{2}$ nails.

Quest. 4. How many inches in length, of a plank that is 9 inches broad, will make a foot square? *Ans.* 16 inches in length.

Quest. 5. If the penny-loaf weigh $10\frac{1}{2}$ ounces when the bushel of wheat

wheat costs $4\frac{1}{2}$ s. what ought the penny-loaf to weigh when the bushel of wheat costs $8\frac{2}{3}$ s.? *Ans.* $5\frac{1}{2}\frac{8}{9}$ ounces.

Quest. 6. If 12 men do a piece of work in $10\frac{2}{3}$ days, in how many days will 5 men do the same? *Ans.* In $21\frac{1}{3}$ days.

The Compound Rule of Three in Vulgar Fractions.

Quest. 1. If $\frac{1}{4}$ acre of grafs be cut down by 2 men in $\frac{1}{3}$ day, how many acres shall be cut down by 6 men in $3\frac{1}{3}$ days?

<i>Men.</i>	<i>acr.</i>	<i>Men.</i>
$\frac{2}{1}$	$\frac{1}{4}$	$\frac{6}{1}$
day $\frac{1}{3}$		$3\frac{1}{3} = \frac{10}{3}$ days.
$\frac{4}{1}$	$\frac{3}{4}$	$\frac{60}{3}$

$$\text{Ans. } \frac{3 \times 3 \times 60}{4 \times 4 \times 3} = \frac{3 \times 60}{4 \times 4} = \frac{3 \times 15}{4} = \frac{45}{4} = 11\frac{1}{4} \text{ acres.}$$

Or thus:

$$\text{Ans. } \frac{3 \times 1 \times 3 \times 6 \times 10}{2 \times 2 \times 4 \times 1 \times 3} = \frac{3 \times 6 \times 10}{2 \times 2 \times 4} = \frac{3 \times 3 \times 5}{2 \times 2} = \frac{45}{4} = 11\frac{1}{4} \text{ acres.}$$

Quest. 2. If 4 men cast a ditch $8\frac{1}{2}$ feet long, $3\frac{1}{3}$ deep, and $2\frac{1}{4}$ broad, in $9\frac{1}{3}$ days, in how many days will 8 men cast a ditch $25\frac{1}{2}$ feet long, $6\frac{2}{3}$ deep, and $4\frac{1}{2}$ broad?

<i>b.</i>	<i>d.</i>	<i>*8</i>	<i>Men.</i>	<i>d.</i>	<i>b.</i>
$2\frac{1}{4}$	$3\frac{1}{3}$	$1.8\frac{1}{2}$	$9\frac{1}{3}$	4^*	$25\frac{1}{2}$
		$\frac{8}{1}$	$\frac{28}{3}$	$\frac{4}{1}$	$6\frac{2}{3}$
$\frac{9}{4}$	$\frac{10}{3}$	$\frac{17}{2}$	$\frac{51}{2}$	$\frac{20}{3}$	$\frac{9}{2}$

$$\frac{9 \times 10 \times 17 \times 8}{4 \times 3 \times 2 \times 1} = \frac{3 \times 5 \times 17 \times 2}{1} = 510 \text{ first term.}$$

$$\frac{4 \times 51 \times 20 \times 9}{1 \times 2 \times 3 \times 2} = \frac{2 \times 51 \times 10 \times 3}{1} = 3060 \text{ third term.}$$

$$\frac{510}{3060} : \frac{28}{3} :: \frac{3060}{255}$$

$$\frac{1 \times 28 \times 3060}{510 \times 3 \times 1} = \frac{14 \times 1020}{255} = \frac{14 \times 80}{3} = 56 \text{ days.}$$

M

Or

Or thus:

$$\begin{aligned}
 & \frac{4 \times 3 \times 2 \times 1 \times 28 \times 4 \times 51 \times 20 \times 9}{9 \times 10 \times 17 \times 8 \times 3 \times 1 \times 2 \times 3 \times 2} = \\
 & \frac{4 \times 3 \times 2 \times 28 \times 4 \times 51 \times 20}{10 \times 17 \times 8 \times 3 \times 2 \times 3 \times 2} = \frac{2 \times 28 \times 4 \times 51 \times 2}{17 \times 2 \times 2 \times 3 \times 2} = \frac{28 \times 4 \times 51}{17 \times 3 \times 2} = \\
 & \frac{14 \times 4 \times 51}{17 \times 3} = \frac{14 \times 4 \times 17}{17} = \frac{14 \times 4}{1} = 56 = 56 \text{ days.}
 \end{aligned}$$

C H A P. IX.

Rules of Practice.

WHEN the first term of a question in the rule of three happens to be unity, the answer may frequently be found more speedily and easily than by a formal stating or working of the rule of three; and the directions to be observed in such operations are called *Rules of Practice*.

The rules of practice naturally follow the doctrine of vulgar fractions, the operation being nothing else but a multiplying the number whose price is required, by such a fraction of a pound, of a shilling, or of a penny, as denotes the rate or price of one.

Thus, if the price of 24 yards, at 6s. 8d. *per* yard, be demanded, the answer is found by multiplying 24 by $\frac{1}{3}$, the fraction of a pound equivalent to 6s. 8d. *viz.* $\frac{24}{3} \times \frac{1}{3} = \frac{24}{3} = 8$ l.

Hence it is obvious, that to multiply a number by a fraction whose numerator is unity, is to divide the said number by the denominator of the fraction. But if the numerator of the fraction be not unity, you must first multiply the given number by the numerator, and then divide the product by the denominator. Thus, if the rate be 13s. 4d. $= \frac{2}{3}$ l. the price of 24 yards is found by saying, $\frac{24}{1} \times \frac{2}{3} = \frac{48}{3} = 16$ l.; or take $\frac{2}{3}$ of the given number twice.

When the fraction denoting the rate happens to be compound, the product or answer is found by dividing the given number by one of the denominators of the compound fraction, the quot by another, and the next quot by the third, &c. Thus, if the rate be 2 farthings $= \frac{1}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ l. the price of 1440 yards is found by saying, $\frac{1440}{2} = 720$, and $\frac{720}{12} = 60$, and $\frac{60}{20} = 3$ l.

When the rate is expressed by two or more simple fractions, connected with the sign +, the product or answer is found by dividing the given number successively by the several denominators, and then adding the quots. Thus, if the rate be 3s. $= \frac{1}{5} + \frac{1}{10}$ l. the price of 80 yards is found by saying, $\frac{80}{5} = 16$, and $\frac{16}{2} = 8$, and $16 + 8 = 24$ l.

The

The fractions equivalent to any number of farthings under 4, to any number of pence under 12, and to any number of shillings under 20, are exhibited in the following tables.

T A B L E I.

Farthings.	of a penny.	of a shilling.	of a pound.
1	$\frac{1}{4}$	$\frac{1}{4}$ of $\frac{1}{12}$	$\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$
2	$\frac{1}{2}$	$\frac{1}{2}$ of $\frac{1}{12}$	$\frac{1}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$
3	$\frac{3}{4}$	$\frac{3}{4}$ of $\frac{1}{12}$	$\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$

TABLE II.

Pen.	of a shill.
1	$\frac{1}{12}$
1 $\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
5	$\frac{1}{4} + \frac{1}{8}$
6	$\frac{1}{2}$
7	$\frac{1}{3} + \frac{1}{4}$
8	$\frac{1}{3} + \frac{1}{3}$
9	$\frac{1}{2} + \frac{1}{4}$
10	$\frac{1}{2} + \frac{1}{3}$
11	$\frac{1}{3} + \frac{1}{3} + \frac{1}{4}$

T A B L E III.

s.	d.	of a pound.	s.	d.	of a pound.
1		$\frac{1}{20}$	9		$\frac{4}{10} + \frac{1}{10}$
1	8	$\frac{1}{12}$	10		$\frac{5}{10}$, or $\frac{1}{2}$
2		$\frac{1}{10}$	11		$\frac{5}{10} + \frac{1}{10}$
2	6	$\frac{1}{8}$	12		$\frac{6}{10}$, or $\frac{3}{5}$
3		$\frac{1}{10} + \frac{1}{10}$	13		$\frac{6}{10} + \frac{1}{10}$
3	4	$\frac{1}{6}$	13	4	$\frac{2}{3}$
4		$\frac{2}{10}$, or $\frac{1}{5}$	14		$\frac{7}{10}$
5		$\frac{2}{10} + \frac{1}{10}$, or $\frac{3}{10}$	15		$\frac{7}{10} + \frac{1}{10}$, or $\frac{3}{4}$
6		$\frac{3}{10}$	16		$\frac{8}{10}$, or $\frac{4}{5}$
6	8	$\frac{1}{3}$	17		$\frac{8}{10} + \frac{1}{10}$
7		$\frac{3}{10} + \frac{1}{10}$	18		$\frac{9}{10}$
8		$\frac{4}{10}$, or $\frac{2}{5}$	19		$\frac{9}{10} + \frac{1}{10}$

The fractions in Table II. become compound fractions of a pound, by annexing (of $\frac{1}{20}$) to each of them. Thus, 1 d. is $\frac{1}{12}$ of $\frac{1}{20}$ l.; and 5 d. is $\frac{1}{4}$ of $\frac{1}{10} + \frac{1}{8}$ of $\frac{1}{20}$ l. &c.

The variety that occurs in the rules of practice arises chiefly from the different rates, or prices, of one thing, as a yard, a pound, an ounce, &c. and may be reduced to the eight cases following, viz.

The rate may be, 1. Farthings under four. 2. Pence under twelve. 3. Pence and farthings. 4. Shillings under twenty. 5. Shillings, pence, and farthings. 6. Pounds. 7. Pounds, shillings, pence, and farthings. 8. The given number may consist of integers and parts.

C A S E I.

When the rate is farthings, under four.

R U L E.

Divide the given number by the denominator of the fraction denoting the rate, as contained in Table I. viz. if the rate be 1 or 2 farthings, divide by 4 or 2, the quot will be pence; and the remainder,

mainder, in dividing by 4, will be farthings, and in dividing by 2, it will be 1 halfpenny: then divide the pence by 12, the quot will be shillings, and the remainder pence: lastly, divide the shillings by 20, the quot will be pounds, and the remainder shillings. But if the rate is 3 farthings, first multiply the given number by the numerator 3, and then divide as above directed.

E X A M P L E S.

Ex. 1.
 $\frac{1}{4}$ 4859, at 1 f.

 $\frac{1}{12}$ 1214 3 f.

 $\frac{1}{20}$ 10|1 2 d.

L. 5 1 2 $\frac{3}{4}$.

Ex. 2.
 $\frac{1}{2}$ 8347, at 2 f.

 $\frac{1}{12}$ 4173 $\frac{1}{2}$ d.

 $\frac{1}{20}$ 34|7 9 d.

L. 17 7 9 $\frac{1}{2}$.

Ex. 3.
1753, at 3 f.
3

 $\frac{3}{4}$ 5259

 $\frac{1}{12}$ 1314 3 f.

 $\frac{1}{20}$ 10|9 6 d.

L. 5 9 6 $\frac{3}{4}$.

Or, because 3 f. = $\frac{1}{2}$ d. + $\frac{1}{4}$ d. or = $\frac{1}{2}$ d. + $\frac{1}{4}$ d. of $\frac{1}{4}$ d. you may work Ex. 3. either of the two ways following.

Meth. 1.
1753, at 3 f.

 $\frac{1}{2}$ 876 2 f.
 $\frac{1}{4}$ 438 1 f.

 $\frac{1}{12}$ 1314 3 f.

 $\frac{1}{20}$ 10|9 6 d.

L. 5 9 6 $\frac{3}{4}$.

Meth. 2.
 $\frac{1}{2}$ 1753, at 3 f.

 $\frac{1}{2}$ 876 2 f.
438 1 f.

 $\frac{1}{12}$ 1314 3 f.

 $\frac{1}{20}$ 10|9 6 d.

L. 5 9 6 $\frac{3}{4}$.

More ways of working the above examples might be assigned, but the most easy and simple methods are the best.

C A S E II.

When the rate is pence, under twelve.

R U L E.

Divide the given number by the denominator of the fraction denoting the rate, as contained in Table II. and you have the answer in shillings; which reduce into pounds, by dividing by 20.

EXAMPLES.

<i>Ex. 1.</i> 818, at 1 d. $\frac{1}{10}$ 6 8 2 d. L. 3 8 2		<i>Ex. 7.</i> 587, at 7 d. 4 d. $\frac{1}{3}$ 195 8 3 d. $\frac{1}{4}$ 146 9 $\frac{1}{20}$ 34 2 5 L. 17 2 5	
<i>Ex. 2.</i> 5316, at 2 d. $\frac{1}{8}$ 88 6 L. 44 6		<i>Ex. 8.</i> 836, at 8 d. 4 d. $\frac{1}{3}$ 278 8 4 d. $\frac{1}{3}$ 278 8 $\frac{1}{20}$ 55 7 4 L. 27 17 4	
<i>Ex. 3.</i> 879, at 3 d. $\frac{1}{4}$ 21 9 9 L. 10 19 9		<i>Ex. 9.</i> 417, at 9 d. 6 d. $\frac{1}{2}$ 208 6 3 d. $\frac{1}{4}$ 104 3 $\frac{1}{20}$ 31 2 9 L. 15 12 9	
<i>Ex. 4.</i> 3097, at 4 d. $\frac{1}{3}$ 103 2 4 L. 51 12 4		<i>Ex. 10.</i> 386, at 10 d. 6 d. $\frac{1}{2}$ 193 4 d. $\frac{1}{3}$ 128 8 $\frac{1}{20}$ 32 1 8 L. 16 1 8	
<i>Ex. 5.</i> 439, at 5 d. 3 d. $\frac{1}{4}$ 109 9 2 d. $\frac{1}{8}$ 73 2 $\frac{1}{20}$ 18 2 11 L. 9 2 11		<i>Ex. 11.</i> 534, at 11 d. 4 d. $\frac{1}{3}$ 178 4 d. $\frac{1}{3}$ 178 3 d. $\frac{1}{4}$ 133 6 $\frac{1}{20}$ 48 9 6 L. 24 9 6	
<i>Ex. 6.</i> 78642, at 6 d. $\frac{1}{2}$ 3932 1 L. 1966 1			

Note 1. The remainders at the first division in all the above examples is the same with the rate. Thus, in Ex. 1. every remainder is 1 d.; in Ex. 3. every remainder is 3 d.; and in Ex. 5. every remainder, in dividing by 4, is 3 d.; and, in dividing by 6, every remainder is 2 d. &c.

Note 2. In all the examples wherein the rate is no aliquot part of a shilling, the question may be solved as many different ways as you can find different fractions equivalent to the rate. Thus, 5 d. = $\frac{1}{4}$ s. + $\frac{1}{8}$ s. or = $\frac{1}{4}$ s. + $\frac{1}{2}$ of $\frac{1}{4}$ s. Or 5 d. = $\frac{1}{8}$ of half a crown, and 1 half crown = $\frac{1}{8}$ l.; that is, 5 d. = $\frac{1}{8}$ of $\frac{1}{8}$ l.; and accordingly Ex. 5. may also be solved the three ways following.

<i>Meth. 1.</i>		<i>Meth. 2.</i>		<i>Meth. 3.</i>	
439, at 5 d.		439, at 5 d.		439, at 5 d.	
$\frac{1}{4}$	146 4	$\frac{1}{4}$	146 4	$\frac{1}{8}$	73 h. cr. 5 d.
$\frac{1}{8}$	36 7	$\frac{1}{8}$	36 7		
$\frac{1}{20}$	18 2 11	$\frac{1}{20}$	18 2 11		L. 9 2 11
	L. 9 2 11		L. 9 2 11		

In like manner, 9 d. = $\frac{1}{2}$ s. + $\frac{1}{2}$ of $\frac{1}{2}$ s. or 9 d. = $\frac{1}{40}$ l. + $\frac{1}{2}$ of $\frac{1}{40}$ l.; and accordingly Ex. 9. may also be solved other two ways, as under.

<i>Meth. 1.</i>		<i>Meth. 2.</i>	
417, at 9 d.		417, at 9 d.	
$\frac{1}{2}$	208 6	$\frac{1}{40}$	10 8 6
$\frac{1}{2}$	104 3	$\frac{1}{2}$	5 4 3
$\frac{1}{20}$	31 2 9		L. 15 12 9
	L. 15 12 9		

Note 3. When the rate is 11 d. you may work as if it were 12 d.; that is, from the given number you may subtract $\frac{1}{12}$ of itself, and

the remainder will be the price in shillings. Thus, $\frac{534 \text{ s.}}{12} = 44 \text{ s.}$

6 d. and 534 s. — 44 s. 6 d. = 489 s. 6 d. = L. 24 : 9 : 6. See Ex. 11.

C A S E III.

When the rate is pence and farthings.

R U L E.

The pence must be some aliquot part of a shilling, and at the same time the farthings some aliquot part of the pence; and if they be not so given, divide the pence into two or more such parts, so as the farthings may be some aliquot part of the lowest division of the pence. Then beginning with the highest division of the pence,

pence, divide by the denominators of the fractions denoting the aliquot parts.

EXAMPLES.

<i>Ex. 1.</i> 1 d. $\frac{1}{12}$ 532, at $1\frac{1}{4}$ d. $\frac{1}{4}$ d. $\frac{1}{4}$ 44 4 11 1 $\frac{1}{20}$ 515 5 L. 2 15 5			<i>Ex. 5.</i> 2 d. $\frac{1}{8}$ 485, at $2\frac{1}{4}$ d. $\frac{1}{4}$ d. $\frac{1}{8}$ 80 10 10 $1\frac{1}{4}$ $\frac{1}{20}$ 910 $11\frac{1}{4}$ L. 4 10 $11\frac{1}{4}$		
<i>Ex. 2.</i> 1 d. $\frac{1}{12}$ 1753, at $1\frac{1}{2}$ d. $\frac{1}{2}$ d. $\frac{1}{2}$ 146 1 73 $0\frac{1}{2}$ $\frac{1}{20}$ 2119 $1\frac{1}{2}$ L. 10 19 $1\frac{1}{2}$ Or rather thus: $1\frac{1}{2}$ d. $\frac{1}{8}$ 1753, at $1\frac{1}{2}$ d. $\frac{1}{20}$ 2119 $1\frac{1}{2}$ L. 10 19 $1\frac{1}{2}$			<i>Ex. 6.</i> 3 d. $\frac{1}{4}$ 3471, at $3\frac{1}{2}$ d. $\frac{1}{2}$ d. $\frac{1}{8}$ 867 9 144 $7\frac{1}{2}$ $\frac{1}{20}$ 10112 $4\frac{1}{2}$ L. 50 12 $4\frac{1}{2}$		
<i>Ex. 3.</i> $1\frac{1}{2}$ d. $\frac{1}{8}$ 1753, at $1\frac{3}{4}$ d. $\frac{1}{4}$ d. $\frac{1}{8}$ 219 $1\frac{1}{2}$ 36 $6\frac{1}{4}$ $\frac{1}{20}$ 2515 $7\frac{3}{4}$ L. 12 15 $7\frac{3}{4}$			<i>Ex. 7.</i> 4 d. $\frac{1}{3}$ 475, at $4\frac{3}{4}$ d. $\frac{1}{2}$ d. $\frac{1}{8}$ 158 4 $\frac{1}{4}$ d. $\frac{1}{2}$ 19 $9\frac{1}{2}$ 9 $10\frac{3}{4}$ $\frac{1}{20}$ 1818 $0\frac{1}{4}$ L. 9 8 $0\frac{1}{4}$		
<i>Ex. 4.</i> 1 d. $\frac{1}{12}$ 859, at $1\frac{1}{8}$ d. $\frac{1}{8}$ d. $\frac{1}{8}$ 71 7 8 11 $1\frac{1}{2}$ f. $\frac{1}{20}$ 810 6 $1\frac{1}{2}$ L. 4 0 6 $1\frac{1}{2}$ f.			<i>Ex. 8.</i> $1\frac{1}{2}$ d. $\frac{1}{8}$ 976, at $5\frac{1}{2}$ d. 4 d. $\frac{1}{3}$ 325 4 $1\frac{1}{2}$ d. $\frac{1}{8}$ 122 $\frac{1}{20}$ 4417 4 L. 22 7 4		
			<i>Ex. 9.</i>		

			<i>Ex. 9.</i>						<i>Ex. 12.</i>		
			3506, at $6\frac{1}{4}$ d.						7012, at $9\frac{1}{4}$ d.		
3 d.	$\frac{1}{4}$		876	6		6 d.	$\frac{1}{2}$		3506		
3 d.	$\frac{1}{4}$		876	6		3 d.	$\frac{1}{4}$		1753		
$\frac{1}{4}$ d.	$\frac{1}{12}$		73	$0\frac{1}{2}$		$\frac{1}{4}$ d.	$\frac{1}{4}$		438	3	
	$\frac{1}{20}$		182	6	$0\frac{1}{2}$		$\frac{1}{20}$		569	7	3
			L. 91 6 $0\frac{1}{2}$						L. 284 17 3		
			<i>Ex. 10.</i>						<i>Ex. 13.</i>		
			520, at $7\frac{1}{4}$ d.						137, at $10\frac{1}{2}$ d.		
4 d.	$\frac{1}{3}$		173	4		6 d.	$\frac{1}{2}$		68	6	
3 d.	$\frac{1}{4}$		130			3 d.	$\frac{1}{2}$		34	3	
$\frac{1}{4}$ d.	$\frac{1}{4}$		32	6		$1\frac{1}{2}$ d.	$\frac{1}{2}$		17	$1\frac{1}{2}$	
	$\frac{1}{20}$		33	5	10		$\frac{1}{20}$		11	9	$10\frac{1}{2}$
			L. 16 15 10						L. 5 19 $10\frac{1}{2}$		
			<i>Ex. 11.</i>						<i>Ex. 14.</i>		
			783, at $8\frac{1}{4}$ d.						2753, at $11\frac{1}{4}$ d.		
6 d.	$\frac{1}{2}$		391	6		6 d.	$\frac{1}{2}$		1376	6	
2 d.	$\frac{1}{3}$		130	6		4 d.	$\frac{1}{3}$		917	8	
$\frac{1}{4}$ d.	$\frac{1}{8}$		16	$3\frac{1}{4}$		1 d.	$\frac{1}{4}$		229	5	
						$\frac{1}{4}$ d.	$\frac{1}{4}$		57	$4\frac{1}{4}$	
	$\frac{1}{20}$		53	8	$3\frac{1}{4}$		$\frac{1}{20}$		258	0	$11\frac{1}{4}$
			L. 26 18 $3\frac{1}{4}$						L. 129 0 $11\frac{1}{4}$		

EXPLICATION.

In *Ex. 1.* I work first for 1 d.; which being $\frac{1}{12}$ s. I divide the given number by the denominator 12, and the quot is shillings, and the remainder pence: then, because 1 farthing is $\frac{1}{4}$ d. I divide the former quot by 4, and the sum of the quots is the price in shillings; which I divide by 20.

In *Ex. 2.* the rate $1\frac{1}{2}$ d. being an aliquot part of a shilling, the second method is shorter and better than the first.

In *Ex. 3.* I work first for $1\frac{1}{2}$ d. and then for $\frac{1}{4}$ d. by taking $\frac{1}{2}$ of the former quot.

In *Ex. 7.* I work first for 4 d. then I work for $\frac{1}{2}$ d.; which being $\frac{1}{8}$ of 4 d. I take $\frac{1}{8}$ of the first quot: lastly, I work for $\frac{1}{4}$ d.; which being $\frac{1}{2}$ of $\frac{1}{2}$ d. I take $\frac{1}{2}$ of the second quot; and, adding the three quots, I have the price in shillings.

In

In Ex. 8. I divide the rate into two parts, *viz.* 4 d. and $1\frac{1}{2}$; each of which being an aliquot part of a shilling, I take first $\frac{1}{4}$, and then $\frac{1}{8}$, of the given number, and then add the quots.

In Ex. 9. I divide the 6 d. into two parts, *viz.* two 3 d. and then the $\frac{1}{4}$ d. is $\frac{1}{12}$ of 3 d. which makes an easy operation; but had I taken $\frac{1}{4}$ of the given number for 6 d. then $\frac{1}{4}$ d. would have been $\frac{1}{24}$ of 6 d. and 24 would have been a troublesome divisor.

Note 1. The above examples, and all others of the like kind, may also be solved by assuming fractions of a pound equivalent to the rate. Thus, in Ex. 6. the rate $3\frac{1}{2}$ d. is divided into 3 d. and $\frac{1}{2}$ d. and $3 \text{ d.} = \frac{1}{80} \text{ l.}$ and $\frac{1}{2} \text{ d.} = \frac{1}{80}$ of 3 d.; wherefore $3\frac{1}{2} \text{ d.} = \frac{1}{80} \text{ l.} + \frac{1}{80}$ of $\frac{1}{80} \text{ l.}$ Again, in Ex. 10. the rate $7\frac{3}{4}$ d. is divided into 4 d. 3 d. and $\frac{3}{4}$ d. and $4 \text{ d.} = \frac{1}{80} \text{ l.}$ and $3 \text{ d.} = \frac{1}{80} \text{ l.}$ and $\frac{3}{4} \text{ d.} = \frac{3}{4}$ of 3 d.; and so $7\frac{3}{4} \text{ d.} = \frac{1}{80} \text{ l.} + \frac{1}{80} \text{ l.} + \frac{3}{4}$ of $\frac{1}{80} \text{ l.}$ And these two examples wrought in this manner will stand as under.

Ex. 6.	
3 d. $\frac{1}{80}$	347 1, at $3\frac{1}{2}$ d.
$\frac{1}{2}$ d. $\frac{1}{80}$	43 7 9
	7 4 $7\frac{1}{2}$
	L. 50 12 $4\frac{1}{2}$

Ex. 10.	
4 d. $\frac{1}{80}$	52 0, at $7\frac{3}{4}$ d.
3 d. $\frac{1}{80}$	8 13 4
$\frac{3}{4}$ d. $\frac{1}{4}$	6 10
	1 12 6
	L. 16 15 10

Note 2. Some, in working the above, or like examples, proceed in the following manner :

Ex. 6.

	s.	L.	s.	d.
3471, at 12 d. is	3471,	or 173	11	
at 6 d. is		86	15	6
at 3 d. is		43	7	9
at $\frac{1}{2}$ d. is		7	4	$7\frac{1}{2}$
3471, at $3\frac{1}{2}$ d. is		L. 50	12	$4\frac{1}{2}$

Ex. 10.

	s.	L.	s.	d.
520, at 12 d. is	520,	or 26		
at 6 d. is		13		
at 1 d. is		2	3	4
at $\frac{1}{2}$ d. is		1	1	8
at $\frac{3}{4}$ d. is			10	10
520, at $7\frac{3}{4}$ d. is		L. 16	15	10

CASE

C A S E IV.

When the rate is shillings, under twenty.

R U L E.

Multiply the given number by the numerators of the fractions contained in Tab. III. and divide the product by the denominators. Or, instead of this general rule, take the two particular ones following:

1. If the rate be an even number of shillings, multiply the given number by half the number of shillings in the rate, always doubling the right-hand figure of the product for shillings, and the rest are pounds.

2. If the rate be an odd number of shillings, work for the next lesser even number of shillings, as above; and for the odd shilling take $\frac{1}{20}$ of the given number.

E X A M P L E S.

1. When the rate is an even number of shillings.

<i>Ex. 1.</i> 436, at 2 s. 1 L. 43, 12 s.	<i>Ex. 4.</i> 48, at 8 s. 4 L. 19, 4 s.	<i>Ex. 7.</i> 326, at 14 s. 7 L. 228, 4 s.
<i>Ex. 2.</i> 127, at 4 s. 2 L. 25, 8 s.	<i>Ex. 5.</i> 87, at 10 s. 5 L. 43, 10 s.	<i>Ex. 8.</i> 48, at 16 s. 8 L. 38, 8 s.
<i>Ex. 3.</i> 56, at 6 s. 3 L. 16, 16 s.	<i>Ex. 6.</i> 420, at 12 s. 6 L. 252	<i>Ex. 9.</i> 52, at 18 s. 9 L. 46, 16 s.

2. When the rate is an odd number of shillings.

<i>Ex. 10.</i> 635, at 1 s. L. 31, 15 s.	<i>Ex. 11.</i> 422, at 3 s. 42 4 21 2 L. 63, 6 s.	<i>Ex. 12.</i> 206, at 5 s. 41 4 10 6 L. 51, 10 s.	<i>Ex. 13.</i> 516, at 7 s. 154 16 25 16 L. 180, 12 s.
--	---	--	--

Ex. 14.

<i>Ex. 14.</i> 124, at 9 s.	<i>Ex. 16.</i> 431, at 13 s.	<i>Ex. 18.</i> 324, at 17 s.
49 12 6 4 	258 12 21 11 	259 4 16 4
L. 55, 16 s.	L. 280, 3 s.	L. 275, 8 s.
<i>Ex. 15.</i> 243, at 11 s.	<i>Ex. 17.</i> 248, at 15 s.	<i>Ex. 19.</i> 425, at 19 s.
121 10 12 3 	173 12 12 8 	382 10 21 5
L. 133, 13 s.	L. 186	L. 403, 15 s.

Note 1. The reason of multiplying by half the number of shillings in the rate will appear by considering, that these are the numerators of the fractions denoting the rate. Thus, 2 s. is $\frac{1}{10}$ l. and 4 s. is $\frac{2}{10}$ l. and 6 s. is $\frac{3}{10}$ l. and each unit in the product is two shillings. The division by the denominator 10 is performed by cutting off the right-hand figure of the product, and the figure so cut off is the remainder; and as each unit in the remainder is two shillings, the double of them is the remainder in shillings.

Note 2. From *Ex. 1.* we may learn, that when the rate is 2 s. the price is found by doubling the right-hand figure of the given number for shillings, and the other figure or figures are pounds.

Note 3. In *Ex. 2.* the price may also be had by taking $\frac{1}{5}$ of the given number; and in this way every remainder will be 4 s.

Note 4. When the rate is 10 s. as in *Ex. 5.* the price may be obtained more easily by taking $\frac{1}{2}$ of the given number; and the remainder, if any, will be 10 s.

Note 5. When the rate is 5 s. as in *Ex. 12.* the price is more readily had by taking $\frac{1}{4}$ of the given number; and in this case every remainder is 5 s. In like manner, when the rate is 15 s. as in *Ex. 17.* the price may be found by taking $\frac{1}{3}$ of the given number, and then $\frac{1}{2}$ of that quot.

Note 6. By reversing the operation, from the price and any even rate given, we may readily find the quantity of goods, *viz.* Multiply the price by 10, that is, to the price annex a cipher, and divide the product by half the rate,

Ex. 1. How many yards, at 14 s. may be bought for 49 l.?
7)490(70 yards. *Ans.*

Ex. 2. How many gallons, at 8 s. may be bought for 500 l.?
4)5000(1250 gallons. *Ans.*

C A S E

C A S E V.

When the rate is shillings and pence, or shillings, pence, and farthings.

R U L E I.

If the rate be shillings and pence, which make an aliquot part of a pound, divide the given number by the denominator of the fraction denoting the rate; the quot is pounds, and each unit of the remainder is equal to the rate.

E X A M P L E S.

$\frac{1}{12}$	<i>Ex. 1.</i> <u>354, at 1s. 8d.</u> L. 29, 10s.	$\frac{1}{3}$	<i>Ex. 4.</i> <u>439, at 6s. 8d.</u> L. 146 6 8
$\frac{1}{3}$	<i>Ex. 2.</i> <u>443, at 2s. 6d.</u> L. 55 7 6		<i>Ex. 5.</i> <u>766, at 13s. 4d.</u> 255 6 8 255 6 8 L. 510 13 4
$\frac{1}{6}$	<i>Ex. 3.</i> <u>346, at 3s. 4d.</u> L. 57 13 4	$\frac{1}{3}$ $\frac{1}{3}$	

Note. The fraction denoting the rate in Ex. 5. being $\frac{2}{3}$, you may also work thus: $766 \times 2 = 1532$, and $3)1532(510\ 13\ 4$.

R U L E II.

If the rate be no aliquot part of a pound, but may be divided into shillings, and some aliquot part or parts, divide it accordingly, work for the parts separately, and then add.

E X A M P L E S.

	<i>Ex. 1.</i> <u>427, at 8s. 6d.</u> 128 2 53 7 6 L. 181 9 6		<i>Ex. 2.</i> <u>540, at 5s. 4d.</u> 90 54 L. 144
6s. $\frac{3}{10}$ 2s. 6d. $\frac{1}{8}$		3s. 4d. $\frac{1}{8}$ 2s. $\frac{1}{10}$	

		<i>Ex. 3.</i>				<i>Ex. 5.</i>	
		386, at 8s. 8d.				796, at 3s. 10d.	
6s. 8d.	$\frac{1}{10}$	128	13 4	3s. 4d.	$\frac{1}{8}$	132	13 4
2s.	$\frac{1}{10}$	38	12	6d.	$\frac{1}{40}$	19	18
		L. 167 5 4				L. 152 11 4	
		<i>Ex. 4.</i>				<i>Ex. 6.</i>	
		386, at 14s. 8d.				394, at 17s. 4d.	
8s.	$\frac{4}{10}$	154	8	6s. 8d.	$\frac{1}{10}$	131	6 8
6s. 8d.	$\frac{1}{10}$	128	13 4	6s. 8d.	$\frac{1}{10}$	131	6 8
		L. 283 1 4		4s.	$\frac{2}{10}$	78	16
						L. 341 9 4	

R U L E III.

If the rate be no aliquot part of a pound, and cannot readily be divided into such parts, divide it into parts whereof one at least may be an aliquot part of a pound, or some number of shillings, and the subsequent part, or parts, each an aliquot part of some prior part. Or,

Multiply the given number by the shillings, and then, for the pence and farthings, work as in Case III.

E X A M P L E S.

		<i>Method 1.</i>				<i>Method 2.</i>	
		<i>Ex. 1.</i>				<i>Ex. 1.</i>	
		3506, at 1s. 3d.				3506, at 1s. 3d.	
1s.	$\frac{1}{10}$	175	6	1s.	$\frac{1}{4}$	876	6
3d.	$\frac{1}{4}$	43	16 6		$\frac{1}{10}$	438	2 6
		L. 219 2 6				L. 219 2 6	
		<i>Ex. 2.</i>				<i>Ex. 2.</i>	
		915, at 1s. 10 $\frac{1}{2}$ d.				95, at 1s. 10 $\frac{1}{2}$ d.	
1s.	$\frac{1}{10}$	4	15	1s.	$\frac{1}{2}$	47	6
6d.	$\frac{1}{2}$	2	7 6	3d.	$\frac{1}{2}$	23	9
3d.	$\frac{1}{2}$	1	3 9	1 $\frac{1}{2}$ d.	$\frac{1}{2}$	11	10 $\frac{1}{2}$
1 $\frac{1}{2}$ d.	$\frac{1}{2}$		11 10 $\frac{1}{2}$		$\frac{1}{20}$	17	8 1 $\frac{1}{2}$
		L. 8 18 1 $\frac{1}{2}$				L. 8 18 1 $\frac{1}{2}$	

Method

Method 1.			Method 2.		
<i>Ex. 3.</i>			<i>Ex. 3.</i>		
485, at 2s. $2\frac{1}{4}$ d.			485, at 2s. $2\frac{1}{4}$ d.		
2s.	$\frac{1}{10}$	48 10	2s.		970
2d.	$\frac{1}{12}$	4' 10	2d.	$\frac{1}{12}$	80 10
$\frac{1}{4}$ d.	$\frac{1}{8}$	10 $1\frac{1}{4}$	$\frac{1}{4}$ d.	$\frac{1}{8}$	10 $1\frac{1}{4}$
		<u>L. 53 0 $11\frac{1}{4}$</u>		$\frac{1}{10}$	<u>106 0 $11\frac{1}{4}$</u>
<i>Ex. 4.</i>			<i>Ex. 4.</i>		
480, at 4s. $8\frac{3}{4}$ d.			480, at 4s. $8\frac{3}{4}$ d.		
4s.	$\frac{1}{5}$	96	4s.		1920
6d.	$\frac{1}{8}$	12	6d.	$\frac{1}{8}$	240
2d.	$\frac{1}{3}$	4	2d.	$\frac{1}{3}$	80
$\frac{1}{2}$ d.	$\frac{1}{4}$	1	$\frac{1}{2}$ d.	$\frac{1}{4}$	20
$\frac{1}{4}$ d.	$\frac{1}{2}$	0 10	$\frac{1}{4}$ d.	$\frac{1}{2}$	10
		<u>L. 113 10</u>		$\frac{1}{10}$	<u>227 0</u>
<i>Ex. 5.</i>			<i>Ex. 5.</i>		
136, at 9s. $2\frac{1}{2}$ d.			136, at 9s. $2\frac{1}{2}$ d.		
8s.	$\frac{4}{10}$	54 8	9s.		1224
1s.	$\frac{1}{10}$	6 16	2d.	$\frac{1}{8}$	22 8
2d.	$\frac{1}{6}$	1 2 8	$\frac{1}{2}$ d.	$\frac{1}{4}$	5 8
$\frac{1}{2}$ d.	$\frac{1}{4}$	5 8		$\frac{1}{10}$	<u>125 2 4</u>
		<u>L. 62 12 4</u>			<u>L. 62 12 4</u>
<i>Ex. 6.</i>			<i>Ex. 6.</i>		
370, at 14s. $2\frac{3}{4}$ d.			370, at 14s. $2\frac{3}{4}$ d.		
12s.	$\frac{6}{10}$	222			148
2s.	$\frac{1}{10}$	37			37
2d.	$\frac{1}{12}$	3 1 8	14s.		5180
$\frac{1}{2}$ d.	$\frac{1}{4}$	15 5	2d.	$\frac{1}{8}$	61 8
$\frac{1}{4}$ d.	$\frac{1}{2}$	7 $8\frac{1}{2}$	$\frac{1}{2}$ d.	$\frac{1}{4}$	15 5
		<u>L. 263 4 $9\frac{1}{2}$</u>	$\frac{1}{4}$ d.	$\frac{1}{2}$	7 $8\frac{1}{2}$
				$\frac{1}{10}$	<u>526 4 $9\frac{1}{2}$</u>
					<u>L. 263 4 $9\frac{1}{2}$</u>

Method 1.

Method 1.		Method 2.	
<i>Ex. 7.</i>		<i>Ex. 7.</i>	
1504, at 19s. 9d.		1504, at 19s. 9d.	
18s. $\frac{9}{10}$	1353 12		13536
1s. $\frac{1}{10}$	75 4		1504
6d. $\frac{1}{2}$	37 12		
3d. $\frac{1}{4}$	18 16	19s. $\frac{1}{2}$	28576
		6d. $\frac{1}{2}$	752
		3d. $\frac{1}{4}$	376
		$\frac{1}{10}$	29704
	L. 1485 4		
			L. 1485 4

Note 1. When the rate is more than 1s. and less than 2s. as in Ex. 1. and 2. there is no occasion, in working by Method 2. to draw a line under the given number: we esteem it so many shillings, and the parts for the pence or farthings are added up with it.

Note 2. In working the above or like examples, some people of inferior skill proceed in the following manner:

Ex. 2. 95, at 1s. 10 $\frac{1}{2}$ d.

	L.	s.	d.
95 shillings make	4	15	
95 six-pences make	2	7	6
95 three-pences make	1	3	9
95 pence make		7	11
95 half-pence make		3	11 $\frac{1}{2}$

Anf. 8 18 1 $\frac{1}{2}$

CASE VI.

When the rate is pounds.

RULE.

Multiply the given number by the rate, and the product is the price in pounds.

EXAMPLES.

<i>Ex. 1.</i> 42, at 2l.	<i>Ex. 2.</i> 13, at 8l.
L. 84	L. 104
<i>Ex. 3.</i> 30, at 3l.	<i>Ex. 4.</i> 48, at 12l.
L. 90	L. 576

CASE

C A S E VII.

When the rate is pounds and shillings, or pounds, shillings, pence, and farthings.

R U L E I.

If the rate be pounds and shillings, multiply the given number by the pounds, and work for the shillings as in Case IV.

E X A M P L E S.

	<i>Ex. 1.</i>
1l.	46, at 1l. 4s.
4s.	9 4
	L. 55 4

	<i>Ex. 2.</i>
	82, at 4l. 10s.
4l.	328
10s.	41
	L. 369

	<i>Ex. 3.</i>
	58, at 3l. 7s.
3l.	174
6s.	17 8
1s.	2 18
	L. 194 6

	<i>Ex. 4.</i>
	26, at 3l. 15s.
3l.	78
14s.	18 4
1s.	1 6
	L. 97 10

Note. When the rate is more than 1l. and less than 2l. as in *Ex. 1.* we have no occasion to draw a line under the given number, it being esteemed so many pounds, and the parts for the shillings or pence are added up with it.

R U L E II.

If the rate be pounds, with shillings and pence that make some aliquot part of a pound, or are divisible into aliquot parts, or into shillings and some aliquot part or parts; then multiply the given number by the pounds, and work for the shillings and pence, as in Case V. Rule I. or II.

E X A M P L E S.

	<i>Ex. 1.</i>
	54, at L. 3 : 2 : 6.
3l.	162
2s. 6d.	6 15
	L. 168 15

	<i>Ex. 2.</i>
	43, at L. 5 : 3 : 4.
5l.	215
3s. 4d.	7 3 4
	L. 222 3 4

Ex. 3.

Ex. 3.
1l. 73, at L. 1:6:8.
6s. 8d. 24 6 8

L. 97 6 8

Ex. 4.
7l. 76, at 7l. 13s. 4d.

6s. 8d. 532
6s. 8d. 25 6 8
6s. 8d. 25 6 8

L. 582 13 4

Ex. 5.
83, at L. 2:8:6.

2l. 166
6s. 24 18
2s. 6d. 10 7 6

L. 201 5 6

Ex. 6.
3l. 92, at L. 3:5:4.

3s. 4d. 276
2s. 15 6 8
9 4

L. 300 10 8

Ex. 7.
37, at L. 4:8:8.

4l. 148
6s. 8d. 12 6 8
2s. 3 14

L. 164 0 8

Ex. 8.
26, at L. 2:17:4.

2l. 52
6s. 8d. 8 13 4
6s. 8d. 8 13 4
4s. 5 4

L. 74 10 8

R U L E III.

If the rate be pounds, with shillings, pence, and farthings, that cannot readily be resolved into aliquot parts of a pound; multiply the given number by the pounds; and then work for the shillings, pence, and farthings, as in Case V. Rule III. Method 1. But if you propose to work for the shillings, pence, and farthings, by Method 2. it will be convenient to do that in the first place, and then work for the pounds.

E X A M P L E S.

Method 1.

Ex. 1.

1l. 213, at L. 1:13:4½.
10s. 106 10
2s. 21 6
1s. 10 13
3d. 2 13 3
1½d. 1 6 7½

L. 355 8 10½

Method 2.

Ex. 1.

213, at L. 1:13:4½.
639
213
13s. 2769
3d. 53 3
1½d. 26 7½
10 284 8 10½
142 8 10½
1l. 213
L. 355 8 10½

N

Method

Method 1.		Method 2.	
Ex. 2.		Ex. 2.	
37, at L. 3 : 8 : 10 $\frac{1}{4}$.		37, at L. 3 : 8 : 10 $\frac{1}{4}$.	
3l.	III	6s.	222
6s.	II 2	2s.	74
2s. 6d.	4 12 6	6d.	18 6
3d.	9 3	3d.	9 3
1d.	3 1	1d.	3 1
$\frac{1}{4}$ d.	9 $\frac{1}{4}$	$\frac{1}{4}$ d.	9 $\frac{1}{4}$
L. 127 7 7 $\frac{1}{4}$		$\frac{1}{10}$	32 7 7 $\frac{1}{2}$
			16 7 7 $\frac{1}{2}$
		3l.	III
		L. 127 7 7 $\frac{1}{4}$	
Ex. 3.		Ex. 3.	
416, at L. 2 : 9 : 3 $\frac{1}{4}$.		416, at L. 2 : 9 : 3 $\frac{1}{4}$.	
2l.	832	9s.	3744
8s.	166 8	3d.	104
1s.	20 16	$\frac{3}{4}$ d.	26
3d.	5 4	$\frac{1}{10}$	387 4
$\frac{1}{4}$ d.	1 6		193 14
L. 1025 14		2l.	832
		L. 1025 14	

Note. A troublesome fraction in the rate may sometimes be avoided, by multiplying the rate, and dividing the quantity by the denominator of the fraction, and then computing the price of the quot from the new rate.

Ex. 1.		d.	s. d.
$\frac{1}{8}$	600, at 7 $\frac{7}{8}$ d.	7 $\frac{7}{8}$ $\times$ 8 = 63d.	= 5 3
75, at 5s. 3d.			
5s.	375		
3d.	18 9		
$\frac{1}{10}$	39 3 9		
L. 19 13 9			

Ex. 2.

Ex. 2.

$\frac{1}{12}$	540, at $3\frac{1}{2}$ d.
	45, at 3 s. 11 d.
3 s.	135
4 d.	15
4 d.	15
3 d.	11 3
$\frac{1}{10}$	176 3
	L. 8 16 3

$3\frac{1}{2} \times 12 = 47 \text{ d.} = 3 \text{ 11}$

Ex. 3.

$\frac{1}{11}$	924, at 4 $5\frac{2}{11}$
	84, at L. 2 : 9 : 4
9 s.	756
4 d.	28
$\frac{1}{10}$	784
2 l.	39 4
	168
	L. 207 4

$4 \ 5\frac{2}{11} \times 11 = 29 \ 4$

CASE VIII.

When the given number consists of integers and parts.

RULE.

Work for the price of the integers as already taught ; and for the part or parts, take a proportional part or parts of the rate.

EXAMPLES.

Ex. 1.

	<i>Yards.</i>
	$720\frac{1}{4}$, at 6 s. 8 d. per yd.
6 s. 8 d.	240
$\frac{1}{4}$ yd.	0 1 8
	L. 240 1 8

Ex. 2.

	<i>Yards.</i>
	$116\frac{1}{2}$, at 4 s. 6 d. per yd.
2 s. 6 d.	14 10
2 s.	11 12
$\frac{1}{2}$ yd.	2 3
	L. 26 4 3

N 2

Ex. 3.

Ex. 3.

	<i>Yards.</i>
	228 $\frac{3}{4}$, at 12s. 11d. per yd.
12 s.	2736
4 d.	76
4 d.	76
3 d.	57
$\frac{1}{2}$ yd.	6 5 $\frac{1}{2}$
$\frac{1}{4}$ yd.	3 2 $\frac{3}{4}$
$\frac{1}{20}$	295 4 8 $\frac{1}{4}$
	L. 147 14 8 $\frac{1}{4}$

Ex. 4.

	<i>Yards.</i>
	23 $\frac{7}{8}$ at 18s. per yd.
18 s.	20 14
$\frac{4}{8}$ yd.	9
$\frac{2}{8}$ yd.	4 6
$\frac{1}{8}$ yd.	2 3
	L. 21 9 9

Ex. 5.

	<i>C. 2.</i>
	365 2, at 5l. 5s. p. C.
5l.	1825
5s.	91 5
2 Q.	2 12 6
	L. 1918 17 6

Ex. 6.

	<i>C. q. lb.</i>
	28 3 14, at 1l. 10s. per C.
1l.	14
10s.	0 15
2 Q.	7 6
1 Q.	3 9
14 lb.	L. 43 6 3

Ex. 7.

	<i>C. 2. lb.</i>
	144 2. 21, at 3l. 17s. 6d. per C.
3l.	432
15s.	108
2s. 6d.	18
2 Q.	1 18 9
14 lb.	9 8 $\frac{1}{4}$
7 lb.	4 10 $\frac{1}{8}$
	L. 560 13 3 $\frac{1}{8}$

Ex. 8.

	<i>T. C. 2.</i>
	73 17 2, at 9l. 12s. 10d. p. tun.
9l.	657
10s.	36 10
2s.	7 6
6d.	1 16 6
2d.	12 2
2d.	12 2
10 C.	4 16 5
5 C.	2 8 2 $\frac{1}{2}$
2 C. 2 Q.	1 4 1 $\frac{1}{4}$
	L. 712 5 6 $\frac{3}{4}$

Ex. 9.

	<i>lb. Tr. oz. dw.</i>
	36 8 16, at 23l. 16s. 4 $\frac{1}{2}$ d. p. lb.
	108
	72
23l.	828
10s.	18
5s.	9
1s.	1 16
4d.	12
$\frac{1}{2}$ d.	1 6
4 oz.	7 18 9 $\frac{1}{2}$
4 oz.	7 18 9 $\frac{1}{2}$
16 dw.	1 11 9 $\frac{1}{16}$
	L. 874 18 10 $\frac{1}{16}$

An operation in the rules of practice may be proved by running over the several steps a second time, by working the same question a different way, or by the rule of three.

Practical Questions.

- | | L. | s. | d. |
|--|-----------------|----|-------------------|
| 1. 126, at $14\frac{3}{4}$ d. | <i>Ans.</i> 7 | 14 | $10\frac{1}{2}$ |
| 2. 478, at 4s. $10\frac{1}{2}$ d. | <i>Ans.</i> 116 | 10 | 3 |
| 3. 50, at L. 3 : 15 : $6\frac{1}{2}$. | <i>Ans.</i> 188 | 17 | 1 |
| 4. $419\frac{1}{2}$ yards, at 4s. $10\frac{1}{4}$ d. per yard. | <i>Ans.</i> 101 | 16 | $3\frac{7}{8}$ f. |
| 5. 75 C. 2 Q. 14 lb. at 15s. $4\frac{1}{2}$ d. per C. | <i>Ans.</i> 58 | 2 | 8 $3\frac{1}{4}$ |
| 6. 8 T. 15 C. 3 Q. 27 lb. at 14 l. per tun. | <i>Ans.</i> 123 | 3 | $10\frac{1}{2}$ |

7. A bankrupt's effects amount to 811 l. 10s.; and he owes

	L.	s.	d.
To A	220	16	6
To B	312		
To C	117	12	6
To D	106	12	6
To E	200	6	
To F	124	12	6

In all 1082

How much can he afford to pay *per* pound, and what must each creditor have?

	L.	s.	d.
A	165	12	$4\frac{1}{2}$
B	234		
C	88	4	$4\frac{1}{2}$
D	79	19	$4\frac{1}{2}$
E	150	4	6
F	93	9	$4\frac{1}{2}$
Total	811	10	

Ans. 15s. *per* pound; and the creditors get as follows:

PART II

DECIMAL ARITHMETIC; or, The Doctrine of DECIMAL FRACTIONS.

CHAPTER I

NOTATION.

A FRACTION having 10, 100, 1000 or unity, with any number of ciphers annexed to it, for a denominator, is called a *decimal fraction*; such as $\frac{7}{10}$, $\frac{25}{100}$, $\frac{34}{1000}$, $\frac{85}{10000}$.

In decimal fractions, as in vulgar, the denominator shews into how many parts the unit or integer is divided, and the numerator shews how many of these parts the fraction contains. Thus, if the fraction be $\frac{9}{10}$, the unit is divided into ten equal parts, and the fraction contains nine of these parts; and consequently, if the unit or integer be a pound Sterling, the value of such a fraction is eighteen shillings.

We may conceive the denominator of a decimal fraction to be formed by dividing the unit into 10 equal parts, and each of these parts into 10 other equal parts, each of these again into 10 other equal parts, and so on, as far as necessary; and hence a decimal fraction will always be so many tenths, or so many tenths of $\frac{1}{10}$, or so many tenths of $\frac{1}{10}$ of $\frac{1}{10}$, &c.; and by reducing the compound fraction to a simple one, we have the decimal. Thus, $\frac{8}{10}$ of $\frac{1}{10}$ of $\frac{1}{10} = \frac{8}{1000}$.

Or we may conceive the denominator of a decimal to be formed by the continual multiplication of unity into 10, as often as there are ciphers in it. Thus, $1 \times 10 = 10$, and $1 \times 10 \times 10 = 100$, and $1 \times 10 \times 10 \times 10 = 1000$, &c. And because the fractions $\frac{9}{10}$, $\frac{99}{100}$, $\frac{999}{1000}$, &c. have the highest numerators possible, it is plain, that the number of figures or places in the numerator of a decimal can never exceed the number of ciphers in the denominator.

It is usual to write down only the numerator of a decimal fraction, omitting the denominator; and when the numerator has the same number of figures or places as the denominator has ciphers, it is done by writing down the figures of the numerator, and prefixing a point, to distinguish them from a whole number. So $\frac{7}{10}$ is written thus, .7; and $\frac{25}{100}$ is written thus, .25. The point thus prefixed is called the *decimal point*.

But when the numerator has not so many figures or places as there are ciphers in the denominator, the defect is supplied by prefixing a cipher for every figure wanting, and then placing the decimal

whole numbers; and also, that the same number may be differently expressed, according as the integer is chosen. Thus, the time since our Saviour's birth may be written thus, 1785; or thus, 178.5; or thus, 17.85; or thus, 1.785; or thus, .1785, according as one year, a decad, a century, a chiliad, or myriad, is used as the integer. Hence arises the superior excellency of decimal arithmetic, above every other sort of numerical computation; as will appear, with convincing evidence, in the sequel.

C H A P. II.

REDUCTION OF DECIMALS.

PROBLEM I.

To reduce a vulgar fraction to a decimal.

R U L E.

To the numerator of the vulgar fraction affix a point or comma, then annex a competent number of ciphers, and divide by the denominator; the quot is the numerator of the decimal, and the ciphers annexed show the number of decimal places.

E X A M P L E I.

Reduce $\frac{1}{2}$ to a decimal.

Here to the numerator 1 I annex one cipher, and dividing by the denominator 2, the quot is 5, and 0 remains; and because a single cipher only was annexed to the numerator, the decimal numerator will consist but of one figure, namely 5; to which, therefore, I prefix the decimal point. So $\frac{1}{2} = .5$.

$$\begin{array}{r} 2 \overline{)1.0(.5} \\ \underline{10} \\ (0) \end{array}$$

Hence appears the reason of the rule: namely, $2 : 1 :: 10 : 5$; that is, as the vulgar denominator to the vulgar numerator, so is the decimal denominator to the decimal numerator.

The truth of the rule will still further appear, if we consider, that the decimal fraction is produced by multiplying the numerator and denominator of the vulgar fraction by the same number, namely, the quot arising from the division of the decimal denominator by the vulgar denominator, or the quot of the decimal numerator divided by the vulgar numerator. Thus, $\frac{10}{2}$ or $\frac{5}{1} = 5$,

$$\text{and } \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = .5.$$

E X-

E X A M P L E II.

Reduce $\frac{3}{4}$ to a decimal.

To the numerator 3 I annex two ciphers; and, dividing by the denominator, the quot gives 75 for the numerator of the decimal, two ciphers having been annexed. So $\frac{3}{4} = .75$.

$$\begin{array}{r} 4 \overline{) 3.00} (.75 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

E X A M P L E III.

Reduce $\frac{1}{16}$ to a decimal.

Here I annex four ciphers; and as the quot gives only three decimal places I supply the defect by prefixing a cipher. So $\frac{1}{16} = .0625$.

$$\begin{array}{r} 16 \overline{) 1.0000} (.0625 \\ \underline{96} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

The reason of prefixing the cipher is obvious; for $\frac{625}{10000} = .0625$, by the manner of notation.

E X A M P L E IV.

Reduce $\frac{7}{3125}$ to a decimal.

Here I annex five ciphers; and as the quot gives only three decimal places, I prefix two ciphers. So $\frac{7}{3125} = .00224$.

$$\begin{array}{r} 3125 \overline{) 7.00000} (.00224 \\ \underline{6250} \\ 7500 \\ \underline{6250} \\ 12500 \\ \underline{12500} \\ 0 \end{array}$$

Though ciphers may be annexed at pleasure, yet it is the ciphers used that determine the number of decimal places in the quot; and at first it is sufficient to annex so many as serve to complete the first dividual, leaving room to annex more as you proceed in the operation; or rather annex the other ciphers to the remainders, without giving them a place in the dividend.

The first dividual also shows whether ciphers ought to be prefixed to the quot, and how many. Thus, if the first dividual take in only one of the annexed ciphers, the figure put in the quot is primes, and no cipher to be prefixed. If the first dividual comprehend two of

of the annexed ciphers, the figure put in the quot is seconds, and one cipher must be prefixed. If the first dividual comprehend three of the annexed ciphers, the figure put in the quot is thirds, and two ciphers must be prefixed, &c. Hence, in reducing a vulgar fraction to a decimal, the natural and easy way is, to place first the decimal point in the quot, and after it a cipher or ciphers, or the quotient-figure, as the first dividual directs.

In reducing a vulgar fraction to a decimal, if 0 at last remains, as in all the above examples, the decimal is precisely equal to the vulgar fraction, and is called a *finite or terminate decimal*.

In finite decimals, the denominator is always some aliquot part of the numerator increased by annexed ciphers; and such decimals take their rise from vulgar fractions whose denominator is 2 or 5, or some power of 2 or 5, or the product of some of their powers.—See powers described in the chapter on Extraction of Roots, in Part III.

The powers of numbers are sometimes expressed by indices or exponents placed at the corners of the numbers. Thus, 2^2 signifies the second power of 2, and 5^3 signifies the third power of 5; and 10^4 signifies the fourth power of 10, &c. The index of the root or first power is seldom expressed.

Any power of 2 multiplied into the like power of 5 gives a product equal to the same power of 10; as appears from the following specimen of the powers of 2, 5, and 10.

$2^1 = 2$	$5^1 = 5$	$2 \times 5 = 10^1 = 10$
$2^2 = 4$	$5^2 = 25$	$4 \times 25 = 10^2 = 100$
$2^3 = 8$	$5^3 = 125$	$8 \times 125 = 10^3 = 1000$
$2^4 = 16$	$5^4 = 625$	$16 \times 625 = 10^4 = 10000$
$2^5 = 32$	$5^5 = 3125$	$32 \times 3125 = 10^5 = 100000$
$2^6 = 64$	$5^6 = 15625$	$64 \times 15625 = 10^6 = 1000000$
$2^7 = 128$	$5^7 = 78125$	$128 \times 78125 = 10^7 = 10000000$
&c.	&c.	&c.

The product of two different powers of 2 and 5, is equal to the product that will arise by raising 10 to the power denoted by the lesser given index, and then multiplying this power of 10 into that power of the other number which is denoted by the difference of the two given exponents. Thus,

$$2^6 \times 5^2 = 64 \times 25 = 10^2 \times 2^4 = 100 \times 16 = 1600$$

$$2^2 \times 5^6 = 4 \times 15625 = 10^2 \times 5^4 = 100 \times 625 = 62500$$

From these remarks it is easy to perceive, that 2 or 5, or any of their powers, or product of their powers, will measure 10 or its powers, viz. 10, 100, &c. or their multiples, such as, 20, 200, 2000, &c. 30, 300, 3000, &c.; and such every numerator becomes by having ciphers annexed; and therefore 2 or 5, or their powers, or product of their powers, used as a denominator, will divide

divide any numerator with a competent number of ciphers annexed, and leave no remainder; and consequently the decimal thence resulting will be finite.

If the numerator of the vulgar fraction be unity, and the denominator any single power of 2 or 5, there will be as many decimal places in the quot as there are units in the index of the given power. Thus, $16 = 2^4$ gives a decimal of four places, viz. $\frac{1}{16} = .0625$; and $125 = 5^3$ gives a decimal of three places, viz. $\frac{1}{125} = .008$.

When the denominator is the product of like powers of 2 and 5; in this case, such a product being equal to the like power of 10, and any power of 10 being equal to 1, with as many ciphers annexed as there are units in the index, it follows, that there will still be as many decimal places in the quot as there are units in the index, either of 2, of 5, or 10. Thus, $8 \times 125 = 2^3 \times 5^3 = 10^3 = 1000$, gives a decimal of three places, viz. $\frac{1}{1000} = .001$.

When the denominator is the product of different powers of 2 and 5, find what power of 10, and what power of 2 or 5, upon being multiplied, will give the same product, as is taught above; and the sum of the indices shews the number of decimal places. Thus, $2^6 \times 5^2 = 10^3 \times 2^4$; and the sum of the indices, $2 + 4 = 6$, gives the number of decimal places, viz. $\frac{1}{100000} = .000025$.

And, in general, to find what number of decimal places any such vulgar fraction will give, divide the denominator by 2, 5, or 10, till the last quotient be 1, and the remainder 0; and the number of divisors shews the number of decimal places. Thus, $\frac{1}{16}$

$$\begin{array}{r} 2)2)2) \end{array}$$

gives a decimal of four places; for $2)16(8(4(2(1$. And $\frac{1}{125}$ gives

$$\begin{array}{r} 5)5) \end{array}$$

a decimal of three places; for $5)125(25(5(1$. And $\frac{1}{1000}$ gives a

$$\begin{array}{r} 10)10) \end{array}$$

decimal of three places; for $10)1000(100(10(1$. And $\frac{1}{10000}$ gives

$$\begin{array}{r} 10)2)2)2)2) \end{array}$$

a decimal of six places; $10)1600(160(16(8(4(2(1$.

If the denominator of a vulgar fraction be neither 2 nor 5, nor any of their powers, nor product of their powers, such a denominator will not divide the numerator with annexed ciphers without a remainder; and the decimal thence resulting is called *infinite*, or *interminate*.

Of infinite or interminate decimals, there are two sorts. For some constantly repeat the same figure; and are called *repeating decimals*, *repeaters*, or *single repetends*. Others repeat a circle of figures; and on that account are called *circulating decimals*, *circulates*, or *compound repetends*.

EXAMPLE V.

Reduce $\frac{1}{3}$ to a decimal.

Here the remainder being still the same, *viz.* 1, the same figure will constantly be repeated in the quot.

In like manner, $\frac{2}{3} = .6$, and $\frac{4}{9} = .4$, $\frac{5}{9} = .5$, $\frac{7}{9} = .7$, $\frac{8}{9} = .8$; and, in general, the denominator 9 gives a continued repetition of the numerator.

Repeating decimals are of two kinds; *viz.* some consist only of the repeating figures, such as the examples above; and these are called *pure repeaters*: others have one or more digits or ciphers betwixt the decimal point and the repeating figure; and these are called *mixt repeaters*; and the digits or ciphers on the left of the repeating figures are called the *finite part* of such decimals.

Pure repeaters take their rise from vulgar fractions whose denominator is 3, or its multiple 9; and are but few in number.

Mixt repeaters derive their origin from vulgar fractions whose denominator is the product of 3 into 2 or 5, or into some of their powers, or product of their powers; and such denominators may be considered as the product of two component parts, whereof one is 2 or 5, or some of their powers, or product of their powers; and hence the finite part. The other component part is 3; and hence the repeating figure.

EXAMPLE VI.

Reduce $\frac{4}{15}$ to a decimal.

Here the repeater is mixt, the finite part being 2, and the repeating figure 6.

$$\begin{array}{r} 15 \overline{) 4.0} (.26 \\ \underline{30} \\ * 100 \\ 90 \\ \underline{} \\ (10) \end{array}$$

We may resolve such denominators into their component parts, and divide the numerator by one of these parts, and then divide the quote by the other. Thus, $15 = 5 \times 3$.

$$\begin{array}{r} 5 \overline{) 4.0} (.8 \\ \underline{40} \\ (0) \end{array} \quad \text{and} \quad \begin{array}{r} 3 \overline{) .8} (.26 \\ \underline{6} \\ * 20 \\ 18 \\ \underline{} \\ (2) \end{array}$$

The number of places in the finite part of a mixt repeater may be ascertained from the number of units in the index of the powers of 2 or 5, as appears plain in the following specimen.

$\frac{1}{2} = \frac{1}{2 \times 3} = .1\dot{6}$	$\frac{1}{15} = \frac{1}{5 \times 3} = .0\dot{6}$
$\frac{1}{12} = \frac{1}{2^2 \times 3} = .08\dot{3}$	$\frac{1}{75} = \frac{1}{5^2 \times 3} = .01\dot{3}$
$\frac{1}{24} = \frac{1}{2^3 \times 3} = .041\dot{6}$	$\frac{1}{175} = \frac{1}{5^3 \times 3} = .002\dot{6}$
$\frac{1}{48} = \frac{1}{2^4 \times 3} = .0208\dot{3}$	$\frac{1}{1875} = \frac{1}{5^4 \times 3} = .0005\dot{3}$

And, universally, to find the number of places in the finite part of such fractions, divide the denominator first by 3, and then divide the quot by 2, 5, or 10, till the last quot be 1, and 0 remain; and the number of divisors, excluding 3, shows the number of places in the finite part. Thus, $\frac{1}{48}$ gives four finite places; for 3)48(16,

2)2)2) and 2)16(8(4(2(1. And $\frac{1}{1875}$ gives also four finite places; for 5)2)5)3)1875(625, and 5)625(125(25(5(1.

Repeating decimals are usually marked by a dash through the right hand figure, as in all the examples above: but some chuse to mark them by a point set over the repeating figure, thus, $\dot{.3}$, $\dot{.26}$. The remainder where the repetition begins is commonly marked with an asterisk.

Because any quotient multiplied by the divisor reproduces the dividend, it follows that any decimal multiplied by the denominator of the vulgar fraction from which it resulted, will reproduce the numerator with the annexed ciphers. Thus, if $.75$, the decimal of $\frac{3}{4}$, be multiplied by 4, it will reproduce the numerator 3 and the two annexed ciphers.

Now, suppose the given decimal to be a repeater; such as $\dot{.3}$, resulting from the vulgar fraction $\frac{1}{3}$, if the repeating decimal be multiplied by the denominator 3, it will by carrying at 9 on the right hand, reproduce the numerator 1 with the annexed cipher. In like manner, if the repeater $\dot{.6} = \frac{2}{3}$, be multiplied by 3, it will, by carrying at 9 on the right hand, reproduce the numerator 2 with the annexed cipher. Again, if the repeater $\dot{.1} = \frac{1}{9}$, be multiplied by the denominator 9, it will, by carrying at 9 on the right hand, reproduce the numerator 1 with the annexed cipher. And, if the mixt repeater $\dot{.26} = \frac{4}{15}$, be multiplied by the denominator 15, it will, by carrying at 9 on the right hand, reproduce the numerator 4 with the two annexed ciphers.

From these remarks we may conclude, that the right-hand figure of every repeating decimal is ninth parts: and the same truth may be evinced by resolving the decimal into its constituent parts, in the following manner.

The

The vulgar fraction $\frac{7}{9}$ reduced to a decimal gives .77, *℄c.*; and this repeater resolved into decimal constituent parts, becomes $\frac{7}{10} + \frac{7}{100} + \frac{7}{1000}$, *℄c.* to infinity. But if we esteem the right-hand figure to be ninth parts, we have $\frac{7}{10} + \frac{7}{9}$ of $\frac{1}{10} = \frac{7}{10} + \frac{7}{90} = \frac{63}{90} + \frac{70}{90} = \frac{133}{90} = \frac{148}{90} = \frac{74}{45} = \frac{7}{9}$, the given vulgar fraction. And as the vulgar fraction $\frac{7}{9}$ gives .77, so $\frac{2}{9}$ gives .22; that is .22 = $\frac{2}{9}$ = 1. And, universally, a series of nines infinitely continued is equal to unity in the place on the left hand: Thus, .999 = 1; and .0999 = .1; and .0099 = .01; and 44.99 = 45.

Hence may be ascertained the value of an infinite series decreasing in a decuple proportion. Thus, $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$, *℄c.* = $\frac{1}{9}$. And $\frac{2}{10} + \frac{2}{100} + \frac{2}{1000}$, *℄c.* = $\frac{2}{9}$.

If the denominator of a vulgar fraction be neither 2 nor 5, nor any power of 2 or 5, nor any product of their powers; nor 3, nor 9, nor any product of 3 into 2 or 5, or into some of their powers, or product of their powers, the decimal resulting from such a vulgar fraction will circulate.

Circulates, like repeaters, are of two sorts, *viz.* pure and mixt. A pure circulate consists of the figures of the circle only; as, .09, 09, *℄c.* or .18, 18, *℄c.* A mixt circulate has a finite part betwixt the decimal point and the figure that begins the circle; as, .0, 45, 45, *℄c.*; or .32, 142857, 142857, *℄c.* Some chuse to distinguish the finite part from the circle, and one circle from another, by a comma, as above. Others dash the first and last figure of the circle. It is likewise usual to mark the remainder where the new circle begins, by affixing an asterisk.

EXAMPLE VII.

Reduce $\frac{1}{11}$ to a decimal.

The denominator 11 gives a pure circle of two figures.

$$\begin{array}{r} 11 \overline{) 1.00(.09,09,} \\ \underline{99} \\ * 100 \\ \underline{99} \\ * 1 \end{array}$$

$\frac{2}{11} = .18, 18,$	$\frac{7}{11} = .63, 63,$
$\frac{3}{11} = .27, 27,$	$\frac{8}{11} = .72, 72,$
$\frac{4}{11} = .36, 36,$	$\frac{9}{11} = .81, 81,$
$\frac{5}{11} = .45, 45,$	$\frac{10}{11} = .90, 90,$
$\frac{6}{11} = .54, 54,$	

It is easy to perceive, that if any of the vulgar fractions in the above specimen have both its numerator and denominator multiplied by 9, there will arise a new vulgar fraction of the same value, whose numerator will be the figures of the circle, and its denominator the like number of 9's. Thus,

$$3 \times 9$$

$$\begin{array}{r} 3 \times 9 \\ \hline 11 \times 9 \end{array} = \frac{27}{99}, \text{ and } \begin{array}{r} 7 \times 9 \\ \hline 11 \times 9 \end{array} = \frac{63}{99}.$$

As the denominator 11, whereof 99 is a multiple, gives a pure circulate of two places, so any denominator whereof 99, or 9999, or 99999, &c. are multiples, will give a pure circulate of three, four, five, &c. places; that is, of as many places as there are 9's in the multiple. And such denominators are all the prime numbers, except 2, 3, and 5, viz. 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, &c.; also their products into three, viz. 21, 33, 39, 51, 57, 69, &c. Such too are all the powers of 3, except 3 and 9, viz. 27, 81, 243, 729, 2187, &c.

The reason is plain; for if any divisor, as 37, divide 999 without a remainder, it will also divide 1000, and leave a remainder of 1, to begin a new circle.

To find how many places the circle will consist of, divide a competent number of 9's by any of the above denominators, continuing the operation till 0 remain; and the number of 9's used will show the number of places.

Thus, 7)999999 six places. Thus, 27)999 three places.

142857

37

The number of figures in a circle, when some power of 3 is the denominator, may also be found thus: Divide the given denominator by 9, and the number of units in the quot will be equal to the number of figures in the circle. Thus, 9)27(3 places. Thus, 9)81(9 places. Thus, 9)729(81 places. Thus, 9)2187(243 places, &c.

I shall now subjoin a further specimen of the pure circulates that arise from the above primes, their products into 3, and powers of 3, and make a few remarks on them.

	Places.
$\frac{1}{7} = .142857, 142857, 142857, 142857, 14$	fix
$\frac{1}{11} = .09, 09, 09, 09, 09, 09, 09, 09, 09, 09, 09,$	two
$\frac{1}{13} = .076923, 076923, 076923, 076923, 07,$	fix
$\frac{1}{17} = .0588235294117647, 0588235294$	fifteen
$\frac{1}{19} = .052631578947368421, 05263157$	eighteen
$\frac{1}{23} = .047619, 047619, 047619, 047619, 04$	fix
$\frac{1}{27} = .0434782608695652173913, 0434$	twenty-two
$\frac{1}{37} = .037, 037, 037, 037, 037, 037, 037,$	three
$\frac{1}{39} = .0344827586206896551724137931, 03$	twenty-eight
$\frac{1}{41} = .032258064516129, 032258064$	fifteen
$\frac{1}{43} = .03, 03, 03, 03, 03, 03, 03, 03, 03, 03, 03,$	two
$\frac{1}{47} = .027, 027, 027, 027, 027, 027, 027,$	three
$\frac{1}{49} = .025641, 025641, 025641, 025641, 02$	fix
$\frac{1}{51} = .02439, 02439, 02439, 02439, 02439,$	five
$\frac{1}{53} = .023255813953488372093, 023$	twenty-one

The

The greatest possible number of places in a circle will always be one less than the number of units in the prime denominator. Thus, 7 dividing any numerator may leave a remainder of 1, 2, 3, 4, 5, or 6, but cannot leave 7; and therefore the number of places in the resulting circle cannot exceed six: for where any former remainder recurs, the circle begins anew.

Of the above circulates, those resulting from the prime numbers, and consisting of an even number of places, have the sum of their figures equal to the product of 9 into half the number of figures in the circle. Thus $.142857 = 27 = 9 \times 3$. And, $.09 = 9 = 9 \times 1$. And, $.0588235294117647 = 72 = 9 \times 8$. For one half of the circle, added to the other half, gives a series of 9's, equal in number to half the figures of the circle.

Thus, 142	and thus, 05882352
857	94117647
999	99999999

If 3 divide a repeater, whose repeating figure is not a multiple of 3, the quot will be a pure circulate of three places. Thus, $3).111(.037$, and $3).855(.185$, and $3).777(.259$,

If 3 divide a pure circulate, the circle not being a multiple of 3, the quot will be a pure circulate of thrice as many places as the circle of the dividend. Thus, $3).037,037,037, (.012345679$,

Mixt circulates take their rise from fractions whose denominators are the prime numbers 7, 11, 13, 17, 19, 23, 29, &c. multiplied into 2, 5, or 10, or into some of their powers, or product of their powers.

EXAMPLE VIII.

Reduce $\frac{2}{7}$ to a decimal.

The denominator $28 = 7 \times 2 \times 2$, gives a mixt circulate, consisting of the finite part 32, and a circle of six figures or places, whose sum is equal to the product of 9 into half the number of figures; that is, $9 \times 3 = 27$.

$ \begin{array}{r} 28)9.0(.32,142857,14 \\ \underline{84} \\ 60 \\ \underline{56} \\ *40 \\ \underline{28} \\ 120 \\ \underline{112} \\ 80 \\ \underline{56} \\ \hline \text{Carried up, } 240 \end{array} $	$ \begin{array}{r} \text{Brought up, } 240 \\ \underline{224} \\ 160 \\ \underline{140} \\ 200 \\ \underline{196} \\ *40 \\ \underline{28} \\ 120 \\ \underline{112} \\ \hline 8 \end{array} $
---	---

A further specimen of mixt circulates, for the learner's instruction, is here subjoined.

$$\frac{1}{14} = \frac{1}{7 \times 2} = .0,714285,714285,714285,$$

$$\frac{3}{11} = \frac{3}{11 \times 2} = .1,36,36,36,36,36,36,36,36,$$

$$\frac{49}{13} = \frac{49}{13 \times 2 \times 2} = .94,230769,230769,230769,$$

$$\frac{67}{85} = \frac{67}{17 \times 5} = .7,8823529411764705,882$$

$$\frac{463}{475} = \frac{463}{19 \times 5 \times 5} = .97,473684210526315789,47$$

$$\frac{278}{390} = \frac{278}{39 \times 10} = .7,128205,128205,128205,$$

$$\frac{18254}{41000} = \frac{18254}{41 \times 10 \times 10 \times 5} = .890,43902,43902,$$

The number of places, both in the finite part and in the circle, may be ascertained thus: Divide the denominator of the vulgar fraction by 10, 5, or 2, as often as possible, and the number of divisors will show the number of places in the finite part; make the last quot a divisor, and the dividend any competent number of 9's; continue the operation till 0 remain, and the number of 9's used will be equal to the number of places in the circle. Thus,

10) 5
10) 20500(2050(205(41; and 41)99999(2439, and 0 remains. So I conclude, that the finite part will consist of three places, and the circle of five.

Universally, any vulgar fraction being given, we may determine whether the decimal thence resulting will be finite or infinite; and if infinite, whether pure or mixt; with the number of places, &c. in the following manner:

Reduce the given vulgar fraction to its lowest terms, then divide the denominator by 10, 5, or 2, as often as possible; and if the last quot be unity, without any remainder, the decimal is finite, and the number of divisors shews the number of decimal places. Thus,

2) 2) 2) 2)
 $\frac{18}{385}$ reduced to its lowest terms is $\frac{3}{55}$; and 10) 160(16(8(4(2(1.
So the decimal is finite, and consists of five places, viz. .01875.

If the last quot be 3, or any power of 3, the resulting decimal will be a mixt repeater, the number of whose finite places will be equal to the number of divisors. Thus, suppose the vulgar frac-

tion in its lowest terms to be $\frac{7}{24}$; now, $2)24(12(6(3$; and accordingly the resulting decimal is a mixt repeater, whose finite part consists of three places, namely .291 $\overline{6}$.

If the last quot cannot be divided by 2, 5, 10, or 3, the resulting decimal will be a mixt circulate; and the way of finding the number of places, both in the finite part and circle, is taught above.

If the denominator of the given vulgar fraction can be divided, neither by 2, 5, nor 10, the resulting decimal will be a pure repeater, or a pure circulate, according as the denominator is 3 or 9, or some of the prime numbers 7, 11, 13, &c.; as has been already explained.

Every vulgar fraction may be reduced to a decimal, finite or infinite; that is, to a finite decimal, to a repeater, or a circulate. For if the denominator divide the numerator with ciphers annexed, so as to leave no remainder, the resulting decimal is finite. If the remaining figure be always the same, the resulting decimal will be a repeater. If neither of these be the case, yet because the divisor is a finite number, the remainder at last must either be the same with the numerator of the vulgar fraction, or the same with some preceding remainder, and then a new circle begins; and consequently the resulting decimal will be a circulate.

Because in circulates, the circle runs on sometimes to 16, 18, 22, 28, 81, 243, &c. places, and because, in decimals of every sort, the finite part runs sometimes on to many places, such circulates, or finite parts, may, without any sensible error, be limited at five or six places, and used as finites: for five decimal places, divide the integer into 100,000 equal parts, and all the loss that can be occasioned by such limitation is less than one hundred thousandth part of the integer. And in most cases, the decimal may be limited at three places, which divide the integer into a thousand equal parts.

Circulates, or finite parts, thus limited, are called *approximate decimals*; and are sometimes marked with + or — annexed, according as the right-hand figure is taken less or greater than just: for in limiting the decimal, if you foresee that the succeeding figure of the quot would be 6 or 7, or any figure above 5, you lessen the error, by increasing the right-hand figure of the approximate by unity.

P R O B. II.

To reduce the parts of Coin, Weight, Measure, Time, &c. to decimals.

This may be done several ways, as follows:

M E-

METHOD I.

Convert the given part or parts to a vulgar fraction of the integer, and then reduce the vulgar fraction to a decimal.

I. M O N E Y.

Ex. 1. Reduce 9 pence to the decimal of a shilling.

$$\begin{array}{r} d. \quad s. \\ 9 = \frac{9}{12} \end{array}$$

and 12)9.0(.75 of a shilling.

$$\begin{array}{r} 84 \\ \hline 60 \\ 60 \\ \hline (0) \end{array}$$

Here the fraction $\frac{9}{12} = \frac{3}{4}$; and the denominator $4 = 2 \times 2$ gives a finite decimal of two places.

Ex. 2. Reduce 9 pence to the decimal of a pound.

$$\begin{array}{r} d. \quad L. \\ 9 = \frac{9}{240} \end{array}$$

240)9.00(.0375 of a pound.

$$\begin{array}{r} 720 \\ \hline 1800 \\ 1680 \\ \hline 1200 \\ 1200 \\ \hline (0) \end{array}$$

The fraction $\frac{9}{240} = \frac{3}{80}$; and the denominator $80 = 10 \times 2 \times 2 \times 2$ gives a finite decimal of four places.

Ex. 3. Reduce 16s. 6d. to the decimal of a pound.

$$\begin{array}{r} s. \quad d. \quad L. \\ 16 \quad 6 = \frac{198}{240} \\ 12 \\ \hline 198 \end{array}$$

$$\begin{array}{r} 240)198.0(.825 L. \\ 1920 \\ \hline 600 \\ 480 \\ \hline 1200 \\ 1200 \\ \hline (0) \end{array}$$

The fraction $\frac{198}{240} = \frac{33}{40} = \frac{33}{4 \times 10}$; and the denominator $40 = 10 \times 2 \times 2$ give a finite decimal of three places.

Ex. 4. Reduce 18s. 4d. to the decimal of a pound.

$$\begin{array}{r} s. \quad d. \quad L. \\ 18 \quad 4 = \frac{220}{240} = \frac{11}{12} \\ 12 \\ \hline 220 \end{array}$$

$$\begin{array}{r} 12)11.0(.916 L. \\ 108 \\ \hline 20 \\ 12 \\ \hline 80 \\ 72 \\ \hline (8) \end{array}$$

The denominator $12 = 2 \times 2 \times 3$ gives a mixt repeater, the finite part consisting of two places.

More examples, one pound being the integer.

Ex.	s.	d.	f.	L.	L.
5.	19	11	3	$= \frac{218}{988} = .998958\bar{8}$	
6.	15	6	1	$= \frac{745}{988} = .776041\bar{6}$	
7.	14	3	2	$= \frac{686}{988} = .71458\bar{8}$	
8.	10	6		$= \frac{31}{40} = .525$	
9.	12	8	1	$= \frac{60}{988} = .634375$	
10.	13	4		$= \frac{2}{3} = .\bar{6}$	
11.	6	8		$= \frac{1}{3} = .\bar{3}$	
12.		7		$= \frac{7}{40} = .0291\bar{6}$	
13.				$= \frac{1}{988} = .001041\bar{6}$	

2. AVOIRDUPOIS WEIGHT.

Ex. 1. Reduce 17 lb. to the decimal of a quarter.

lb. $\frac{2}{4}$
 $17 = \frac{17}{1}$

$28) 17.0(.60,714285,71 \frac{2}{168}$

Here the denominator 28, being the product of the prime number $7 \times 2 \times 2$, gives a mixt circulate, the finite part consisting of two places, and the circle of six figures; for $7)999999(142857$, and leaves no remainder.

$\frac{*200}{196}$

$\frac{40}{28}$

$\frac{120}{112}$

$\frac{80}{56}$

$\frac{240}{224}$

$\frac{160}{140}$

$\frac{*200}{196}$

$\frac{40}{28}$

$\frac{120}{112}$

Ex. 2.

Ex. 2. Reduce 3 Q. 15 lb. to the decimal of a C.

$$\begin{array}{r}
 \text{Q. lb. C.} \\
 3 \text{ } 15 = \frac{99}{112} \\
 28 \\
 \hline
 99
 \end{array}
 \qquad
 \begin{array}{r}
 112 \overline{) 99.0} (.8839,285714,28 \text{ C.} \\
 \underline{896} \\
 940 \\
 \underline{896} \\
 440 \\
 \underline{336} \\
 1040 \\
 \underline{1008} \\
 *320 \\
 \underline{224} \\
 960 \\
 \underline{896} \\
 640 \\
 \underline{560} \\
 800 \\
 \underline{784} \\
 160 \\
 \underline{112} \\
 480 \\
 \underline{448} \\
 *320 \\
 \underline{224} \\
 960 \\
 \underline{896} \\
 (64)
 \end{array}$$

The denominator 112, being the product of the prime number $7 \times 2 \times 2 \times 2 \times 2$, gives a mixt circulate; the finite part consisting of four places, and the circle of six figures.

More examples, one C. or 112 lb. being the integer.

Ex.	Q.	lb.	oz.	C.	C.
3.	1	27	15	$= \frac{895}{112} = .49944196,428571,42$	
4.	3	14		$= \frac{7}{8} = .875$	
5.	2	21		$= \frac{11}{16} = .6875$	
6.		16		$= \frac{1}{7} = .142857,14$	
7.		14		$= \frac{1}{8} = .125$	
8.		13		$= \frac{13}{112} = .1160,714285,71$	
9.				$1 = \frac{1}{112} = .00055803,571428,57$	
				O 3	3. TROY

3. TROY WEIGHT.

Ex. 1. Reduce 16 grains to the decimal of an ounce.

Gr. oz.

$$16 = \frac{16}{480} = \frac{4}{120} = \frac{1}{30}$$

$$30 \overline{) 1.00} (.0390$$

The denominator $30 = 10 \times 3$ gives a mixt repeater, the finite part consisting of one place.

(10)

Ex. 2. Reduce 7 oz. 13 dw. to the decimal of a pound Troy.

oz. dw. lb.

$$7 \text{ } 13 = \frac{7 \times 20 + 13}{20} = \frac{153}{20}$$

20

153

$$80 \overline{) 51.0} (.6375$$

480

300

240

600

560

400

400

(0)

The denominator $80 = 10 \times 2 \times 2 \times 2$ gives a finite decimal of four places, as formerly observed.

More examples, one pound Troy being the integer.

Ex. oz. dw. gr. lb.

$$3. \text{ } 11 \text{ } 19 \text{ } 23 = .99982638$$

$$4. \text{ } 7 \text{ } 10 = .625$$

$$5. \text{ } 18 = .075$$

$$6. \text{ } 1 = .00017361$$

4. WINE-MEASURE.

Ex. 1. Reduce 3 pints to the decimal of a gallon.

Pts. gall.

$$3 = \frac{3}{8}$$

$$8 \overline{) 3.0} (.375 \text{ gall.}$$

24

60

56

40

40

(0)

The denominator $8 = 2 \times 2 \times 2$ gives a finite decimal of three places.

Ex. 2.

Ex. 2. Reduce 25 gallons 7 pints to the decimal of a hoghead.

Gall. pts. hhd.

$$25 \text{ } 7 = \frac{2507}{128} = \frac{6268}{32} = \frac{313}{16}$$

25
8

207

$$56)23.0(.410,714285,7 \text{ hhds.}$$

224

60

56

*400

392

80

56

240

224

160

112

480

448

320

280

*400

392

(8)

The denominator $56 = 7 \times 2 \times 2 \times 2$ gives a mixt circulate; the finite part consisting of three places, and the circle of six figures.

More examples, one hoghead being the integer.

Ex. Gall. pts. hhd.

$$3. \text{ } 62 \text{ } 7 = .998,015873,01$$

$$4. \text{ } 33 \text{ } 5 = .533,73015,7$$

5. LONG MEASURE.

Ex. 1. Reduce 5 inches to the decimal of a foot.

Inch. f.

$$5 = \frac{5}{12}$$

$$12)5.0(.41\bar{6} \text{ foot.}$$

48

20

12

*80

72

(8)

The denominator $12 = 2 \times 2 \times 3$ gives a mixt repeater, as was already observed.

Ex. 2. Reduce 1 foot 8 inches to the decimal of a yard.

F. in.

$$\begin{array}{r} 1 \quad 8 = \frac{10}{12} = \frac{10}{12} = \frac{5}{6} \\ 12 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 9) 5.0(.8 \text{ yard.} \\ 45 \\ \hline (5) \end{array}$$

The denominator 9 gives a pure repeater, the quot being a continued repetition of the numerator.

More examples, one yard being the integer.

Ex. F. in. yd.

$$3. \quad 2 \quad 11 = .97\bar{2}$$

$$4. \quad 1 \quad 9 = .58\bar{3}$$

6. T I M E.

Ex. 1. Reduce 17 hours to the decimal of a day.

H. day.

$$17 = \frac{17}{24}$$

$$\begin{array}{r} 24) 17.0(.708\bar{3} \text{ day,} \\ 168 \\ \hline \end{array}$$

The denominator $24 = 2 \times 2 \times 2 \times 3$ gives a mixt repeater, the finite part consisting of three places.

$$\begin{array}{r} 200 \\ 192 \\ \hline *80 \\ 72 \\ \hline (8) \end{array}$$

Ex. 2. Reduce 12 hours 36 minutes to the decimal of a day.

H. min. day.

$$\begin{array}{r} 12 \quad 36 = \frac{756}{1440} = \frac{378}{720} = \frac{63}{120} = \frac{21}{40} \\ 60 \\ \hline 756 \end{array}$$

$$\begin{array}{r} 40) 21.0(.525 \text{ day,} \\ 200 \\ \hline \end{array}$$

$$\begin{array}{r} 100 \\ 80 \\ \hline \end{array}$$

The denominator $40 = 10 \times 2 \times 2$ gives a finite decimal of three places.

$$\begin{array}{r} 200 \\ 200 \\ \hline (0) \end{array}$$

More examples, one day being the integer.

Ex. H. min. day.

$$3. \quad 23 \quad 59 = .9993\bar{0}\bar{8}$$

$$4. \quad 5 \quad 45 = .2395\bar{8}\bar{3}$$

M E T H O D II.

Here there are two cases.

1. If the given parts consist of one denomination, in this case reduce the given parts to a decimal of the next superior denomination; again, reduce this decimal to another decimal of the next superior denomination; and proceed in this manner till you bring it to a decimal of the integer required.

A decimal of a lower denomination is reduced to a decimal of a higher, if you divide it by the number of units in the lower which makes an unit of the higher. Thus, the decimal of a farthing is reduced to the decimal of a penny, by dividing it by 4; and the decimal of a penny is reduced to the decimal of a shilling, by dividing it by 12; and the decimal of a shilling is reduced to the decimal of a pound, by dividing it by 20.

Thus, let it be required to reduce 9d. to the decimal of a pound Sterling. To perform this, I first reduce 9d. to the decimal of a shilling, by dividing by 12; and then divide the decimal of a shilling by 20, as follows:

$$\begin{array}{rcl} d. & s. & \\ 9 = \frac{9}{12}, & \text{and} & 12) 9.0 \end{array} \quad \begin{array}{r} 20) L. \\ .75 \end{array} \quad \begin{array}{r} d. \\ .0375 = 9 \end{array}$$

In dividing by 20, because the first dividend takes in two decimal places, I set 0 after the decimal point, the quotient-figure 3 being decimal seconds.

$$\begin{array}{r} 84 \quad 60 \\ \hline 60 \quad 150 \\ 60 \quad 140 \\ \hline (0) \quad 100 \\ \quad 100 \\ \hline (0) \end{array}$$

2. If the given parts be of different denominations, work by the following

R U L E S.

1. Place the numbers that make an unit of their respective superior denominations, under one another, as so many divisors; setting those of the lowest denomination uppermost.

2. On the right of the divisors place the several given parts, as so many dividends, and the lowest of the parts uppermost.

3. Annex ciphers to the uppermost dividend; or, rather, suppose ciphers annexed; then divide, placing the quot on the right of the following dividend. Again, divide this mixt number by the following divisor, placing the quot on the right of the subsequent dividend. Proceed in the same manner, till you have divided by every divisor, and the last quot is the decimal required.

E X.

EXAMPLE I.

Reduce 14s. 9d. 3f. to the decimal of a pound.

Here the numbers that make an unit of their respective superior denominations, *viz.* 4, 12, 20, are placed under one another, as so many divisors; and opposite to them the given parts 3f. 9d. 14s. are placed under one another as dividends; and the several divisions being finished, the last quot. .740625 is a decimal of a pound equal to 14s. 9d. 3f.

$$\begin{array}{r|l}
 4 & 3 \\
 \hline
 12 & 9.75 \\
 \hline
 20 & 14.8125 \\
 \hline
 & .740625 \text{ L.}
 \end{array}$$

The reason of the rules is obvious: for the divisors are the component parts of 960, the number of farthings in a pound Sterling; and therefore, if we divide by these component parts, as in the above example, we shall have the same decimal as would be obtained by reducing the given parts to farthings, and then dividing by 960.

Ciphers on the right of any divisor may be cut off, as in the above example; or quite omitted, as in the following examples: such omission being compensated by the ciphers annexed to the dividend.

When the quot of any division is a repeater, or circulate, the next division must be carried on, not by annexing ciphers, but by annexing the repeated figure or figures of the circle; as in the examples following.

EXAMPLE II.

Reduce 18s. 4d. 1f. to the decimal of a pound.

Here, in dividing by 12, the quot is a mixt repeater; and therefore, in dividing by 20, that is, by 2, after the dividend is exhausted, I annex the repeating figure 6, and the quot, or decimal sought, is a mixt repeater.

$$\begin{array}{r|l}
 4 & 1 \\
 \hline
 12 & 4.25 \\
 \hline
 20 & 18.3541\bar{6} \\
 \hline
 & .917708\bar{3} \text{ L.}
 \end{array}$$

EXAMPLE III.

Reduce 3 Q. 18 lb. to the decimal of a C.

Here, because 28 is a troublesome divisor, I divide by the component parts 4 and 7, and the quot is a mixt circulate.

In dividing the circulate by 4, after the dividend is exhausted, instead of annexing ciphers, I annex the figures of the circle, and the quot, or decimal sought, comes out another mixt circulate.

$$\begin{array}{r|l}
 4 & 1.8 \\
 \hline
 7 & 4.5 \\
 \hline
 4 & 3.6,428571,42 \\
 \hline
 & .910,714285,71
 \end{array}$$

E X.

EXAMPLE IV.

Reduce 10 oz. 18 dw. 16 gr. to the decimal of a lb. Troy.

Here, again, instead of dividing by 24, I divide by the component parts 4 and 6.

$$\begin{array}{r|l}
 24 \left\{ \begin{array}{l} 4 \\ 6 \end{array} & \begin{array}{l} 16 \\ 4 \end{array} \\
 \hline
 2 & 18.6 \\
 \hline
 12 & 10.93 \\
 \hline
 & .91
 \end{array}$$

The examples left, in Method I. for the learner's exercise, may be here resumed, to be reduced by this method.

METHOD III.

When the given parts consist of several denominations, we may find the decimal to each of these denominations separately; and the sum of the decimals thus found is the decimal required.

EXAMPLE I.

Reduce 16s. 6d. to the decimal of a pound.

$$\begin{array}{rcl}
 & L. & L. \\
 16s. & = \frac{16}{20} & = .8 \\
 6d. & = \frac{6}{240} & = .025 \\
 \hline
 & .825 & = 16 \text{ s. } 6 \text{ d.}
 \end{array}$$

EXAMPLE II.

Reduce 13s. 4d. 2f. to the decimal of a pound.

$$\begin{array}{rcl}
 & L. & L. \\
 13s. & = \frac{13}{20} & = .65 \\
 4d. & = \frac{4}{240} & = .01666 \\
 2f. & = \frac{2}{960} & = .002083 \\
 \hline
 & .66875 & = 13 \text{ s. } 4 \text{ d. } 2 \text{ f.}
 \end{array}$$

EXAMPLE III.

Reduce 8 oz. 19 dw. 8 gr. to the decimal of a lb. Troy.

$$\begin{array}{rcl}
 & lb. & lb. \text{ Troy.} \\
 8 \text{ oz.} & = \frac{8}{12} & = .66666 \\
 19 \text{ dw.} & = \frac{19}{240} & = .07916 \\
 8 \text{ gr.} & = \frac{8}{5760} & = .00138 \\
 \hline
 & .74722 & = 8 \text{ oz. } 19 \text{ dw. } 8 \text{ gr.}
 \end{array}$$

The

EXAMPLE I.

Reduce 14s. 9d. 3f. to the decimal of a pound.

Here the numbers that make an unit of their respective superior denominations, *viz.* 4, 12, 20, are placed under one another, as so many divisors; and opposite to them the given parts 3f. 9d. 14s. are placed under one another as dividends; and the several divisions being finished, the last quot. .740625 is a decimal of a pound equal to 14s. 9d. 3f.

$$\begin{array}{r|l}
 4 & 3 \\
 \hline
 12 & 9.75 \\
 \hline
 20 & 14.8125 \\
 \hline
 & .740625 \text{ L.}
 \end{array}$$

The reason of the rules is obvious: for the divisors are the component parts of 960, the number of farthings in a pound Sterling; and therefore, if we divide by these component parts, as in the above example, we shall have the same decimal as would be obtained by reducing the given parts to farthings, and then dividing by 960.

Ciphers on the right of any divisor may be cut off, as in the above example; or quite omitted, as in the following examples: such omission being compensated by the ciphers annexed to the dividend.

When the quot of any division is a repeater, or circulate, the next division must be carried on, not by annexing ciphers, but by annexing the repeated figure or figures of the circle; as in the examples following.

EXAMPLE II.

Reduce 18s. 4d. 1f. to the decimal of a pound.

Here, in dividing by 12, the quot is a mixt repeater; and therefore, in dividing by 20, that is, by 2, after the dividend is exhausted, I annex the repeating figure 6, and the quot, or decimal sought, is a mixt repeater.

$$\begin{array}{r|l}
 4 & 1 \\
 \hline
 12 & 4.25 \\
 \hline
 20 & 18.3541\bar{6} \\
 \hline
 & .917708\bar{3} \text{ L.}
 \end{array}$$

EXAMPLE III.

Reduce 3Q. 18lb. to the decimal of a C.

Here, because 28 is a troublesome divisor, I divide by the component parts 4 and 7, and the quot is a mixt circulate.

In dividing the circulate by 4, after the dividend is exhausted, instead of annexing ciphers, I annex the figures of the circle, and the quot, or decimal sought, comes out another mixt circulate.

$$\begin{array}{r|l}
 4 & 1.8 \\
 \hline
 7 & 4.5 \\
 \hline
 4 & 3.6,428571,42 \\
 \hline
 & .910,714285,71
 \end{array}$$

EXAMPLE IV.

Reduce 19 oz. 18 dw. 16 gr. to the decimal of a lb. Troy.

$$\begin{array}{r|l} 24 \left\{ \begin{array}{l} 4 \\ 6 \\ 2 \\ 12 \end{array} & \begin{array}{l} 16 \\ 4 \\ 18.6 \\ 10.93 \end{array} \\ \hline & .91 \end{array}$$

Here, again, instead of dividing by 24, I divide by the component parts 4 and 6.

The examples left, in Method 1. for the learner's exercise, may be here resumed, to be reduced by this method.

METHOD III.

When the given parts consist of several denominations, we may find the decimal to each of these denominations separately; and the sum of the decimals thus found is the decimal required.

EXAMPLE I.

Reduce 16s. 6d. to the decimal of a pound.

$$\begin{array}{rcl} L. & L. & \\ 16s. = \frac{16}{20} = .8 & & \\ 6d. = \frac{6}{240} = .025 & & \\ \hline .825 = 16 & 6 & \end{array}$$

EXAMPLE II.

Reduce 13s. 4d. 2f. to the decimal of a pound.

$$\begin{array}{rcl} L. & L. & \\ 13s. = \frac{13}{20} = .65 & & \\ 4d. = \frac{4}{240} = .016666 & & \\ 2f. = \frac{2}{960} = .002083 & & \\ \hline .66875 = 13 & 4 & 2 \end{array}$$

EXAMPLE III.

Reduce 8 oz. 19 dw. 8 gr. to the decimal of a lb. Troy.

$$\begin{array}{rcl} lb. & lb. \text{ Troy.} & \\ 8 \text{ oz.} = \frac{8}{12} = .66666 & & \\ 19 \text{ dw.} = \frac{19}{240} = .07916 & & \\ 8 \text{ gr.} = \frac{8}{5760} = .00138 & & \\ \hline .74722 = 8 & 19 & 8 \end{array}$$

The

The same answer may also be found by adding the vulgar fractions, and then reducing the vulgar fraction of their sum to a decimal.

Thus, in Ex. 1. we may say, $\frac{16}{10} + \frac{6}{40} = \frac{3840}{4800} + \frac{120}{4800} = \frac{3960}{4800} = \frac{396}{480} = \frac{132}{160} = \frac{66}{80} = \frac{33}{40} = .825$.

METHOD IV.

The decimal of given parts may frequently be known by inspection, without any operation: for because $\frac{1}{4} = .25$, $\frac{1}{2} = .5$, $\frac{3}{4} = .75$, $\frac{1}{3} = .3$, $\frac{2}{3} = .6$, $\frac{1}{5} = .2$, $\frac{1}{6} = .1\bar{6}$, $\frac{2}{5} = .4$, &c. it follows, that if the given parts be equal to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, &c. of the integer, the correspondent decimal will be .25, .5, .75, &c.

Thus, 3 inches = $\frac{1}{4}$ foot, = .25; and 6 inches = $\frac{1}{2}$ foot, = .5; and 9 inches = $\frac{3}{4}$ foot, = .75; and 4 inches = $\frac{1}{3}$ foot = .3; and 8 inches = $\frac{2}{3}$ foot, = .6.

In like manner, 4s. = $\frac{1}{5}$ L. = .2; and 6s. 8d. = $\frac{1}{3}$ L. = .3; and 13s. 4d. = $\frac{2}{5}$ L. = .6.

But the decimal answering to any number of shillings, pence, and farthings, may be readily found by inspection, to three places, in the following manner, *viz.*

Half the greatest even number of shillings gives the figure for the place of primes; the number of farthings in the given pence and farthings, increased by unity, when their sum amounts to 24, or upwards, generally affords two figures more for seconds and thirds; but if the number of farthings in the given pence and farthings amount only to a single digit, that digit goes to the place of thirds, and 0 fills the place of seconds. Here observe, if the given number of shillings be odd, the place of seconds must be increased by 5.

EXAMPLES.

Ex.	s.	d.	f.	L.
1.	10	6	3	=.528
2.	14	1	1	=.705
3.	19	7	3	=.982
4.	17	6		=.875
5.	1			=.05
6.		2	1	=.009

In Example 1. the half of 10s. gives 5 for primes; the 6d. 3f. make 27 farthings; which amounting to 24, I increase by unity, and have 28 for seconds and thirds.

In Example 2. the half of 14s. gives 7 for primes, and 1d. 1f. make 5 farthings; which being a single digit, is so many thirds, and I put 0 in the place of seconds.

In

In Example 3. the half of 19 s. gives 9 for primes, and 7 d. 3. f. make 31 farthings; which amounting to 24, I increase by unity, and have 32; but 19 s. being an odd number, I increase the place of seconds by 5, and so have 82 for seconds and thirds.

In Example 4. the half of 17 s. gives 8 for primes, and 6 d. makes 24 farthings; which, increased by unity, is 25; and because 17 s. is an odd number, I increase the place of seconds by 5, and so have 75 for seconds and thirds.

In Example 5. the half of 1 s. gives 0 for primes; and 1 s. being an odd number, gives 5 for seconds; and as there are no pence or farthings, there can be no thirds.

In Example 6. no shillings gives 0 for primes, and 2 d. 1 f. make 9 farthings; which, being a single digit, goes to the place of thirds, and 0 fills the place of seconds.

The reason of increasing the number of farthings by unity, when their sum amounts to 24, or upwards, is obvious: for a pound in farthings is 960, and in thousandth parts is 1000; consequently, $500=480$, and $50=48$, and $25=24$. If therefore to every 24 farthings unity be added, we change the farthings, or 960 parts, into 1000th parts.

The reason of adding 5 to the place of seconds for every odd shilling is also evident: for $1 \text{ s.} = \frac{1}{20} \text{ L.} = \frac{5}{100} = .05$.

A decimal of three places is exact enough in most computations; but if the complete decimal be required, it may be found as follows.

Take half the number of farthings in the pence and farthings, rejecting sixpence if there be one; to the half number of farthings annex ciphers, or rather suppose ciphers annexed; divide by 12 till 0 remain, or till the figure in the quot repeat; and the quot annexed to the three figures formerly found is the decimal complete. The former six examples completed in this manner, follow.

Ex.	s.	d.	f.
1.	10	6	3=.528125
2.	14	1	1=.7052083
3.	19	7	3=.9822916
4.	17	6	=.875
5.	1		=.05
6.		2	1=.009375

In Ex. 1. rejecting the 6 d. the half of the farthings is 1.5, and $12)1.5(125$; and this quot annexed to 528 completes the decimal, which is finite, there being no remainder.

In Ex. 2. the half of the farthings in 1 d. 1 f. is 2.5, and $12)2.5(2083$; which quot, annexed to 705, completes the decimal, being a mixt repeater.

In

In Ex. 3. rejecting 6 d. there remains 1 d. 3 f.; the half of which in farthings is 3.5, and $12)3.5(2916$; which quot, annexed to 982, completes the decimal, being another mixt repeater. The remaining three examples give finite decimals.

The reason of the above operation is obvious. For we reject every 6 d. because the farthings in 6 d. was converted to thousandths parts, by adding unity when the first three figures of the decimal were found: but the farthings in the other pence and farthings, being carried to the quot without any correction, are defective, and may be completed by saying, As 24 to 1, so the number of given farthings to the additional part; or, As 12 to 1, so half the number of farthings to the additional part; that is, we divide half the number of farthings by 12, and the quot annexed to the three figures first found, completes the decimal.

The decimals of pence and farthings may also be completed in the manner following.

Multiply by 4 the figures in the place of seconds and thirds, if less than 25; or multiply by 4, their excess, above 25, or above 50, or above 75; to the product add 1 for every 24 it contains; and you will have two places more to the decimal. If the fourth and fifth figures thus found be still imperfect, proceed, in like manner, to multiply them, or their excesses, by 4, to the product adding 1 for every 24 it contains, and you will gain two places more to the decimal; which will now either be finite, or will terminate in a repeating figure.

Thus, in Ex. 1. the excess of 28 above 25 is 3, and $3 \times 4 = 12$, the two subsequent figures of the decimal. But the decimal is still imperfect: for all decimals of this sort are imperfect, except when the last two figures are 25, or 50, or 75, or when the decimal terminates in a repeating figure. Accordingly I proceed to find other two figures, saying, $12 \times 4 = 48$, and $48 + 2 = 50$, which completes the decimal.

In Ex. 2. I say, $05 \times 4 = 20$; but the decimal being still imperfect, I say, $20 \times 4 = 80$, and $80 + 3 = 83$. So the decimal repeats.

In Ex. 3. the excess of 82 above 75 is 7, and $7 \times 4 = 28$, and $28 + 1 = 29$. Again, $29 - 25 = 4$, and $4 \times 4 = 16$.

In Ex. 6. I say, $9 \times 4 = 36$, and $36 + 1 = 37$. Again, $37 - 25 = 12$, and $12 \times 4 = 48$, and $48 + 2 = 50$.

The reason of the rule now assigned will appear by considering, that if 6 d. or 24 farthings, require the addition of an unit, to complete the decimal, every single farthing will require the addition of $\frac{1}{24}$ of an unit; but $\frac{1}{24} = .0416$. If, therefore, the imperfect decimal for the farthings be multiplied by .0416, the product will be the additional part. Thus, .001, the expression for one farthing, multiplied by .0416, gives .0000416 for the additional part.

.001

.0000416

And .0010416 is the decimal of 1 farthing complete.

But .0416 being a troublesome multiplier, it is better to use the multiplier $.04 = \frac{1}{25}$; and as a twenty-fifth part is less than a twenty-fourth part, by 1 in 24, it is plain, that if you multiply by .04, or simply by 4, adding 1 for every 24 in the product, and annex the product thus increased to the imperfect expression for the farthings, you will have the same decimal as if you had multiplied by .0416.

M E T H O D V.

The decimal for the parts of coin, weights, and measures, may easily and expeditiously be obtained, from the following tables, constructed for that purpose: in using of which, take the decimals for the several given parts; add them, as taught in Addition of Decimals, and their sum is the decimal required.

E X A M P L E I.

Required the decimal of a pound, for 15s. $7\frac{1}{2}$ d.

$$\begin{aligned} 15 \text{ s.} &= .75 \\ 7 \text{ d.} &= .02916666 \\ \frac{1}{2} \text{ d.} &= .00364583 \end{aligned}$$

Ans. .78281250 L.

E X A M P L E II.

Required the decimal of one C. for 2 Q. 12lb. 9oz.

$$\begin{aligned} 2 \text{ Q.} &= .5 \\ 12 \text{ lb.} &= .10714285, 714285, \\ 9 \text{ oz.} &= .00502232, 142857, \end{aligned}$$

Ans. .61216517, 857142, C.

E X A M P L E III.

Required the decimal of a lb. Troy, for 10 oz. 7 dw. 1 gr.

$$\begin{aligned} 10 \text{ oz.} &= .83333333 \\ 7 \text{ dw.} &= .02916666 \\ 1 \text{ gr.} &= .00017361 \end{aligned}$$

Ans. .86267361 lb. Troy.

The decimals in the tables are given complete, to answer such purposes as may require accuracy; but the practitioner may limit them to four, five, six, or any number of places, as he sees cause. The tables follow.

I. M O.

I. M O N E Y.

TABLE I. One pound the integer.

N.	Shillings.	pence.	farthings.	half-farthings.
1	.05	.00416	.0010416	.00052083
2	.1	.0083	.002083	.0010416
3	.15	.0125	.003125	.0015625
4	.2	.016		.002083
5	.25	.02083		.002083
6	.3	.025		.003125
7	.35	.02916		.00364583
8	.4	.03		
9	.45	.0375		
10	.5	.0416		
11	.55	.04583		
12	.6			
13	.65			
14	.7			
15	.75			
16	.8			
17	.85			
18	.9			
19	.95			

TABLE II. One shilling the integer.

N.	Pence.	farthings.	half-farthings.
1	.083	.02083	.010416
2	.16	.0416	.02083
3	.25	.625	.03125
4	.33		.0416
5	.416		.052083
6	.5		.0625
7	.583		.072916
8	.66		
9	.75		
10	.83		
11	.916		

2. AVOIR.

2. AVOIRDUPOIS WEIGHT.

TABLE I. One tun the integer.

N.	G.	Q.	lb.	N.	G.	lb.
1	.05	.0125	.0004464,285714,	15	.75	.0066964,285714,
2	.1	.025	.0008928,571428,	16	.8	.0071428,571428,
3	.15	.0375	.0013392,857142,	17	.85	.0075892,857142,
4	.2		.0017857,142857,	18	.9	.0080357,142857,
5	.25		.0022321,428571,	19	.95	.0084821,428571,
6	.3		.0026785,714285,	20		.0089285,714285,
7	.35		.003125	21		.009375
8	.4		.0035714,285714,	22		.0098214,285714,
9	.45		.0040178,571428,	23		.0102678,571428,
10	.5		.0044642,857142,	24		.0107142,857142,
11	.55		.0049107,142857,	25		.0111607,142857,
12	.6		.0053571,428571,	26		.0116071,428571,
13	.65		.0058035,714285,	27		.0120535,714285,
14	.7		.00625			

TABLE II. One C. or 112 lb. the integer.

N.	Q.	lb.	oz.	N.	lb.
1	.25	.00892857,142857,	.00055803,571428,	16	.14285714,285714,
2	.5	.01785714,285714,	.00111607,142857,	17	.15178571,428571,
3	.75	.02678571,428571,	.00167410,714285,	18	.16071428,571428,
4		.03571428,571428,	.00223214,285714,	19	.16964285,714285,
5		.04464285,714285,	.00279017,857142,	20	.17857142,857142,
6		.05357142,857142,	.00334821,428571,	21	.1875
7		.0625	.00390625	22	.19642857,142857,
8		.07142857,142857,	.00446428,571428,	23	.20535714,285714,
9		.08035714,571428,	.00502232,142857,	24	.21428571,428571,
10		.08928571,428571,	.00558035,714285,	25	.22321428,571428,
11		.09821428,571428,	.00613839,285714,	26	.23214285,714285,
12		.10714285,714285,	.00669642,857142,	27	.24107142,857142,
13		.11607142,857142,	.00725446,428571,		
14		.125	.0078125		
15		.13392857,142857,	.00837053,571428,		

TABLE III One lb. the integer.

N.	oz.	drams.	N.	oz.	drams.	N.	oz.	drams.
1	.0625	.00390625	6	.375	.0234375	11	.6875	.04296875
2	.125	.0078125	7	.4375	.02734375	12	.75	.046875
3	.1875	.01171875	8	.5	.03125	13	.8125	.05078125
4	.25	.015625	9	.5625	.03515625	14	.875	.0546875
5	.3125	.01953125	10	.625	.0390625	15	.9375	.05859375

3. TROY WEIGHT.

TABLE I. One lb. the integer.

N.	oz.	dw.	grains.	N.	dw.	grains.
1	.083	.00416	.00017361	13	.05416	.00225694
2	.16	.0083	.0003472	14	.0583	.0024308
3	.25	.0125	.00052083	15	.0625	.00260416
4	.3	.016	.000694	16	.06	.0027
5	.416	.02083	.00086808	17	.07083	.00295138
6	.5	.025	.0010416	18	.075	.003125
7	.583	.02916	.00121527	19	.07916	.00329861
8	.6	.03	.00138	20		.003472
9	.75	.0375	.0015625	21		.00364583
10	.83	.0416	.0017361	2		.0038194
11	.916	.04583	.00190972	23		.00399308
12		.05	.002083			

TABLE II. One ounce the integer.

N.	dw.	grains.	N.	dw.	grains.
1	.05	.002083	13	.65	.027083
2	.1	.00416	14	.7	.02916
3	.15	.00625	15	.75	.03125
4	.2	.0083	16	.8	.03
5	.25	.010416	17	.85	.035416
6	.3	.0125	18	.9	.0375
7	.35	.014583	19	.95	.039583
8	.4	.016	20		.0416
9	.45	.01875	21		.04375
10	.5	.02083	22		.04583
11	.55	.022916	23		.047916
12	.6	.25			

4. APOTHECARIES WEIGHT.

TABLE. One lb. the integer.

N.	oz.	drams.	scruples.	grains.	N.	grains.
1	.083	.010416	.003472	.00017362	12	.002083
2	.16	.02083	.00694	.0003472	13	.00225694
3	.25	.03125		.00052083	14	.0024308
4	.3	.0416		.000694	15	.00260416
5	.416	.052083		.00086808	16	.0027
6	.5	.0625		.0010416	17	.00295138
7	.583	.072916		.00121527	18	.003125
8	.6			.00138	19	.00329862
9	.75			.0015625		
10	.83			.0017362		
11	.916			.00190972		

5. WOOL-WEIGHT.

TABLE I. One last the integer.

N.	Sacks.	stones.	lb.	N.	stones.
1	.083	.0032,051282,	.0002,289377,	14	.0448,717948,
2	.16	.0064,102564,	.0004,578754,	15	.0480,769230,
3	.25	.0096,153846,	.0006,868131,	16	.0512,820512,
4	.3	.0128,205128,	.0009,157509,	17	.0544,871794,
5	.416	.0160,256410,	.0011,446886,	18	.0576,923076,
6	.5	.0192,307692,	.0013,736263,	19	.0608,974358,
7	.583	.0224,358974,	.0016,025641,	20	.0641,025641,
8	.6	.0256,410256,	.0018,315018,	21	.0673,076923,
9	.75	.0288,461538,	.0020,604395,	22	.0705,128205,
10	.83	.0320,512820,	.0022,893772,	23	.0737,179487,
11	.916	.0352,564102,	.0025,183150,	24	.0769,230769,
12		.0384,615384,	.0027,472527,	25	.0801,282051,
13		.0416	.0029,761904,		

TABLE II. One lack the integer.

N.	Stones.	lb.	N.	stones.
1	.03,846153,	.00,274725,	14	.53,846153,
2	.07,692307,	.00,549450,	15	.57,692307,
3	.11,538461,	.00,824175,	16	.61,538461,
4	.15,384615,	.01,098901,	17	.65,384615,
5	.19,230769,	.01,373626,	18	.69,230769,
6	.23,076923,	.01,648351,	19	.73,076923,
7	.26,923076,	.01,923076,	20	.76,923076,
8	.30,769230,	.02,197802,	21	.80,769230,
9	.34,615384,	.02,472527,	22	.84,615384,
10	.38,461538,	.02,747252,	23	.88,461538,
11	.42,307692,	.03,021978,	24	.92,307692,
12	.46,153846,	.03,296703,	25	.96,153846,
13	.5	.03,571428,		

6. DRY MEASURE.

TABLE I.
One load the integer.

N.	Q.	Buf.	Gallons.
1	.2	.025	.003125
2	.4	.05	.00625
3	.6	.075	.009375
4	.8	.1	.0125
5		.125	.015625
6		.15	.01875
7		.175	.021875

TABLE II.
One quarter the integer.

Buf.	Gallons.
.125	.015625
.25	.03125
.375	.046875
.5	.0625
.625	.078125
.75	.09375
.875	.109375

TABLE III.
One bushel the integer.

Gall.	Pints.
.125	.015625
.25	.03125
.375	.046875
.5	.0625
.625	.078125
.75	.09375
.875	.109375

7. WINE.

7. WINE-MEASURE.

TABLE I. One tun the integer.

N. Hbds.	Gallons.	N. Gallons.	N. Gallons.
1 .25	.00,396825,	22.08,730158,	43.17,003492,
2 .5	.00,793650,	23.09,126984,	44.17,460317,
3 .75	.01,190476,	24.09,523809,	45.17,857142,
4	.01,587301,	25.09,920634,	46.18,253968,
5	.01,984126,	26.10,317460,	47.18,650793,
6	.02,380952,	27.10,714285,	48.19,047619,
7	.02,777777,	28.11,111111,	49.19,444444,
8	.03,174603,	29.11,507936,	50.19,841269,
9	.03,571428,	30.11,904761,	51.20,238095,
10	.03,968253,	31.12,301587,	52.20,634920,
11	.04,365079,	32.12,698412,	53.21,031746,
12	.04,761904,	33.13,095238,	54.21,428571,
13	.05,158730,	34.13,492063,	55.21,825396,
14	.05,555555,	35.13,888888,	56.22,222222,
15	.05,952380,	36.14,285714,	57.22,619047,
16	.06,349206,	37.14,682539,	58.23,015873,
17	.06,746031,	38.15,079365,	59.23,412698,
18	.07,142857,	39.15,476190,	60.23,809523,
19	.07,539682,	40.15,873015,	61.24,206349,
20	.07,936507,	41.16,269841,	62.24,603174,
21	.08,333333,	42.16,666666,	

TABLE II. One hoghead the integer.

N.	Gallons.	N.	Gallons.	Pints.
60	.952,380952,	1	.015,873015,	.001,984126,
50	.793,650793,	2	.031,746031,	.003,968253,
40	.634,920634,	3	.047,619047,	.005,952380,
30	.476,190476,	4	.063,492063,	.007,936507,
20	.317,460317,	5	.079,365079,	.009,920634,
10	.158,730158,	6	.095,238095,	.011,904761,
		7	.111,111111,	.013,888888,
		8	.126,984126,	
		9	.142,857142,	

8. BEER-MEASURE.

TABLE I. One tun the integer.				TABLE II. One hoghead the integer.		
N.	Hbds.	Firkins.	Gallons.	Fir.	Gallons.	Pints.
1	.25	.0416	.004,629,	.16	.0185,185,	.0023,148,
2	.5	.083	.009,259,	.32	.0370,370,	.0046,296,
3	.75	.125	.013,888,	.5	.0555,555,	.0069,444,
4		.16	.018,518,	.66	.0740,740,	.0092,592,
5		.2083	.023,148,	.83	.0925,925,	.0115,740,
6			.027,777,		.1111,111,	.0138,888,
7			.032,407,		.1296,296,	.0162,037,
8			.037,037,		.1481,481,	

TABLE III. One hoghead of 51 gallons, 8 pints each, the integer.

N.	Gallons.	N.	Pints.
50	.980,3921568627450980,	1	.002,4509803921568627,
40	.784,3137254901960784,	2	.004,9019607843137254,
30	.588,2352941176470588,	3	.007,3529411764705882,
20	.392,1568627450980392,	4	.009,8039215686274509,
10	.196,0784313725490196,	5	.012,2549019607843137,
9	.176,4705882352941176,	6	.014,7058823529411764,
8	.156,8627450980392156,	7	.017,1568627450980392,
7	.137,2559019607843137,		
6	.117,6470588235294117,		
5	.098,0392156862745098,		
4	.078,4313725490196078,		
3	.058,8235294117647058,		
2	.039,2156862745098039,		
1	.019,6078431372549019,		

TABLE IV. One Scots hoghead the integer.

N.	Gall.	pints.	N.	Gall.	pints.	N.	Gall.
1	.0625	.0078125	6	.375	.046875	11	.6875
2	.125	.015625	7	.4375	.0546875	12	.75
3	.1875	.0234375	8	.5		13	.8125
4	.25	.03125	9	.5625		14	.875
5	.3125	.0390625	10	.625		15	.9375

9. CLOTH.

9. CLOTH-MEASURE.

TABLE I. One yard the integer.			TABLE II. One ell Flemish the integer.			TABLE III. One ell English the integer.		
N.	℥.	Nails.	N.	℥.	Nails.	N.	℥.	Nails.
1	.25	.0625	1	.3	.083	1	.2	.05
2	.5	.125	2	.6	.16	2	.4	.1
3	.75	.1875	3		.243	3	.6	.15
						4	.8	

10. LONG MEASURE.

TABLE I. One mile the integer.

N.	Fur.	chains.	poles.	links.	N.	links.	N.	links.
1	.125	.0125	.003125	.000125	10	.00125	19	.002375
2	.25	.025	.00625	.00025	11	.001375	20	.0025
3	.375	.0375	.009375	.000375	12	.0015	21	.002625
4	.5	.05		.0005	13	.001625	22	.00275
5	.625	.0625		.000625	14	.00175	23	.002875
6	.75	.075		.00075	15	.001875	24	.003
7	.875	.0875		.000875	16	.002		
8		.1		.001	17	.002125		
9		.1125		.001125	18	.00225		

TABLE II. One chain the integer.

TABLE III. One pole the integer.

N	Pol.	lin.	N	lin.	N	lin.	N	Yards.	feet.	inches.	N	inches.
1	.25	.01	10	.10	19	.19	1	.1,81,	.0,60,	.0,05,	10	.0,50,
2	.5	.02	11	.11	20	.20	2	.3,63,	.1,2,	.0,10,	11	.0,55,
3	.75	.03	12	.12	21	.21	3	.5,45,		.0,15,		
4		.04	13	.13	22	.22	4	.7,27,		.0,20,		
5		.05	14	.14	23	.23	5	.9,09,		.0,25,		
6		.06	15	.15	24	.24	6			.0,30,		
7		.07	16	.16			7			.0,35,		
8		.08	17	.17			8			.0,40,		
9		.09	18	.18			9			.0,45,		

TABLE IV. One yard
the integer.

N.	Feet.	inches.
1	.3	.027
2	.6	.054
3		.081
4		.108
5		.135
6		.162
7		.189
8		.216
9		.243
10		.270
11		.297

TABLE V. One foot
the integer.

N	Inch.	lines.
1	.083	.00694
2	.16	.0138
3	.25	.02083
4	.3	.027
5	.416	.03472
6	.5	.0416
7	.583	.04861
8	.66	.0555
9	.75	.0625
10	.83	.0694
11	.916	.07638

TABLE VI. One Scots mile the integer.

N.	Furl.	chains.	falks.	ells.
1	.125	.0125	.003125	.00052083
2	.25	.025	.00625	.0010416
3	.375	.0375	.009375	.00156249
4	.5	.05		.00208332
5	.625	.0625		.00260417
6	.75	.075		
7	.875	.0875		
8		.1		
9		.1125		

II. LAND.

II. LAND-MEASURE.

TABLE I. One acre the integer.

N. Rds.	sq. poles.	. links.	N.	sq. links.
1	.25	.00001	40	.0004
2	.5	.00002	50	.0005
3	.75	.00003	60	.0006
4	.025	.00004	70	.0007
5	.03125	.00005	80	.0008
6	.0375	.00006	90	.0009
7	.04375	.00007	100	.001
8	.05	.00008	200	.002
9	.05625	.00009	300	.003
10	.0625	.0001	400	.004
20	.125	.0002	500	.005
30	.1875	.0003	600	.006

TABLE II. One Scots acre the integer.

N. Rds.	sq. falls.	sq. ells.
1	.25	.00017367
2	.5	.0003472
3	.75	.00052083
4	.025	.000694
5	.03125	.00086808
6	.0375	.0010416
7	.04375	.00121527
8	.05	.001388
9	.05625	.0015625
10	.0625	.0017367
20	.125	.003472
30	.1875	.0052083

12. SUPERFICIAL MEASURE.

TABLE I. One square yard the integer.

N.	Sq. feet.	square inches.	N.	Square inches.
1	.1	.0007,716049382,	10	.0077,160493827,
2	.2	.0015,432098765,	20	.0154,329987654,
3	.3	.0023,148148148,	30	.0231,481481481,
4	.4	.0030,864197530,	40	.0308,641975308,
5	.5	.0038,580246913,	50	.0385,802469135,
6	.6	.0046,296296296,	60	.0462,962962962,
7	.7	.0054,012345679,	70	.0540,123456790,
8	.8	.0061,728395061,	80	.0617,283950611,
9	.9	.0069,444444444,	90	.0694,444444444,
			100	.0771,604938271,

TABLE

TABLE II. One foot the integer.

N. <i>sq. inch.</i>	N. <i>sq. inch.</i>	N. <i>sq. inch.</i>	N. <i>sq. inch.</i>
1 .00694	6 .0416	20 .138	70 .4862
2 .0138	7 .04862	30 .2083	80 .8
3 .02083	8 .08	40 .27	90 .625
4 .027	9 .0625	50 .3472	100 .694
5 .03472	10 .0694	60 .416	

13. SOLID MEASURE.

TABLE I. One cubic yard the integer.

N. <i>Cu. F.</i>	N. <i>Cu. F.</i>	N. <i>Cu. F.</i>	N. <i>Cu. F.</i>	N. <i>Cu. F.</i>
1 .037,	7 .259,	13 .481,	19 .703,	25 .925,
2 .074,	8 .296,	14 .518,	20 .740,	26 .962,
3 .111,	9 .333,	15 .555,	21 .777,	
4 .148,	10 .370,	16 .592,	22 .814,	
5 .185,	11 .407,	17 .629,	23 .851,	
6 .222,	12 .444,	18 .666,	24 .888,	

TABLE II. One cubic foot the integer.

N. <i>Cubic inches.</i>	N. <i>Cubic inches.</i>	N. <i>Cubic inches.</i>
1 .000578,703,	10 .005787,037,	100 .057870,370,
2 .001157,407,	20 .011574,074,	200 .115740,740,
3 .001736,111,	30 .017361,111,	300 .173611,111,
4 .002314,814,	40 .023148,148,	400 .231481,481,
5 .002893,518,	50 .028935,185,	500 .289351,851,
6 .003472,222,	60 .034722,222,	600 .347222,222,
7 .004050,925,	70 .040509,259,	700 .405092,592,
8 .004629,629,	80 .046296,296,	800 .462962,962,
9 .005208,333,	90 .052083,333,	900 .520833,333,
		1000 .578703,703,

14. A C I R C L E.

TABLE I. One circle the integer.

N.	Sig.	deg.	min.
1	.083	.0027	.00004,629,
2	.16	.008	.00009,259,
3	.25	.0083	.00013,888,
4	.3	.01	.00018,518,
5	.416	.0138	.00023,148,
6	.5	.016	.00027,777,
7	.583	.0194	.00032,407,
8	.6	.02	.00037,037,
9	.75	.025	.00041,666,
10	.83	.027	.00046,296,
11	.916		
20		.08	.00092,592,
30			.00138,888,
40			.00185,185,
50			.00231,481,

TABLE II. One sign the integer.

N.	Deg.	min.	sec.
1	.03	.0008	.00000,925,
2	.06	.001	.00001,851,
3	.1	.0016	.00002,777,
4	.13	.002	.00003,703,
5	.16	.0027	.00004,629,
6	.2	.003	.00005,555,
7	.23	.0038	.00006,481,
8	.26	.004	.00007,407,
9	.3	.005	.00008,333,
10	.3	.008	.00009,259,
20	.6	.01	.00018,518,
30		.016	.00027,777,
40		.02	.00037,037,
50		.027	.00046,296,

15. T I M E.

TABLE I. One year of 365 days the integer.

N.	Days.	N.	Days.	N.	Days.
1	.00,27397260,	8	.02,19178082,	60	.16,43835616,
2	.00,54794520,	9	.02,46575342,	70	.19,17808219,
3	.00,82191780,	10	.02,73972602,	80	.21,91780821,
4	.01,09589041,	20	.05,47945205,	90	.24,65753424,
5	.01,36986301,	30	.08,21917808,	100	.27,39726027,
6	.01,64383561,	40	.10,95890410,	200	.54,79452054,
7	.01,91780821,	50	.13,69863013,	300	.82,19178082,

TABLE

TABLE II. One year of 13 months, 28 days each, the integer.

N.	Months.	weeks.	days.	N.	Months.
1	.07,692307,	.01,923076,	.00,274725,	7	.53,846153,
2	.15,384615,	.03,846153,	.00,549450,	8	.61,538461,
3	.23,076923,	.05,769230,	.00,824175,	9	.69,230769,
4	.30,769230,		.01,098901,	10	.76,923076,
5	.38,461538,		.01,373626,	11	.84,615384,
6	.46,153846,		.01,648351,	12	.92,307692,

TAB. III. One month of 28 days the integer.

N.	W.	days.
1	.25	.03,571428,
2	.5	.07,142857,
3	.75	.10,714285,
4		.14,285714,
5		.17,857142,
6		.21,428571,

TAB. IV. One day the integer.

N.	Hour.	minutes.	N.	Minutes.
1	.0416	.000694	30	.02083
2	.083	.00138	40	.027
3	.125	.002083	50	.03472
4	.16	.0027		
5	.2083	.003472		
6	.25	.00416		
7	.2916	.004861		
8	.3	.005		
9	.375	.00625		
10	.416	.00694		
20	.83	.0138		

TABLE V. One hour, or one degree, the integer.

N.	Min.	sec.	N.	Min.	sec.
1	.016	.00027	10	.16	.0027
2	.03	.00054	20	.3	.0054
3	.05	.00083	30	.5	.0083
4	.08	.0011	40	.8	.011
5	.083	.00138	50	.83	.0138
6	.1	.0016			
7	.116	.00194			
8	.13	.002			
9	.15	.0025			

PROBLEM III.

To reduce the remainder of a division to a decimal.

R U L E.

The remainder being the numerator, and the divisor the denominator of a vulgar fraction, after placing the decimal point on the right of the integral part of the quot, annex ciphers to the remainder; then continue the division till 0 remain, or till the quot repeat or circulate, or till you think proper to limit the decimal; and the number on the right of the point is a decimal of the integer expressed in the quot.

Example 1.

Divide 5131. among 36 men.

$$\begin{array}{r}
 36 \overline{) 5131} \quad L. \\
 \underline{36} \\
 153 \\
 \underline{144} \\
 \text{Rem. } 90 \\
 72 \\
 \underline{} \\
 180 \\
 \underline{180} \\
 \text{Rem. } 0 \\
 (0)
 \end{array}$$

Example 2.

Divide 176s. among 24 boys.

$$\begin{array}{r}
 24 \overline{) 176} \\
 \underline{168} \\
 \text{Rem. } 80 \\
 72 \\
 \underline{} \\
 (8)
 \end{array}$$

Example 3.

Divide 267 guineas among 33 sailors.

$$\begin{array}{r}
 33 \overline{) 267} \quad g. \\
 \underline{264} \\
 \text{Rem. } 300 \\
 297 \\
 \underline{} \\
 3
 \end{array}$$

Example 4.

Divide 409 C. into 81 equal parcels.

$$\begin{array}{r}
 81 \overline{) 409} \quad C. \\
 \underline{405} \\
 \text{Rem. } 400 \\
 324 \\
 \underline{} \\
 760 \\
 729 \\
 \underline{} \\
 310 \\
 243 \\
 \underline{} \\
 670 \\
 648 \\
 \underline{} \\
 (22)
 \end{array}$$

P R O B.

P R O B. IV.

To reduce a decimal to value.

R U L E.

Multiply the given decimal by the number of parts of the next inferior denomination contained in an unit of the integer; and from the product point off so many figures to the right hand as there are places in the given decimal. On the left hand of the point are parts, and on the right a decimal of one of these parts; which decimal must be reduced in the same manner to the next inferior denomination, and from that to the next; and so on to the lowest; the several figures on the left of the points are parts; and if there be still some figure or figures on the right, they are a decimal of the lowest of the parts.

Example 1.

Reduce .875l. to value.

$$\begin{array}{r}
 \text{L.} \quad \text{s.} \quad \text{d.} \\
 .875 = 17 \quad 6 \\
 \quad \quad 20 \\
 \hline
 \text{s. } 17.500 \\
 \quad \quad 12 \\
 \hline
 \text{d. } 6.0
 \end{array}$$

Example 2.

Reduce .769l. to value.

$$\begin{array}{r}
 \text{L.} \quad \text{s.} \quad \text{d.} \quad \text{f.} \\
 .769 = 15 \quad 4 \quad 2 \\
 \quad \quad 20 \\
 \hline
 \text{s. } 15.3820 \\
 \quad \quad 12 \\
 \hline
 \text{d. } 4.584 \\
 \quad \quad 4 \\
 \hline
 \text{f. } 2.336
 \end{array}$$

Example 3.

Reduce .9375 C. to value.

$$\begin{array}{r}
 \text{C.} \quad \text{Q.} \quad \text{lb.} \\
 .9375 = 3 \quad 21 \\
 \quad \quad 4 \\
 \hline
 \text{Q. } 3.7500 \\
 \quad \quad 28 \\
 \hline
 600 \\
 \quad 150 \\
 \hline
 \text{lb. } 21.00
 \end{array}$$

Example 4.

Reduce .84362 lb. Troy to value.

$$\begin{array}{r}
 \text{lb. Troy.} \quad \text{oz.} \quad \text{dw.} \quad \text{gr.} \\
 .84362 = 10 \quad 2 \quad 11 \\
 \quad \quad 12 \\
 \hline
 \text{oz. } 10.12344 \\
 \quad \quad 20 \\
 \hline
 \text{dw. } 2.46880 \\
 \quad \quad 24 \\
 \hline
 18752 \\
 \quad 9376 \\
 \hline
 \text{gr. } 11.2512
 \end{array}$$

This

This problem is the converse of Problem II. Method II. and the one operation serves as a proof of the other. The reason of pointing the product, as the rule directs, is plain. For, in Ex. 1. as $1000 : 875 :: 20 : 17$; that is, as the decimal denominator to the decimal numerator, so the vulgar denominator to the vulgar numerator.

In Ex. 1. and 3. the full value of the decimal comes out in parts, the decimal being quite exhausted; but in Ex. 2. besides the parts, there is a decimal of a farthing, viz. .336 f.; and in Ex. 4. besides the parts, we have a decimal of a grain, viz. .2512 grains.

M O R E E X A M P L E S.

5. .8761. = 17 s. 6 d. 0.96 f.
6. .74 guineas = 15 s. 6 d. 1.92 f.
7. .43569 C. = 1 Q. 20 lb. 12.75648 oz.
8. .91112 lb. Troy = 10 oz. 18 dw. 16.0512 gr.
9. .95 yard = 2 feet 10.2 inches.
10. .746 acre = 2 roods 39 falls 12.96 ells Scots.

If the given decimal be a repeater, pure or mixt, in order to obtain the complete value, carry at 9 on the right hand; and if the multiplier have a cipher on the right, instead of annexing the cipher, annex the right-hand figure of the product. If the multiplier repeat, work as taught in multiplication of repeating decimals.

Ex. 1.

L. s. d.
 $\text{.}\text{6} = 13 \quad 4$
 20

s. 13.8
 12

d. 4.0

Ex. 2.

L. s. d.
 $\text{.}\text{98} = 18 \quad 8$
 20

s. 18.66
 12

d. 8.0

Ex. 3.

Mark. s. d. f.
 $3)\text{.}4453125 = 5 \quad 11 \quad 1$
 13.3

13359375

4453125

5.7890625

1484375

s. 5.9375000

12

d. 11.2500

4

f. 1.00

Ex. 4.

Ex. 4.

$$\begin{array}{r} \text{C. } \text{Q. } \text{lb.} \\ .3 = 1 \text{ } 9\frac{1}{2} \\ 4 \end{array}$$

$$\begin{array}{r} \text{Q. } 1.3 \\ 28 \\ \hline 28 \\ 66 \end{array}$$

lb. 9.3

Ex. 5.

$$\begin{array}{r} \text{lb. Troy. oz. dw.} \\ .62083 = 7 \text{ } 9 \\ 12 \end{array}$$

$$\begin{array}{r} \text{oz. } 7.45000 \\ 20 \\ \hline \text{dw. } 9.00 \end{array}$$

If the given decimal be a circulate, in order to obtain the value accurately, we may work as taught in addition and multiplication of circulates.

Ex. 1.

$$\begin{array}{r} \text{C. } \text{Q. } \text{lb.} \\ .6696,428571, = 2 \text{ } 19 \\ 4 \end{array}$$

$$\begin{array}{r} \text{Q. } 2.6785,714285, \\ 28 \\ \hline 5 \text{ } 4285,714285, \\ 13 \text{ } 5714,285714, \end{array}$$

lb. 19.0000000000

Ex. 2.

$$\begin{array}{r} \text{Pole. yd. ft. in.} \\ .87, = 4 \text{ } 2 \text{ } 6 \\ 5.5 \end{array}$$

$$\begin{array}{r} 4.39 \\ 43.93 \\ \hline \text{yds. } 4.833 \\ 3 \end{array}$$

feet. 2.500

inch. 6.0

But such operations being commonly tedious, it is more usual to take part of the circulate as an approximate, and find the value nearly by the general rule explained above.

The decimal of a pound Sterling may be reduced to value by inspection, in the following manner.

Double the figure in the place of primes for shillings; and if the figure in the place of seconds be 5, or exceed 5, reckon 1 shilling more; and rejecting 5 in the second place, the figures in the second and third places are so many farthings, abating 1 for every 25.

EXAMPLES.

	L.	s.	d.	f.
1.	.718	= 14	4	2
2.	.759	= 15	2	1
3.	.894	= 17	10	3
4.	.92	= 18	5	
5.	.68	= 13	7	1
6.	.072	= 1	5	2
7.	.9276	= 18	6	2
8.	.87638	= 17	6	1

In Example 1. the figure 7 doubled gives 14 s.; the two following figures 18 are farthings, equal to 4 d. 2 f.

In Example 2. the figure 7 doubled gives 14 s. and 5 in the place of seconds gives 1 shilling more, in all 15 s.; and the other figure 9 is farthings, viz. 2 d. 1 f.

In Example 3. the figure 8 in the place of primes, and 5 in the place of seconds give 17 s.; the remaining figures 44, abating 1, are farthings, viz. 10 d. 3 f.

In Examples 4. and 5. the place of thirds must be supposed to be filled with a cipher; and so, in Ex. 4. we have 20 f. = 5 d.; and in Ex. 5. we have 30 f. — 1 = 7 d. 1 f.

In Example 6. the place of primes affords no shillings; but we have 1 s. from the place of seconds.

In Examples 7. and 8. the figures to the right of the place of thirds, are rejected, as being in value less than a farthing.

When the figures in the second and third place to be converted into farthings are 25, the answer, by inspection, comes out exact, viz. 24 f. or 6 d.; but in all other cases, the answer, by inspection, is too great, no allowance or correction being made till the convertible number amount to 25, and afford a deduction of 1 farthing complete. Hence, by inspection, we have frequently 1 farthing more than by the common method; but the two methods will agree, or give the same answer, if, from the figures to be turned into farthings, we subtract their 25th part, esteeming the remainder farthings and decimal parts of a farthing.

Thus, $.718 \text{ l.} = 14 \text{ s. } 4 \text{ d. } 2 \text{ f.}$ by inspection; but by the common method, and by inspection corrected, the answer comes out 1 farthing less, as follows.

Q

Common

Common method.

L.
 .718
 20

 s. 14.360
 12

 d. 4.32
 4

 f. 1.28

Inspection corrected.

If 25 : 1 :: 18 : .72.
 that is, 25)18.0(.72
 175

 50 and 18
 50 .72

 (0) 17.28
 d. f.
 And 17.28 = 4 1.28

To conclude, instead of dividing by 25, we may multiply by .04; and then the exact value of any decimal of a pound Sterling may be found as follows:

From the primes and seconds set off the shillings; multiply the remainder by 4, setting the product two places to the right; subtract the product from the first remainder; and from the second remainder point off so many places to the right as there are figures in the first remainder. The number on the left of the point is farthings, and the figures on the right are a decimal of a farthing.

Example 1.

s. d. f.
 .718 l. = 14 4 1.28

 1. Rem. .18
 72 = 18 × 4

 2. Rem. 17.28

Example 2.

s. d. f.
 .7691 l. = 15 4 2.336

 1. Rem. .191
 764 = 191 × 4

 2. Rem. 18.336

The value of a decimal is easily and readily found from the following tables, calculated for that purpose.

The tables are constructed by finding the value of unity in the place of primes, seconds, thirds, fourths, &c.; that is, by finding the value of .1, of .01, of .001, .0001, &c. and placing these values under the top-figures 1, 2, 3, 4, &c. The values of 2 in the place of primes, seconds, thirds, &c. are found by doubling the values of unity already found; the values of 3 are found by adding the values of 2 to the values of 1; the values of 4 are found by adding the values of 3 to the values of 1; and in like manner are found the values of the other digits.

In using the tables, take the value of the figure in the place of primes from under the top-figure 1; take the value of the figure in the place of seconds from under the top-figure 2, &c. The sum of these values is the total value of the given decimal.

I. M O N E Y.

E X A M P L E I.

Required the value of .775 l. Sterling.

	s.	d.	f.
.7	=	14	
.07	=	1	4 3.2
.005	=	1	0.8

Total value 15 6

The value of the first and second figures of the given decimal may generally with ease be taken out at once : thus,

	s.	d.	f.
.77	=	15	4 3.2
.005	=	1	0.8

Total value 15 6

E X A M P L E II.

Required the value of .795 l. Sterling.

	s.	d.	f.
.79	=	15	9 2.4
.005	=	1	0.8

Total value 15 10 3.2

If the given decimal consist of more places than there are top-figures in the table, in this case the values under the top-figure 5 are found by moving the decimal points under the top-figure 4, one place more to the left; and the values under the top-figure 6 are found by moving the decimal points under 4, two places more to the left, &c.

E X A M P L E III.

Required the value of .634375 l. Sterling.

	s.	d.	f.
.63	=	12	7 0.8
.004	=		3.84
.0003	=		.288
.00007	=		.0672
.000005	=		.0048

Total value 12 8 1.0000

Q 2

18

If the given decimal be a repeater, pure or mixt, take successive values of the repeating figure, till the columns of the decimals begin to reiterate; that is, till the column of primes be reiterated in the column of seconds, or till the column of seconds be reiterated in the column of thirds, &c. Begin the addition at the column on the left of the first reiteration; in adding this column carry at 9, and you have the value complete.

EXAMPLE IV.

Required the value of .31. Sterling.

	s.	d.	f.
.33	= 6	7	0.8
.003	=		2.88
.0003	=		.288
.00003	=		.0288

Value 6 8 0.0 complete.

In the above example, I take successive values of the repeating figure 3, till I perceive that the column of primes is reiterated in the column of seconds; then I add the column of primes, as being the column on the left of the first reiteration; I carry at 9, and find the complete value of the given decimal to be 6s. 8d. The columns on the right of the primes are rejected, as being, when completed by assuming more successive values of 3, so many pure repetitions of the column of primes.

EXAMPLE V.

Required the value of .9161. Sterling.

	s.	d.	f.
.91	= 18	2	1.6
.006	=	1	1.76
.0006	=		.576
.00006	=		.0576

Value 18 4 0.0 complete.

EXAMPLE VI.

Required the value of .99895831. Sterling.

	s.	d.	f.
.99	= 19	9	2.4
.008	=	1	3.68
.0009	=		.864
.00005	=		.0480
.000008	=		.00768
.0000003	=		.000288
.00000003	=		.0000288
.000000003	=		.00000288

Value 19 11 3.00000 complete.

E X.

EXAMPLE VII.

Required the value of .867l. Sterling.

	s.	d.	f.
.86	= 17	2	1.6
.007	=		.96
.0001	=		.096

Value 17 2 2.6 complete.

EXAMPLE VIII.

Required the value of .847l. Sterling.

	s.	d.	f.
.84	= 16	9	2.4
.007	=	1	2.72
.0007	=		.672
.00007	=		.0672

Value 16 11 1.86 complete.

EXAMPLE IX.

Required the value of .458s.

	d.	f.
.4	= 4	3.2
.05	=	2.4
.008	=	.384
.0008	=	.0384
.00008	=	.00384

Value 5 2.026 complete.

2. AVOIRDUPOIS WEIGHT.

When the given decimal is a circulate, as often happens in Avoirdupois weight, wool-weight, wine and beer measure, long measure of one pole the integer, superficial and solid measure, a circle, and time; in this case, the exact value of the given decimal may be found by completing the circles of the separate values of the figures that constitute the circle of the given decimal, and adding as directed in addition of circulates. Here observe, that the said circles may always be completed from the decimals under the top-figure that denotes the number of places in the circle of the given decimal, or from the decimals under any top-figure that is higher than it, and frequently by a simple reiteration of the figures in the finite part.

EXAMPLE I.

Required the value of .07,142857, C.

	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
.07	= 7	13	7.04
.001	=	1	12.672
.0004	=		11.4688
.00002	=		.57344
.000008	=		.229376
.0000005	=		.143360
.00000007	=		.200704

Thus it stands as taken from the tables; but after separating the circles from the finite parts, and completing the circles from the decimals under the top-figure 6, and adding all into one sum, it will stand as follows,

<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
7	13	7.04
	1	12.67,202867,
		11.46,881146,
		.57,344057,
		.22,937622,
		.1,433601,
		. ,200704,
7	15	15.99,999999,

Value 8 0 0.00 complete.

The sum of the circles being a series of nines, their value is unity; which being accordingly added to the finite part, completes the value of the given decimal.

Though most of the other tables have only four top-figures, yet the circles of the several figures that constitute the circle of the given decimal may be found, by moving the decimal points under the top-figure 4 toward the left, till you have the value of the several figures under the top-figure that denotes the number of places in the circle of the given decimal; and then the several circles may be completed the same way, as in the example above. For further illustration take the following example of wine-measure.

EXAMPLE II.

Required the value of .81,349206, tun.

Hbds.

	<i>Hbds.</i>	<i>g.</i>	<i>p.</i>
.8	= 3	12	4.80,
.01	=	2	4.16,
.003	=		6.04,800604,
.0004	=		.80,640080,
.00009	=		.18,144018,
.000002	=		. ,403200,
.00000006	=		. , 12096,

3 15 7.99,999999,
1,

Value 3 16 0 complete.

But few cases require such accuracy. The value of a decimal may generally be found to sufficient exactness from the tables, by taking the separate values of the first four, or at most of the first six figures, and using two decimal places only. Example I. done in this manner follows.

Required the value of .07,142857, C.

	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
.07	= 7	13	7.04
.001	=	1	12.67
.0004	=		11.46
.00002	=		.57
.000008	=		.22

Value 7 15 15.96 nearly.

EXAMPLE III.

Required the value of .86216517,857142, C.

	<i>Q.</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
.8	= 3	5	9	9.6
.06	=	6	11	8.32
.002	=		3	9.34
.0001	=			2.86
.00006	=			1.72
.000005	=			.14

Value 3 12 8 15.98 nearly.

Value 3 12 9 complete.

Q4

3. T R O Y

3. TROY WEIGHT.

EXAMPLE I.

Required the value of .778125 lb.

	oz. dw. gr.	
.7	= 8	8
.07	=	16 19.2
.008	=	1 22.08
.0001	=	.576
.00002	=	.1152
.000005	=	.0288

Value 9 6 18,0000 complete.

EXAMPLE II.

Required the value of .43 ounce.

	dw. gr.	
.43	= 8	14.4
.003	=	1.44
.0003	=	.144
.00003	=	.0144

Value 8 16.0 complete.

EXAMPLE III.

Required the value of .52690972 lb.

	oz. dw. gr.	
.52	= 6	4 19.2
.006	=	1 10.56
.0009	=	5.18

Value 6 6 10.94 nearly.

Value 6 6 11 complete.

There is no occasion to carry the exemplification further; all the varieties that occur in using the tables being already explained. The tables follow.

I. M O N E Y.

TABLE I. One pound the integer.

1.	2.	3.	4.
s.	s. d. f.	d. f.	f.
1 2	2 1.6	.96	.096
2 4	4 3.2	1.92	.192
3 6	7 0.8	2.88	.288
4 8	9 2.4	3.84	.384
5 10	1 0 0.0	0.80	.480
6 12	1 2 1.6	1.76	.576
7 14	1 4 3.2	2.72	.672
8 16	1 7 0.8	3.68	.768
9 18	1 9 2.4	0.64	.864

TABLE II. One shilling the integer.

1.	2.	3.	4.
d. f.	d. f.	f.	f.
1 1 0.8	.48	.048	.0048
2 2 1.6	.96	.096	.0096
3 3 2.4	1.44	.144	.0144
4 4 3.2	1.92	.192	.0192
5 6 0.0	2.40	.240	.0240
6 7 0.8	2.88	.288	.0288
7 8 1.6	3.36	.336	.0336
8 9 2.4	3.84	.384	.0384
9 10 3.2	0.32	.432	.0432

2. AVOIRDUPOIS WEIGHT.

TABLE I. One tun the integer.

1.	2.	3.	4.	5.	6.
C.	C. 2. lb. oz.	lb. oz.	lb. oz.	oz.	oz.
1 2	22 6.4	2 3.84	3.584	.3584	.03584
2 4	1 16 12.8	4 7.68	7.168	.7168	.07168
3 6	2 11 3.2	6 11.52	10.752	1.0752	.10752
4 8	3 5 9.6	8 15.36	14.336	1.4336	.14336
5 10	1 0 0 0.0	11 3.2	1 1.920	1.7920	.1792
6 12	1 0 22 6.4	13 7.04	1 5.504	2.1504	.21504
7 14	1 1 16 12.8	15 10.88	1 9.088	2.5088	.25088
8 16	1 2 11 3.2	17 14.72	1 12.672	2.8672	.28672
9 18	1 3 5 9.6	20 2.56	2 0.256	3.2256	.32256

TABLE

TABLE II. One C. or 112 lb. the integer.

	1.				2.			3.			4.		5.		6.	
	lb. oz. dr.				lb. oz. dr.			lb. oz. dr.			oz. dr.		dr.		dr.	
1	11	3	3.2		1	1	14.72	1	12.672		2.8672		.28672		.028672	
2	22	6	6.4		2	3	13.44	3	9.344		5.7344		.57344		.057344	
3	1	5	9 9.6		3	5	12.16	5	6.016		8.6016		.86016		.086016	
4	1	16	12 12.8		4	7	10.88	7	2.688		11.4688		1.14688		.114688	
5	2	0	0 0.0		5	9	9.6	8	15.360		14.3360		1.43360		.143360	
6	2	11	3 3.2		6	11	8.32	10	12.032	1	1.2032		1.72032		.172032	
7	2	22	6 6.4		7	13	7.04	12	8.704	1	4.0704		2.00704		.200704	
8	3	5	9 9.6		8	15	5.76	14	5.376	1	6.9376		2.29376		.229376	
9	3	16	12 12.8		10	1	4.48	1	0 2.048	1	9.8048		2.58048		.258048	

TABLE III. One lb. the integer.

	1.			2.		3.		4.	
	oz. dr.			oz. dr.		dr.		dr.	
1	1	9	.6	2	.56		.256		.0256
2	3	3	.2	5	.12		.512		.0512
3	4	12	.8	7	.68		.768		.0768
4	6	6	.4	10	.24	1	.024		.1024
5	8	0	.0	12	.8	1	.28		.128
6	9	9	.6	15	.36	1	.536		.1536
7	11	3	.2	1	1 .92	1	.792		.1792
8	12	12	.8	1	4 .48	2	.048		.2048
9	14	6	.4	1	7 .04	2	.304		.2304

3. TROY-WEIGHT.

TAB. I. One lb. the integer.

	1.			2.		3.		4.	
	oz. dw. gr.			oz. dw. gr.		dw. gr.		gr.	
1	1	4		2	9.6		5.76		.576
2	2	8		4	19.2		11.52		1.152
3	3	12		7	4.8		17.28		1.728
4	4	16		9	14.4		23.04		2.304
5	6	0		12		1	4.8		2.88
6	7	4		14	9.6	1	10.56		3.456
7	8	8		16	19.2	1	16.32		4.032
8	9	12		19	4.8	1	22.08		4.608
9	10	16		1	14.4	2	3.84		5.184

TAB. II. One ounce the integer.

	1.			2.		3.		4.	
	dw. dw. gr. gr.			dw. dw. gr. gr.		gr. gr.		gr. gr.	
1	2		4.8		.48		.048		
2	4		9.6		.96		.096		
3	6		14.4		1.44		.144		
4	8		19.0		1.92		.192		
5	10	1	00		2.40		.240		
6	12	1	4.8		2.88		.288		
7	14	1	9.6		3.36		.336		
8	16	1	14.4		3.84		.384		
9	18	1	19.2		4.32		.432		

4. APOTHECARIES WEIGHT.

TABLE. One lb. the integer.

1.					2.			3.	4.
3 3 9 gr.					3 3 9 gr.			9 gr.	gr.
1	1	1	1	16		2	17.6	5.76	.576
2	2	3	0	12	1	2	15.2	11.52	1.152
3	3	4	2	8	2	2	12.8	17.28	1.728
4	4	6	1	4	3	2	10.4	3.04	2.304
5	6	0	0	0	4	2	8.0	8.8	2.880
6	7	1	1	16	5	2	5.6	14.56	3.456
7	8	3	0	12	6	2	3.2	0.32	4.032
8	9	4	2	8	7	2	0.8	6.08	4.608
9	10	6	1	4	8	2	18.4	11.84	5.184

5. WOOL-WEIGHT.

TABLE I. One last the integer.

1.				2.		3.	4.
Sac. ft. lb.				S. ft. lb.		ft. lb.	lb.
1	1	5	2.8	3	1.68	4.368	.4368
2	2	10	5.6	6	3.36	8.736	.8736
3	3	15	8.4	9	5.04	13.104	1.3104
4	4	20	11.2	12	6.72	1 3.472	1.7472
5	6	0	0.0	15	8.40	1 7.840	2.1840
6	7	5	2.8	18	10.08	1 12.208	2.6208
7	8	10	5.6	21	11.76	2 2.576	3.0576
8	9	15	8.4	24	13.44	2 6.944	3.4944
9	10	20	11.2	27	15.12	2 11.312	3.9312

TABLE

TABLE II. One sack the integer.

	1.	2.	3.	4.
	<i>St. lb.</i>	<i>St. lb.</i>	<i>lb.</i>	<i>lb.</i>
1	2 8.4	3.64	.364	.0364
2	5 2.8	7.28	.728	.0728
3	7 11.2	10.92	1.092	.1092
4	10 5.6	1 0.56	1.456	.1456
5	13 0.0	1 4.20	1.820	.1820
6	15 8.4	1 7.84	2.184	.2184
7	18 2.8	1 11.48	2.548	.2548
8	20 11.2	2 1.12	2.912	.2912
9	23 5.6	2 4.76	3.276	.3276

6. DRY MEASURE.

TAB. I. One load the integer.

	1.	2.	3.	4.
	<i>℔. b. B. g.</i>	<i>Gall.</i>	<i>Gall.</i>	
1	4 3.2	.32	.032	
2	1 0 6.4	.64	.064	
3	1 4 1 1.6	.96	.096	
4	2 0 1 4.8	1.28	.128	
5	2 4 2 0.0	1.60	.160	
6	3 0 2 3.2	1.92	.192	
7	3 4 2 6.4	2.24	.224	
8	4 0 3 1.6	2.56	.256	
9	4 4 3 4.8	2.88	.288	

TAB. II. One quarter the integer.

	1.	2.	3.	4.
	<i>B. g. pts.</i>	<i>G. pts.</i>	<i>Pts.</i>	<i>Pts.</i>
1	6 3.2	5.12	.512	.0512
2	1 4 6.4	1 2.24	1.024	.1024
3	2 3 1.6	1 7.36	1.536	.1536
4	3 1 4.8	2 4.48	2.048	.2048
5	4 0 0.0	3 1.60	2.560	.2560
6	4 6 3.2	3 6.72	3.072	.3072
7	5 4 6.4	4 3.84	3.584	.3584
8	6 3 1.6	5 0.96	4.096	.4096
9	7 1 4.8	5 6.08	4.608	.4608

TABLE III. One bushel the integer.

	1.	2.	3.	4.
	<i>Gall. pts.</i>	<i>Pts.</i>	<i>Pts.</i>	<i>Pts.</i>
1	6.4	.64	.064	.0064
2	1 4.8	1.28	.128	.0128
3	2 3.2	1.92	.192	.0192
4	3 1.6	2.56	.256	.0256
5	4 0.0	3.20	.320	.0320
6	4 6.4	3.84	.384	.0384
7	5 4.8	4.48	.448	.0448
8	6 3.2	5.12	.512	.0512
9	7 1.6	5.76	.576	.0576

7. WINE MEASURE.

TABLE I. One tun the integer.

	1.			2.		3.		4.
	Hbds.	gall.	pts.	Gall.	pts.	Gall.	pts.	Pts.
1		25	1.6	2	4.16		2.016	.2016
2		50	3.2	5	0.32		4.032	.4032
3	1	12	4.8	7	4.48		6.048	.6048
4	1	37	6.4	10	0.64	1	0.064	.8064
5	2	0	0.0	12	4.80	1	2.080	1.0080
6	2	25	1.6	15	0.96	1	4.096	1.2096
7	2	50	3.2	17	5.12	1	6.112	1.4112
8	3	12	4.8	20	1.28	2	0.128	1.6128
9	3	37	6.4	22	5.44	2	2.144	1.8144

TABLE II. One hoghead the integer.

	1.			2.		3.		4.
	Gall.	pts.	Gall.	pts.	Pts.	Gall.	pts.	Pts.
1	6	2.4		5.04	.504		.0504	
2	12	4.8	1	2.08	1.008		.1008	
3	18	7.2	1	7.12	1.512		.1512	
4	25	1.6	2	4.16	2.016		.2016	
5	31	4.0	3	1.20	2.520		.2520	
6	37	6.4	3	6.24	3.024		.3024	
7	43	8.8	4	3.28	3.528		.3528	
8	50	3.2	5	0.32	4.032		.4032	
9	56	5.6	5	5.36	4.536		.4536	

8. BEER.

8. BEER-MEASURE.

TABLE I. One tun the integer.

	1.			2.		3.		4.
	<i>Hbds. gall. pts.</i>			<i>Gall. pts.</i>		<i>Gall. pts.</i>		<i>Pts.</i>
1		21	4.8	2	1.28		1.728	.1728
2		43	1.6	4	2.56		3.456	.3456
3	1	10	6.4	6	3.84		5.184	.5184
4	1	32	3.2	8	5.12		6.912	.6912
5	2	0	0.0	10	6.40	1	0.640	.8640
6	2	21	4.8	12	7.68	1	2.368	1.0368
7	2	43	1.6	15	0.96	1	4.096	1.2096
8	3	10	6.4	17	2.24	1	5.824	1.3824
9	3	32	3.2	19	3.52	1	7.552	1.5552

TABLE II. One hoghead the integer.

	1.			2.		3.		4.
	<i>Gall. pts.</i>			<i>Gall. pts.</i>		<i>Pts.</i>		<i>Pts.</i>
1	5	3.2		4.32		.432		.0432
2	10	6.4	1	0.64		.864		.0864
3	16	1.6	1	4.96		1.296		.1296
4	21	4.8	2	1.28		1.728		.1728
5	27	0.0	2	5.60		2.160		.2160
6	32	3.2	3	1.92		2.592		.2592
7	37	6.4	3	6.24		3.024		.3024
8	43	1.6	4	2.56		3.456		.3456
9	48	4.8	4	6.88		3.888		.3888

TABLE

TABLE III. One hoghead of 51 gallons, 8 pints each, the integer.

	1.	2.	3.	4.
	Gal. pts.	Gal. pts.	Pints.	Pints.
1	5 0.8	4.08	.408	.0408
2	10 1.6	1 0.16	.816	.0816
3	15 2.4	1 4.24	1.224	.1224
4	20 3.2	2 0.32	1.632	.1632
5	25 4.0	2 4.40	2.040	.2040
6	30 4.8	3 0.48	2.448	.2448
7	35 5.6	3 4.56	2.856	.2856
8	40 6.4	4 0.64	3.264	.3264
9	45 7.2	4 4.72	3.672	.3672

TABLE IV. One Scots hoghead the integer.

	1.	2.	3.	4.
	Gal. pts.	Gal. pts.	Pints.	Pints.
1	1 4.8	1.28	.128	.0128
2	3 1.6	2.56	.256	.0256
3	4 6.4	3.84	.384	.0384
4	6 3.2	5.12	.512	.0512
5	8 0.0	6.40	.640	.0640
6	9 4.8	7.68	.768	.0768
7	11 1.6	1 0.96	.896	.0896
8	12 6.4	1 2.24	1.024	.1024
9	14 3.2	1 3.52	1.152	.1152

9. CLOTH-MEASURE.

TAB. I. One yard the integer.

	1.	2.	3.	4.
	Q. n. N.	N.	N.	N.
1	1.6	.16	.016	.0016
2	3.2	.32	.032	.0032
3	4.8	.48	.048	.0048
4	6.4	.64	.064	.0064
5	8.0	.80	.080	.0080
6	9.6	.96	.096	.0096
7	11.2	1.12	.112	.0112
8	12.8	1.28	.128	.0128
9	14.4	1.44	.144	.0144

TAB. II. One ell Flemish the integer.

	1.	2.	3.	4.
	Q. n. N.	N.	N.	N.
1	1.2	.12	.012	.0012
2	2.4	.24	.024	.0024
3	3.6	.36	.036	.0036
4	4.8	.48	.048	.0048
5	6.0	.60	.060	.0060
6	7.2	.72	.072	.0072
7	8.4	.84	.084	.0084
8	9.6	.96	.096	.0096
9	10.8	1.08	.108	.0108

TABLE III. One ell English the integer.

	1.		2.	3.	4.
	<i>Qrs.</i>	<i>n.</i>	<i>N.</i>	<i>N.</i>	<i>N.</i>
1		2	.2	.02	.002
2	1	6	.4	.04	.004
3	1	2	.6	.06	.006
4	2	0	.8	.08	.008
5	2	2	1.0	.10	.010
6	3	0	1.2	.12	.012
7	3	2	1.4	.14	.014
8	4	0	1.6	.16	.016
9	4	2	1.8	.18	.018

10. LONG MEASURE.

TABLE I. One mile the integer.

	1.		2.	3.	4.
	<i>Fur. ch.</i>	<i>Ch. p. l.</i>	<i>P. l.</i>	<i>L.</i>	
1	8	3	5	8	.8
2	1 6	1 2 10	16	1	1.6
3	2 4	2 1 15	24	2	2.4
4	3 2	3 0 20	31	7	3.2
5	4 0	4 0 0	41	15	4.0
6	4 8	4 3 5	51	23	4.8
7	5 6	5 2 10	62	31	5.6
8	6 4	6 1 15	72	39	6.4
9	7 2	7 0 20	82	47	7.2

TABLE II. One chain the integer.

	1.	2.	3.	4.
	<i>P. l. L.</i>	<i>L.</i>	<i>L.</i>	
1	10	1	.1	.01
2	20	2	.2	.02
3	30	3	.3	.03
4	40	4	.4	.04
5	50	5	.5	.05
6	60	6	.6	.06
7	70	7	.7	.07
8	80	8	.8	.08
9	90	9	.9	.09

TABLE

TABLE III. One pole the integer.

	1.			2.	3.	4.
	<i>Yds. ft. inch.</i>			<i>Ft. inch.</i>	<i>Inch.</i>	<i>Inch.</i>
1	1		7.8	1.98	.198	.0198
2	1	0	3.6	3.96	.396	.0396
3	1	1	11.4	5.94	.594	.0594
4	2	0	7.2	7.92	.792	.0792
5	2	2	3.0	9.90	.990	.0990
6	3	0	10.8	11.88	1.188	.1188
7	3	2	6.6	1.86	1.386	.1386
8	4	1	2.4	3.84	1.584	.1584
9	4	2	10.2	5.82	1.782	.1782

TABLE IV. One yard the integer.

	1.	2.	3.	4.
	<i>Ft. in.</i>	<i>Inch.</i>	<i>Inch.</i>	<i>Inch.</i>
1	3.6	.36	.036	.0036
2	7.2	.72	.072	.0072
3	10.8	1.08	.108	.0108
4	1 2.4	1.44	.144	.0144
5	1 6.0	1.80	.180	.0180
6	1 9.6	2.16	.216	.0216
7	2 1.2	2.52	.252	.0252
8	2 4.8	2.88	.288	.0288
9	2 8.4	3.24	.324	.0324

TABLE V. One foot the integer.

	1.	2.	3.	4.
	<i>In. lin.</i>	<i>In. lin.</i>	<i>Lin.</i>	<i>Lin.</i>
1	1 2.4	1.44	.144	.0144
2	2 4.8	2.88	.288	.0288
3	3 7.2	4.32	.432	.0432
4	4 9.6	5.76	.576	.0576
5	5 0.0	7.20	.720	.0720
6	6 2.4	8.64	.864	.0864
7	7 4.8	10.08	1.008	.1008
8	8 7.2	11.52	1.152	.1152
9	9 9.6	12.96	1.296	.1296

TABLE VI. One Scots mile the integer.

	1.			2.			3.			4.		
	<i>Fur. cb.</i>			<i>Ch. fa. ell.</i>			<i>Fa. ell.</i>			<i>Ells.</i>		
1		8		3	1.2		1.92			.192		
2	1	6		1	2 2.4		3.84			.384		
3	2	4		2	1 3.6		5.76			.576		
4	3	2		3	0 4.8	1	1.68			.768		
5	4	0		4	0 0.0	1	3.60			.960		
6	4	8		4	3 1.2	1	5.52			1.152		
7	5	6		5	2 2.4	2	1.44			1.344		
8	6	4		6	1 3.6	2	3.36			1.536		
9	7	2		7	0 4.8	2	5.28			1.728		

II. LAND-MEASURE.

TABLE I. One acre the integer.

	1.			2.			3.			4.		
	<i>R. sq. p.</i>			<i>Sq. p. sq. yd.</i>			<i>Sq. p. sq. yd.</i>			<i>Sq. yd.</i>		
1		16		1	18.15		4.84			.484		
2		32		3	6.05		9.68			.968		
3	1	8		4	24.20		14.52			1.452		
4	1	24		6	12.10		19.36			1.936		
5	2	0		7	30.25		24.20			2.420		
6	2	16		9	18.15		29.04			2.904		
7	2	32		11	6.05	1	3.63			3.388		
8	3	8		12	24.20	1	8.47			3.872		
9	3	24		14	12.10	1	13.31			4.356		

TABLE II. One Scots acre the integer.

	1.			2.			3.			4.		
	<i>R. sq. f.</i>			<i>Sq. f. sq. el.</i>			<i>Sq. f. sq. el.</i>			<i>Sq. el.</i>		
1		16		1	21.6		5.76			.576		
2		32		3	6.2		11.52			1.152		
3	1	8		4	27.8		17.28			1.728		
4	1	24		6	13.4		23.04			2.304		
5	2	0		7	35.0		28.80			2.880		
6	2	16		9	20.6		34.56			3.456		
7	2	32		11	42.2	1	4.32			4.032		
8	3	8		12	28.8	1	10.08			4.608		
9	3	24		14	13.4	1	15.84			5.184		

12. SUPERFICIAL MEASURE.

TABLE I. One square yard the integer.

		I.	2.	3.	4.
		<i>Sq. ft. sq. in.</i>	<i>Sq. in.</i>	<i>Sq. in.</i>	<i>Sq. in.</i>
1		129.6	12.96	1.296	.1296
2	1	115.2	25.92	2.592	.2592
3	2	100.8	38.88	3.888	.3888
4	3	86.4	51.84	5.184	.5184
5	4	72.0	64.80	6.480	.6480
6	5	57.6	77.76	7.776	.7776
7	6	43.2	90.72	9.072	.9072
8	7	28.8	103.68	10.368	1.0368
9	8	14.4	116.64	11.664	1.1664

TABLE II. One square foot the integer.

		I.	2.	3.	4.
		<i>Sq. in. sq. li.</i>	<i>S. in. sq. lin.</i>	<i>S. in. sq. lin.</i>	<i>Sq. lin.</i>
1	14	57.6	1 63.36	20.736	2.0736
2	28	115.2	2 126.72	41.472	4.1472
3	43	28.8	4 46.08	62.208	6.2208
4	57	86.4	5 109.44	82.944	8.2944
5	72	00.0	7 28.80	103.680	10.3680
6	86	57.6	8 92.16	124.416	12.4416
7	100	115.2	10 11.52	1 1.152	14.5152
8	115	28.8	11 74.88	1 21.888	16.5888
9	129	86.4	12 138.24	1 42.624	18.6624

13. SOLID MEASURE.

TABLE I. One cubic yard the integer.

	1.		2.		3.		4.	
	<i>C. ft. c. in.</i>		<i>C. f. c. in.</i>		<i>C. in.</i>		<i>C. in.</i>	
1	2	1209.6		466.56		46.656		4.6656
2	5	691.2		933.12		93.312		9.3312
3	8	172.8		1399.68		139.968		13.9968
4	10	1382.4	1	138.24		186.624		18.6624
5	13	864.0	1	604.80		233.280		23.3280
6	16	345.6	1	1071.36		279.936		27.9936
7	18	1555.2	1	1537.92		326.592		32.6592
8	21	1036.8	2	276.48		373.248		37.3248
9	24	518.4	2	743.04		419.904		41.9904

TABLE II. One cubic foot the integer.

	1.		2.		3.		4.	
	<i>C. in. c. lin.</i>		<i>C. in. c. lin.</i>		<i>C. in. c. lin.</i>		<i>C. in. c. lin.</i>	
1	172	1382.4	17	483.84	1	1257.984		298.5984
2	345	1036.8	34	967.68	3	787.968		597.1968
3	518	691.2	51	1451.52	5	317.952		895.7952
4	691	345.6	69	207.36	6	1575.936		1194.3936
5	864	000.0	86	691.20	8	1105.920		1492.9920
6	1036	1382.4	103	1175.04	10	635.904	1	63.5904
7	1209	1036.8	120	1628.88	12	163.888	1	362.1888
8	1382	691.2	138	414.72	13	1423.872	1	660.7872
9	1555	345.6	155	898.56	15	953.856	1	959.3856

14. A C I R C L E.

TABLE I. One circle the integer.

	1.	2.	3.	4.	5.	6.
	<i>Si. de.</i>	<i>Si. de. m.</i>	<i>Deg. mi.</i>	<i>Min.</i>	<i>Min.</i>	<i>Min.</i>
1	1 6	3 36	21.6	2.16	.216	.0216
2	2 12	7 12	43.2	4.32	.432	.0432
3	3 18	10 48	1 4.8	6.48	.648	.0648
4	4 24	14 24	1 26.4	8.64	.864	.0864
5	6 0	18 00	1 48.0	10.80	1.080	.1080
6	7 6	21 36	2 9.6	12.96	1.296	.1296
7	8 12	25 12	2 31.2	15.12	1.512	.1512
8	9 18	28 48	2 52.8	17.28	1.728	.1728
9	10 24	1 2 24	3 14.4	19.44	1.944	.1944

TABLE II. One sign the integer.

	1.	2.	3.	4.	5.	6.
	<i>Deg.</i>	<i>Deg. min.</i>	<i>Min. sec.</i>	<i>Min. sec.</i>	<i>Sec.</i>	<i>Sec.</i>
1	3	18	1 48	10.8	1.08	.108
2	6	36	3 36	21.6	2.16	.216
3	9	54	5 24	32.4	3.24	.324
4	12	1 12	7 12	43.2	4.32	.432
5	15	1 30	9 0	54.0	5.40	.540
6	18	1 48	10 48	1 4.8	6.48	.648
7	21	2 6	12 36	1 15.6	7.56	.756
8	24	2 24	14 24	1 26.4	8.64	.864
9	27	2 42	16 12	1 37.2	9.72	.972

15. T I M E.

TABLE I. One year of 365 days the integer.

	1.	2.	3.	4.
	<i>Days.</i>	<i>Days.</i>	<i>Days.</i>	<i>Days.</i>
1	36.5	3.65	.365	.0365
2	73.0	7.30	.730	.0730
3	109.5	10.95	1.095	.1095
4	146.0	14.60	1.460	.1460
5	182.5	18.25	1.825	.1825
6	219.0	21.90	2.190	.2190
7	255.5	25.55	2.555	.2555
8	292.0	29.20	2.920	.2920
9	328.5	32.85	3.285	.3285

TABLE II. One year of 13 months, 28 days each, the integer.

	1.					2.			3.	4.
	<i>M. w. d. b.</i>					<i>M. w. d. b.</i>			<i>D. b.</i>	<i>H.</i>
1	1	1	1	1	9.6		3	15.36	8.736	.8736
2	2	2	2	2	19.2	1	0	6.72	17.472	1.7472
3	3	3	3	4	4.8	1	3	22.08	1 2.208	2.6208
4	5	0	5		14.4	2	0	13.44	1 10.944	3.4944
5	6	2	0		0.0	2	4	4.80	1 19.680	4.3680
6	7	3	1		9.6	3	0	20.16	2 4.416	5.2416
7	9	0	2		19.2	3	4	11.52	2 13.152	6.1152
8	10	1	4		4.8	1	0	1 2.88	2 21.888	6.9888
9	11	2	5		14.4	1	0	4 18.24	3 6.624	7.8624

TABLE III. One month of 28 days the integer.

	1.			2.		3.	4.
	<i>W. d. b.</i>			<i>D. b.</i>		<i>H.</i>	<i>H.</i>
1	2	19.2		6.72		.672	.0672
2	5	14.4		13.44		1.344	.1344
3	1	9.6		20.16		2.016	.2016
4	1	4	4.8	1	2.88	2.688	.2688
5	2	0	0.0	1	9.60	3.360	.3360
6	2	2	19.2	1	16.32	4.032	.4032
7	2	5	14.4	1	23.04	4.704	.4704
8	3	1	9.6	2	5.76	5.376	.5376
9	3	4	4.8	2	12.48	6.048	.6048

TABLE IV. One day the integer.

	1.		2.		3.	4.
	<i>H. m.</i>		<i>H. m.</i>		<i>Min.</i>	<i>Min.</i>
1	2	24		14.4	1.44	.144
2	4	48		28.8	2.88	.288
3	7	12		43.2	4.32	.432
4	9	36		57.6	5.76	.576
5	12	00	1	12.0	7.20	.720
6	14	24	1	26.4	8.64	.864
7	16	48	1	40.8	10.08	1.008
8	19	12	1	55.2	11.52	1.152
9	21	36	2	9.6	12.96	1.296

TABLE V. One hour, or one degree, the integer.

	1.		2.		3.	4.
	<i>Min. M. sec.</i>		<i>Min. M. sec.</i>		<i>Sec.</i>	<i>Sec.</i>
1	6		36		3.6	.36
2	12	1	12		7.2	.72
3	18	1	48		10.8	1.08
4	24	2	24		14.4	1.44
5	30	3	00		18.0	1.80
6	36	3	36		21.6	2.16
7	42	4	12		25.2	2.52
8	48	4	48		28.8	2.88
9	54	5	24		32.4	3.24

P R O B. V.

To reduce a decimal to its primitive vulgar fraction.

C A S E I.

When the given decimal is finite.

R U L E.

Divide both numerator and denominator of the given decimal by their greatest common measure; the quot is the vulgar fraction required.

Thus, $.875 = \frac{875}{1000} = \frac{7}{8}$. For $875 \overline{)1000}(1$
 $\phantom{875 \overline{)1000}}875$

Greatest common measure $125 \overline{)875}(7$
 $\phantom{125 \overline{)875}}875$

And $125 \overline{)1000}(8$
 $\phantom{125 \overline{)1000}}1000$
 $\phantom{125 \overline{)1000}}(0)$

Or, instead of dividing by the greatest common measure, you may divide by 2, 5, 10, or by any number that will divide both numerator and denominator without a remainder, continuing the operation till the fraction be reduced to its lowest terms.

Thus, $.0625 = \frac{625}{10000} = \frac{1}{16}$. For $\begin{matrix} 5) & 5) & 5) & 5) \\ 625 & 125 & 25 & 5 \\ \hline 10000 & 2000 & 400 & 80 & 16 \end{matrix}$

C A S E II.

When the given decimal is a pure repeater, or a pure circulate.

R U L E.

Make the repeating figure, or the figures of the circle, the numerator of the vulgar fraction; the denominator is 9 for the repeating figure, or 9 for every figure of the circle; and then, if occasion require, reduce this fraction to its lowest terms.

Thus, $.3 = \frac{3}{9} = \frac{1}{3}$, and $.6 = \frac{6}{9} = \frac{2}{3}$, and $.9 = \frac{9}{9} = 1$.

Again, $.27 = \frac{27}{99} = \frac{1}{3}$, and $.714285 = \frac{714285}{999999} = \frac{1}{7}$.

C A S E III.

When the given decimal is a mixt repeater, or a mixt circulate.

R U L E.

From the mixt repeater, or mixt circulate, subtract the finite part, and the remainder is the numerator of the vulgar fraction; the denominator is 9 for the repeating figure, or 9 for every figure of

of the circle, with as many ciphers annexed as there are figures in the finite part.

Thus, $.03 = \frac{3}{100} = \frac{1}{33}$, and $.16 = \frac{16}{100} = \frac{4}{25}$, and $.083 = \frac{83}{1000} = \frac{1}{12}$.

Again, $.0,45, = \frac{45}{1000} = \frac{1}{22}$, and $.3,18, = \frac{318}{1000} = \frac{7}{11}$, and $.03,571428, = \frac{3571428}{10000000} = \frac{1}{28}$.

The reason of the rule may be shewn thus : Esteem the finite part of the last example an integer, and then the mixt number $\frac{3571428}{999999}$ will be equal to the given circulate. Again, reduce this mixt number to an improper fraction, *viz.* multiply the integer 3 by the denominator 999999, and to the product add the numerator, as directed in reduction of vulgar fractions.

Multiply the integer 3 into 999999 by the method of multiplying any number by 9, 99, 999, &c. taught in multiplication of integers, and to the product add the numerator, and the sum shall be the numerator of the improper fraction, as in the margin.

$$\begin{array}{r} 3000000 \\ 3 \\ \hline 2999997 \\ 571428 \\ \hline 3571425 \text{ num.} \end{array}$$

Now it is evident, that the same numerator will be found, if, in the upper line, instead of the six ciphers, I place the figures of the circle, and from them subtract 3, the finite part.

$$\begin{array}{r} 3571428 \\ 3 \\ \hline 3571425 \text{ num.} \end{array}$$

To the numerator thus found, the denominator is 999999 ; and so the vulgar fraction is $\frac{3571428}{999999}$. But we esteemed 3 an integer ; whereas, in fact, it is $\frac{3}{1000}$; and so our vulgar fraction will be a hundred times greater than it ought to be. To correct this error, we must multiply the denominator by 100, which is done by annexing two ciphers to it ; and the true fraction comes out to be $\frac{3571428}{99999900}$, as by the rule.

Because this rule is of great importance, and will often occur in practice, I shall here subjoin some more examples.

Ex. 1. Reduce $.041\bar{6}$ to a vulgar fraction.

$$\begin{array}{r} .041\bar{6} \\ 41 \\ \hline \text{Num. } 375 \\ \text{Den. } 9000 \end{array} = \frac{1}{24}$$

Ex. 2. Reduce $.58\bar{3}$ to a vulgar fraction.

$$\begin{array}{r} .58\bar{3} \\ 58 \\ \hline \text{Num. } 525 \\ \text{Den. } 900 \end{array} = \frac{7}{12}$$

Ex. 3.

Ex. 3. Reduce .1,153846, to a vulgar fraction.

$$\begin{array}{r} .1,153846, \\ \text{I} \\ \hline \text{Num. } 153845 \\ \hline \text{Den. } 9999990 \end{array} = \frac{1}{28}$$

Ex. 4. Reduce .53,571428, to a vulgar fraction.

$$\begin{array}{r} .53,571428, \\ 53 \\ \hline \text{Num. } 53571375 \\ \hline \text{Den. } 99999900 \end{array} = \frac{1}{28}$$

In this manner too, may any mixt number, consisting of an integer with a repeater or circulate, be reduced to an improper vulgar fraction; but no ciphers are to be annexed to the denominator for the figures of the integer.

Ex. 1. Reduce 8.3 to an improper vulgar fraction.

$$\begin{array}{r} 8.3 \\ 8 \\ \hline \text{Num. } 75 \\ \hline \text{Den. } 9 \end{array} = \frac{25}{3} = 8\frac{1}{3}$$

Ex. 3. Reduce 65.296, to an improper vulgar fraction.

$$\begin{array}{r} 65.296, \\ 65 \\ \hline \text{Num. } 65231 \\ \hline \text{Den. } 999 \end{array} = 65\frac{8}{17}$$

Ex. 2. Reduce 4.16 to an improper vulgar fraction.

$$\begin{array}{r} 4.16 \\ 41 \\ \hline \text{Num. } 375 \\ \hline \text{Den. } 90 \end{array} = \frac{75}{18} = \frac{25}{6} = 4\frac{1}{6}$$

Ex. 4. Reduce 8.32,142857, to an improper vulgar fraction.

$$\begin{array}{r} 8.32,142857, \\ 832 \\ \hline \text{Num. } 832142025 \\ \hline \text{Den. } 99999900 \end{array} = 8\frac{9}{14}$$

Approximate decimals, being imperfect, cannot be exactly reduced back to the vulgar fractions from which they resulted. But if the approximate be completed by annexing to it a vulgar fraction, whereof the remainder of the division is the numerator, and the divisor the denominator, you shall have a mixt number, which you may reduce to an improper vulgar fraction; then to the denominator annex as many ciphers as there are figures in the approximate; and this fraction reduced to its lowest terms, will be the primitive vulgar fraction required.

P R O B. VI.

To reduce unlike circles to others that are similar and conterminous.

Similar or like circles are such as consist of an equal number of places.

Thus,

Thus, .27, and .09, are similar circles, as consisting of two places each. But .63, and .148, are unlike; the former consisting of two, and the latter of three places.

Conterminous circles are such as begin and end at the same distance from the decimal point.

Thus, .153846, and .384615, are conterminous; because they both begin at the place of primes, and have an equal number of places. And .0,714285, and .7,857142, are conterminous; because they both begin at the place of seconds, and have the same number of places. But .81, and .1,36, are not conterminous; the former beginning at the place of primes, and the latter at the place of seconds. Again, .63, and .481, are not conterminous, because they have not the same number of places; for circles cannot be conterminous, unless they be at the same time similar.

Unlike circles are reduced to similar ones by the following

R U L E.

Find the least multiple of the numbers denoting the number of places in the several given circles, and extend each of the given circles to as many places as there are units in the least multiple.

Thus, to reduce the unlike circles .63, and .148, to similar ones, I extend both circles to six places, because 6 is the least multiple of 2 and 3, the number of places in the given circles.

$$\begin{array}{l} .63, = .636363, \\ .148, = .148148, \end{array}$$

Again, to reduce .72, and .02439, to similar circles, I extend each of them to ten places, because 10 is the least multiple of 2 and 5, the number of places in the given circle.

$$\begin{array}{l} .72, = .7272727272, \\ .02439, = .0243902439, \end{array}$$

In a circle, any one of the circulating figures may be made the first of the circle. Thus 7.592 may be expressed thus, 7.5,925; or thus, 7.59,259; and that without changing its value: consequently a pure circulate may put on the form of a mixt circulate, if one or more figures on the left be set aside for the finite part; thus, .72, = .7,27, where .7 is the finite part; and thus, .57,1428, = .57,142857; and thus, .36, = .363,63.

That the value is not changed, may be thus demonstrated.

$$.7,27, = \frac{727}{999} = \frac{72}{99} = .72,$$

Hence two or more given circles may be made conterminous, by the following

R U L E.

Set aside, by a comma on the left, as many figures as there are places in the longest finite part, and then prolong the several circles to as many places as will make them similar.

Ex. I.

Ex. 1. To make .54,63, and .9,148, conterminous. $.54,63, = .54,6363,$
 $.9,148, = .91,481481,$

Here, because 54, the longest finite part, consists of two places, I set aside .91, in the other circulate, for a finite part, and then I prolong both circles to six places, which renders them similar.

Ex. 2. To make the three circles in the margin conterminous. $.17,857142, = .17,857142,$
 $.592, = .59,259259,$

Here, because the first circulate has a finite part of two places, I set off two figures on the left from each of the other two circles, and then I prolong each circle to six places, which renders them similar. $.54, = .54,545454,$

Ex. 3. To make the three following circles conterminous.

Here, because the first circulate has a finite part of three places, I set off three figures in each of the other two for finite parts, and then, to render them similar, I extend the figures of the circles to six places. $.714,285714, = .714,285714,$
 $.3,571428, = .357,142857,$
 $.36, = .363,636363,$

C H A P. III.

ADDITION OF DECIMALS.

RULE I. Place the given decimals so that the points may stand directly under each other, and consequently tenths under tenths, hundredths under hundredths, &c. ; then, if the given decimals be all finite or approximate, add them as integers, inserting the decimal point directly under the column of points. The figures on the left of the point are integers, and those on the right are a decimal of the integer, consisting of as many places as there are figures in the longest of the given decimals.

The operation is the same here as in addition of vulgar fractions ; for a cipher on the right of a decimal does not change its value : if, therefore, ciphers be annexed, so as to give every decimal the same number of places, as is done in the margin, they will by this means be reduced to a common denominator, viz. 1000.

Ex. 1.

$$\begin{array}{r} .75 = .750 \\ .895 = .895 \\ .5 = .500 \\ .625 = .625 \\ .725 = .725 \\ \hline 3.495 = 3\overline{495} \end{array}$$

Ex. 2.

Ex. 2.

$$\begin{array}{r} 45\frac{1}{4} = 45.25 \\ 68\frac{1}{8} = 68.125 \\ 72\frac{1}{2} = 72.5 \\ 24\frac{3}{4} = 24.75 \\ 36\frac{5}{8} = 36.625 \\ 28\frac{1}{8} = 28.0625 \\ \hline 274\frac{1}{8} = 274.7375 \end{array}$$

Ex. 3.

L.	s.	d.	L.
42	17	6	= 42.875
8	8	7 $\frac{1}{2}$	= 8.43125
12	5	6	= 12.275
28	14	6	= 28.725
7	10	6	= 7.525
99	16	7 $\frac{1}{2}$	= 99.83125

Note, If the decimals to be added are of different denominations, first reduce them to one denomination, and then add. The reason is, because like things only can be added or subtracted.

Ex. What is the sum of .725 l. and .625 s.

Here I may either reduce the decimal of a shilling to that of a pound, or I may reduce the decimal of a pound to that of a shilling.

First, reduce the decimal of a shilling to that of a pound, by reduction-ascending, viz. divide by 20, as follows :

L.	
20)	.62500(.03125
	.725
Sum,	.75625 = 15 s. 1 $\frac{1}{2}$ d.

Secondly, reduce the decimal of a pound to that of a shilling, by reduction-descending ; that is, multiply by 20, as follows :

	.725 l.
The answer here is	20
the same as before.	
	s. 14.500
	.625
	s. 15.125 sum.
	12
	d. 1.500
	4
	f. 2.0

A P P R O X I M A T E S.

If the decimals to be added run on to a great many places, it will be sufficient in most cases to use only four or five places, and observe

observe to increase the figure at which you break off by an unit, if the rejected figure on the right exceed 5. And in adding such approximates, omit the right-hand figure of the sum, as uncertain, but take in the carriage. Follows an example at large, and the same contracted.

Ex. 4. at large.

$$\begin{array}{r} 12.2352946 \\ 8.15789325 \\ 7.086968435 \\ 6.32143482 \\ 4.75 \\ \hline \end{array}$$

38.551591105

contracted.

$$\begin{array}{r} 12.23529+ \\ 8.15789+ \\ 7.08697- \\ 6.32143+ \\ 4.75 \\ \hline \end{array}$$

38.5515 certain.

RULE II. When all or any of the given decimals are repeaters, give every repeater the same number of places, and one place more than the longest finite; and for every nine in the right-hand column carry 1, or to its sum add 1 for every nine, and then carry at ten.

Ex. 1.

$$\begin{array}{r} 748\frac{1}{2} = 748.33 \\ 653\frac{2}{3} = 653.\overline{66} \\ 84\frac{5}{8} = 84.\overline{71} \\ 25\frac{5}{8} = 25.8\overline{3} \\ 37\frac{5}{8} = 37.8\overline{5} \\ 8\frac{1}{8} = 8.1\overline{6} \\ \hline \end{array}$$

1557 $\frac{2}{3}$ = 1557. $\overline{66}$

Ex. 2.

<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>L.</i>
26	14	1 $\frac{1}{4}$	= 26.705208 $\overline{3}$
45	10	7 $\frac{1}{4}$	= 45.532291 $\overline{6}$
24	10	6 $\frac{1}{4}$	= 24.528125
32	15	0	= 32.75
37	13	4	= 37. $\overline{666666}$
42	18	8	= 42.933333

210 2 3 $\frac{1}{4}$ = 210.1156250

Ex. 3.

<i>s.</i>	<i>d.</i>	<i>L.</i>
13	4	= .06
6	8	= .33
3	4	= .1 $\overline{6}$
10	0	= .5
4	0	= .2
6	0	= .3

2 3 4 = 2.1 $\overline{6}$

Ex. 4.

<i>oz. Troy.</i>
4.35208 $\overline{3}$
7.45625
8.908 $\overline{3}$ 33
9.503333
5.0375
9.04791 $\overline{6}$

44.30541 $\overline{6}$

Ex. 5.

<i>Feet.</i>
6.08 $\overline{3}$
5.333
8.41 $\overline{6}$
4.25
7.75
3.91 $\overline{6}$

35.750

Ex. 6.

<i>Days.</i>
342.2541 $\overline{6}$
238.33333
316.41 $\overline{66}$
45.20833
35.291 $\overline{66}$
40.125

1017.6291 $\overline{6}$

In Example 1. the sum of the right-hand column is 24, which contains 9 twice, and 6 over; so I set down 6 and carry 2: or to the sum 24 I add 2, for the two nines, which makes 26; so I set down 6 and carry 2. I proceed with the rest as in integers.

The

The sums, differences, and products of interminate decimals are always interminate, unless they end in a cipher, as in Ex. 2.

From Ex. 3. it appears, that addition may sometimes, by the means of repeating decimals, be performed with more ease and fewer figures, than by finite numbers.

A repeating digit is the numerator of a vulgar fraction, whose denominator is 9; and hence, in adding a column of repeating digits, every 9 of the sum is $\frac{9}{9}$, or an unit, to be carried; and what is over a just number of nines is so many ninth parts.

Or, if to the sum of a column of repeating digits, 1 for every 9 contained in it be added, we then carry 1 for every ten; but what is over a just number of tens will still continue to be ninth parts.

If in any example the repeating figures happen all to be reiterated, the carriage from the right-hand column adjusts the column on the left, or makes every ten of them equal to an unit of the next superior column, &c. Thus, if we imagine a column of the repeating figures reiterated on the right of any example, the carriage from it would adjust the right-hand column of the example.

RULE III. When all or any of the given decimals are circulates, make all the circles conterminous, find the number of tens to be carried from the left-hand column of the circles, add this carriage to the right-hand column, and proceed as in addition of integers.

If repeaters be mixed with the circulates, give the repeaters the form of circulates, by extending the repeating figures till they become conterminous with the other circles; as in Ex. 3. and 4.

If finite decimals are joined with the circulates, extend the finite parts of all the circulates to as many places as there are figures in the longest finite; as in Ex. 4.

E X A M P L E I.

$$\begin{aligned}\frac{3}{7} &= .428571, = .428571, \\ \frac{6}{7} &= .857142, = .857142, \\ \frac{3}{11} &= .45, = .454545, \\ \frac{10}{17} &= .370, = .370370, \\ &\hline &2.110630\end{aligned}$$

In order to find the carriage from the left-hand column of the circles, I add the column next to it on the right, saying, $7 + 5 + 5 + 2 = 19$; from which I carry 1, and say, $1 + 3 + 4 + 8 + 4 = 20$; from which I carry 2, and go on to add the right-hand column of the circle, saying, the carriage $2 + 5 + 2 + 1 = 10$; so I set down 0, and carry 1, and proceed with the rest as in integers.

The adding the carriage from the left-hand column of the circles to

to the column on the right hand, arises from the flux of numbers; for as the circles repeat infinitely, if we suppose a new set of the same circles to be repeated upon the right of our examples, it is plain, that in adding them the carriage from the left-hand column of the new set would naturally fall into the right-hand column of our example.

The operation here is the same as in addition of vulgar fractions; for every circle is the numerator of a vulgar fraction, whose common denominator is 999999; and if the circles or numerators be added, without minding any carriage from the left-hand column, the sum will be 2110628.

$$\begin{array}{r} \text{And } 999999)2110628(2\overset{110628}{999999} \\ \underline{1999998} \\ 110630 \end{array}$$

But, by pointing off from the sum of the circles six figures towards the right, we divide by 1000000, instead of dividing by 999999; which gives indeed the same quot, but makes the remainder too small.

Now, that the carriage-figure from the left-hand column of the circles, is the integral part of the quot, and at the same time the difference betwixt the true and false remainder, is evident; for the quotient figure 2, multiplied into the two divisors 1000000 and 999999, gives two products, whose difference is 2; and consequently, if the greatest product, viz. $2 \times 1000000 = 2000000$, be subtracted from the dividend, the result will want 2 of the true remainder. To prevent such errors, and to put the work on a sure footing, find the carriage from the left-hand column of the circles, add this carriage to the right-hand column, divide the sum by 1000000, and you will have a true quot, and a true remainder. The learner may look back to division of integers, where the method of dividing by 9,99,999, &c. is explained.

Hence it follows, that if we add the circles as they stand, without minding any carriage from the left, and to the sum add the excrescent figure on the left of the decimal point, we shall have the full sum of the circles, both as to the integral and fractional part, as in the margin.

$$\begin{array}{r} .428571, \\ .857142, \\ .454545, \\ .370370, \\ \hline 2.110628, \\ \quad 2 \\ \hline 2.110630, \end{array}$$

Pure repeaters, being the numerators of vulgar fractions, whose denominator is 9 as often taken as the digit is repeated, may be added in the same manner as circles. But in examples clear of circulates, the method prescribed in Rule II. is preferable.

In

In adding circles and pure repeaters by the method now explained, it will sometimes happen that the fractional part of the sum will be a series of nines, as in the margin: and in this case, the numerator of the fraction being the same with the denominator, its value will be unity; and accordingly 1 must be added to the integral part. But in adding pure repeaters by the method in Rule II. this cannot happen.

.857142,	.8566,
.571428,	.8566,
.857142,	.8566,
.714285,	
	1.9999,
2.999999,	1
1	
	2.0000
3.000000	

By way of proof, I shall here add all the vulgar fractions in Example I. and reduce their sum to a mixt number, continuing the division to a decimal.

$$\frac{1}{4} + \frac{6}{7} + \frac{1}{11} + \frac{12}{17} = \frac{6237}{14353} + \frac{12474}{14353} + \frac{6615}{14353} + \frac{1320}{14353} = \frac{10716}{14353}$$

$$14553 \overline{) 30716} (2.110630,$$

$$29106$$

$$*16100$$

$$14553$$

$$15470$$

$$14553$$

$$91700$$

$$87318$$

$$43820$$

$$43659$$

$$*16100$$

Here the dividial being the same with the second, a new circle begins.

EXAMPLE II.

$$\frac{3}{14} = .3,571428, = .35,714285,$$

$$\frac{1}{10} = .0,384615, = .03,846153,$$

$$\frac{12}{17} = .67,857142, = .67,857142,$$

$$\frac{1}{11} = .27, = .27,272727,$$

$$\frac{4}{17} = .148, = .14,814814,$$

$$1.49,505124,$$

Here I make all the circles conterminous; and having found the carriage from the left-hand column of the circles, I add it up with the right-hand column, and then proceed as in adding integers.

S

The

The reason of making the circles conterminous is, because, by this means, like things come to be added; for the finite parts are all tenth parts, but the circles are all ninth parts.

EXAMPLE III.

$$\begin{array}{rcl}
 \frac{49}{52} & = .94,230769, & = .94,230769, \\
 \frac{278}{390} & = .7,128205, & = .71,282051, \\
 \frac{1}{3} & = .3 & = .33,333333, \\
 \frac{2}{3} & = .6 & = .66,666666, \\
 \frac{1}{6} & = .16 & = .16,666666, \\
 \frac{1}{12} & = .08\bar{3} & = .08,333333, \\
 \hline
 & & 2.90,512820,
 \end{array}$$

Here I give the repeaters the form of circulates, and, by extending the repeating figure, I make their circles conterminous with the other circles.

That repeaters reduced to the form of circulates retain their former value, may be thus demonstrated.

$$\begin{array}{r}
 .33,333333, \\
 \quad \quad 33 \\
 \hline
 \text{Num. } 33333300 \\
 \hline
 \text{Den. } 99999900
 \end{array}
 = \frac{333333}{999999} = \frac{1}{3} = \frac{1}{3}$$

EXAMPLE IV.

$$\begin{array}{rcl}
 L. & & L. \\
 18\frac{11}{14} & = 18.7,857142, & = 18.785,714285, \\
 25\frac{2}{7} & = 25.81, & = 25.818,181818, \\
 14\frac{1}{2} & = 14.5 & = 14.5 \\
 29\frac{3}{4} & = 29.75 & = 29.75 \\
 35\frac{7}{8} & = 35.875 & = 35.875 \\
 24\frac{1}{5} & = 24.01\bar{3} & = 24.013,333333, \\
 32\frac{7}{9} & = 32.\bar{7} & = 32.777,777777, \\
 \hline
 & & 181.520,007215,
 \end{array}$$

Here I extend the finite parts of all the circulates and repeaters to three places, because 875, the longest finite, consists of three places; and by this means the circles, which are all ninth parts, come to be added by themselves.

To ascertain the carriage from the left-hand column of the circles, you must go back to the second column on the right of it, saying, $7 + 3 + 1 + 4 = 15$, one carried, &c.

CHAP. IV.

SUBTRACTION OF DECIMALS.

RULE I. Place the minor under the major, so that the points may be in one column; and then, if the given decimals be finite or approximate, work as in subtraction of integers.

If the major and minor have not the same number of places, imagine the void places to be filled up with ciphers.

Example 1.

$$\begin{array}{r} \text{L. s. d.} \quad \text{L.} \\ \text{From } 48 \ 10 \ 6 = 48.525 \\ \text{Sub. } 18 \ 12 \ 8\frac{1}{4} = 18.634375 \\ \hline \text{Rem. } 29 \ 17 \ 9\frac{3}{4} = 29.890625 \end{array}$$

Example 2.

$$\begin{array}{r} \text{C. } \text{Q. lb.} \quad \text{C.} \\ \text{From } 54 \ 2 \ 21 = 54.6875 \\ \text{Sub. } 36 \ 3 \ 14 = 36.875 \\ \hline \text{Rem. } 17 \ 3 \ 7 = 17.8125 \end{array}$$

A P P R O X I M A T E S.

In subtracting approximates, neglect the right-hand figure of the remainder, as uncertain; but an unit borrowed on the right must be repaid, as in Ex. 1. 2. and 3. following.

Ex. 1.

$$\begin{array}{r} \text{From } 783.0625 \\ \text{Sub. } 495.28571 + \\ \hline \text{Rem. } 287.7767 \text{ certain.} \end{array}$$

Ex. 3.

$$\begin{array}{r} \text{From } 12\frac{1}{9} = 12.05263 + \\ \text{Sub. } 4\frac{2}{9} = 4.10526 + \\ \hline \text{Rem. } 7\frac{8}{9} = 7.9473 \text{ certain.} \end{array}$$

Ex. 2.

$$\begin{array}{r} \text{From } 549.4643 - \\ \text{Sub. } 78.0875 \\ \hline \text{Rem. } 471.376 \text{ certain.} \end{array}$$

Ex. 4.

$$\begin{array}{r} \text{From } 17\frac{2}{7} = 17.11765 - \\ \text{Sub. } 8\frac{5}{7} = 8.29412 - \\ \hline \text{Rem. } 8\frac{4}{7} = 8.8235 \text{ certain.} \end{array}$$

RULE II. If one of the given decimals be a repeater, and the other a finite decimal, give the repeater one place more than the finite decimal, and in subtracting borrow 9 on the right hand.

But if both major and minor repeat, give them an equal number of places, and then subtract as above.

Ex. 1.

$$\begin{array}{r} \text{From } .7145833 \\ \text{Sub. } .634375 \\ \hline \text{Rem. } .0802083 \end{array}$$

Ex. 2.

$$\begin{array}{r} .525 \\ .3333 \\ \hline .1916 \\ \text{S } 2 \end{array}$$

Ex. 3.

$$\begin{array}{r} .9989583 \\ .0291666 \\ \hline .9697916 \end{array}$$

In

The reason of making the circles conterminous is, because, by this means, like things come to be added; for the finite parts are all tenth parts, but the circles are all ninth parts.

EXAMPLE III.

$$\begin{array}{rcl}
 \frac{49}{11} = .94,230769, & = & .94,230769, \\
 \frac{278}{390} = .7,128205, & = & .71,282051, \\
 \frac{1}{3} = .3 & = & .33,333333, \\
 \frac{2}{3} = .6 & = & .66,666666, \\
 \frac{1}{8} = .125 & = & .125,000000, \\
 \frac{1}{12} = .083 & = & .083,333333, \\
 \hline
 & & 2.90,512820,
 \end{array}$$

Here I give the repeaters the form of circulates, and, by extending the repeating figure, I make their circles conterminous with the other circles.

That repeaters reduced to the form of circulates retain their former value, may be thus demonstrated.

$$\begin{array}{r}
 .33,333333, \\
 \quad \quad 33 \\
 \hline
 \text{Num. } 33333300 \\
 \hline
 \text{Den. } 99999900
 \end{array}
 = \frac{333333}{999999} = \frac{1}{3} = \frac{1}{3}$$

EXAMPLE IV.

$$\begin{array}{rcl}
 L. & & L. \\
 18\frac{11}{4} = 18.7,857142, & = & 18.785,714285, \\
 25\frac{9}{11} = 25.81, & = & 25.818,181818, \\
 14\frac{1}{2} = 14.5 & = & 14.5 \\
 29\frac{3}{4} = 29.75 & = & 29.75 \\
 35\frac{7}{8} = 35.875 & = & 35.875 \\
 24\frac{1}{15} = 24.013 & = & 24.013,333333, \\
 32\frac{1}{9} = 32.7 & = & 32.777,777777, \\
 \hline
 & & 181.520,007215,
 \end{array}$$

Here I extend the finite parts of all the circulates and repeaters to three places, because 875, the longest finite, consists of three places; and by this means the circles, which are all ninth parts, come to be added by themselves.

To ascertain the carriage from the left-hand column of the circles, you must go back to the second column on the right of it, saying, $7 + 3 + 1 + 4 = 15$, one carried, &c.

CHAP. IV.

SUBTRACTION OF DECIMALS.

RULE I. Place the minor under the major, so that the points may be in one column; and then, if the given decimals be finite or approximate, work as in subtraction of integers.

If the major and minor have not the same number of places, imagine the void places to be filled up with ciphers.

Example 1.

$$\begin{array}{r} \text{L. s. d.} \quad \text{L.} \\ \text{From } 48 \text{ } 10 \text{ } 6 = 48.525 \\ \text{Sub. } 18 \text{ } 12 \text{ } 8\frac{1}{4} = 18.634375 \\ \hline \text{Rem. } 29 \text{ } 17 \text{ } 9\frac{3}{4} = 29.890625 \end{array}$$

Example 2.

$$\begin{array}{r} \text{C. } \text{Q. lb.} \quad \text{C.} \\ \text{From } 54 \text{ } 2 \text{ } 21 = 54.6875 \\ \text{Sub. } 36 \text{ } 3 \text{ } 14 = 36.875 \\ \hline \text{Rem. } 17 \text{ } 3 \text{ } 7 = 17.8125 \end{array}$$

A P P R O X I M A T E S.

In subtracting approximates, neglect the right-hand figure of the remainder, as uncertain; but an unit borrowed on the right must be repaid, as in Ex. 1. 2. and 3. following.

Ex. 1.

$$\begin{array}{r} \text{From } 783.0625 \\ \text{Sub. } 495.28571 + \\ \hline \text{Rem. } 287.7767 \text{ certain.} \end{array}$$

Ex. 3.

$$\begin{array}{r} \text{From } 12\frac{1}{9} = 12.05263 + \\ \text{Sub. } 4\frac{2}{9} = 4.10526 + \\ \hline \text{Rem. } 7\frac{1}{9} = 7.9473 \text{ certain.} \end{array}$$

Ex. 2.

$$\begin{array}{r} \text{From } 549.4643 - \\ \text{Sub. } 78.0875 \\ \hline \text{Rem. } 471.376 \text{ certain.} \end{array}$$

Ex. 4.

$$\begin{array}{r} \text{From } 17\frac{2}{7} = 17.11765 - \\ \text{Sub. } 8\frac{1}{7} = 8.29412 - \\ \hline \text{Rem. } 8\frac{1}{7} = 8.8235 \text{ certain.} \end{array}$$

RULE II. If one of the given decimals be a repeater, and the other a finite decimal, give the repeater one place more than the finite decimal, and in subtracting borrow 9 on the right hand.

But if both major and minor repeat, give them an equal number of places, and then subtract as above.

Ex. 1.

$$\begin{array}{r} \text{From } .7145833 \\ \text{Sub. } .634375 \\ \hline \text{Rem. } .0802083 \end{array}$$

Ex. 2.

$$\begin{array}{r} .525 \\ .3333 \\ \hline .1916 \\ \text{S } 2 \end{array}$$

Ex. 3.

$$\begin{array}{r} .9989583 \\ .0291866 \\ \hline .9697916 \end{array}$$

In

In Ex. 1. and 2. I give the repeater one place more than the finite decimal, and by this means I obtain the repeating figure of the remainder. But in Ex. 3. I give the two repeaters an equal number of places.

In Ex. 2. and 3. I borrow 9 on the right hand.

It may be a proper exercise for the learner to reduce back the decimals that occur in these three examples to the vulgar fractions from which they result; then subtract the vulgar fractions, and reduce the remainders to decimals.

RULE III. If both the given decimals be circulates, make the circles conterminous, and work as in integers; only, if, in the left-hand column of the circles, you foresee, that in subtracting the figure of the minor from that of the major, one must be borrowed, in this case add 1 to the right-hand figure of the minor, and then subtract.

If one of the given decimals be a circulate, and the other a repeater, give the repeater the form of a conterminous circulate, and then subtract as above.

If one of the given decimals be a circulate, and the other a finite decimal, extend the finite part of the circulate to as many places as there are figures in the finite decimal, and then subtract.

EXAMPLE I.

$$\begin{array}{r} \text{From } \frac{25}{7} = .925,925, = .925925, \\ \text{Sub. } \frac{4}{7} = .571,428, = .571428, \\ \hline \text{Rem. } .354497, \end{array}$$

EXAMPLE II.

$$\begin{array}{r} \text{From } \frac{9}{4} = .6,428571, = .64,285714, \\ \text{Sub. } \frac{5}{8} = .17,857142, = .17,857142, \\ \hline \text{Rem. } .46,428571, \end{array}$$

In this last example, because, in the left-hand column of the circles, 8 cannot be subtracted from 2 without borrowing; therefore I add 1 to the right-hand figure of the minor, and say, $1 + 2 = 3$, and 3 from 4, and 1 remains. The reason is obvious: for, supposing the circles reiterated on the right of the example, it would be 8 from 2 I cannot, but 8 from 12, and 4 remains; 1 borrowed, and 2, make 3, &c.

EXAMPLE III.

$$\begin{array}{r} \text{From } \frac{11}{4} = .9,285714, = .9,285714, \\ \text{Sub. } \frac{2}{3} = .6,666666, \\ \hline \text{Rem. } .2,619047, \end{array}$$

E X-

E X A M P L E IV.

$$\text{From } 18\frac{1}{3} = 18.08\bar{3} \quad = 18.08,33333,$$

$$\text{Sub. } 4\frac{3}{4} = 4.3,571428, \quad = 4.35,714285,$$

$$\text{Rem. } 13.72,619047,$$

In the above two examples I give the repeaters the form of continuous circulates.

E X A M P L E V.

$$\text{From } \frac{5}{13} = .384615, \quad = .384,615384,$$

$$\text{Sub. } \frac{1}{8} = .125 \quad = .125$$

$$\text{Rem. } .259,615384,$$

E X A M P L E VI.

$$\text{From } \frac{3}{4} = .75 \quad = .75$$

$$\text{Sub. } \frac{1}{14} = .0,714285, \quad = .07,142857,$$

$$\text{Rem. } .67,857142,$$

In the last two examples I extend the finite part of the circulate to as many places as there are figures in the finite decimal, by which means like things come to be subtracted, and I obtain the exact circle of the remainder.

C H A P. V.

M U L T I P L I C A T I O N O F D E C I M A L S.

In multiplication and division, there may happen nine varieties, arising from the different nature of the numbers that may occur in the operation; and these are of three sorts, viz. integers, mixt numbers, and pure decimals.

Now, since the multiplier or divisor may be of three kinds, and the multiplicand or dividend of as many, there must of consequence be nine varieties; which are these following:

An integer may multiply or divide

{ an integer,
a mixt number,
a pure decimal.

A mixt number may multiply or divide

{ an integer,
a mixt number,
a pure decimal.

A pure decimal may multiply or divide

{ an integer,
a mixt number,
a pure decimal.

Of these varieties the first belongs properly to vulgar arithmetic, the other eight occur in decimal operations.

But in multiplication and division of decimals, there will occur other nine varieties, arising likewise from the nature of the numbers; which may either be finite, repeating, or circulating.

And since the multiplier or divisor may be of three sorts, and the multiplicand or dividend of as many, there must of course be nine varieties; and these are so obvious, that it would be losing time here to enumerate them.

Before I enter on multiplication, I shall lay down a rule for pointing the product, which is of a general nature, and extends to decimals of every sort, whether finite, repeating, or circulating; and is as follows.

GENERAL RULE.

Give so many decimal places to the product, on the right, as are in both factors; and if the product has not so many figures, supply that defect by prefixing ciphers.

We now proceed to multiplication,

RULE I. If both factors be finite or approximate, work exactly as in multiplication of integers.

Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.	Ex. 5.
.585	.125	.0375	6.875	.8765
.75	.25	.05	.25	12.5
3925	625	.001875	34375	43825
5495	250		13750	105180
.58875	.03125		1.71875	10.95625
Ex. 6.	Ex. 7.	Ex. 8.	Ex. 9.	Ex. 10.
5.25	85	.95	4.75	75
3.5	.5	7	6	2.5
2625	42.5	6.65	28.50	365
1575				146
18.375				182.5

In Ex. 2. and 3. the product not affording so many decimal places as are in the multiplicand and multiplier, I supply the defect by prefixing ciphers.

The reason of giving as many decimal places to the product as are in both factors, appears by considering that the operation is the

the same here as in multiplication of vulgar fractions. Thus,
 $.785 \times .75 = \frac{785}{1000} \times \frac{75}{100} = \frac{58875}{100000} = .58875$. And thus, $6.875 \times$
 $.25 = \frac{6875}{1000} \times \frac{25}{100} = \frac{171875}{40000} = \frac{171875}{100000} = 1.71875$.

To multiply by 10, 100, 1000, &c. move the decimal point so many places toward the right hand as there are ciphers in the multiplier.

Thus :

$$\begin{aligned} .4375 \times 10 &= 4.375 \\ .4375 \times 100 &= 43.75 \\ .4375 \times 1000 &= 437.5 \\ .4375 \times 10000 &= 4375. \end{aligned}$$

And thus :

$$\begin{aligned} 6.875 \times 10 &= 68.75 \\ 6.875 \times 100 &= 687.5 \\ 6.875 \times 1000 &= 6875. \\ 6.875 \times 10000 &= 68750. \end{aligned}$$

A P P R O X I M A T E S.

In multiplying approximates, the certain places of the product may be determined by one or other of the two rules following, *viz.*

1. If both factors be approximates, the uncertain places of the product will be one more than the number of places in the longest factor.

2. If one of the factors be finite, and the other approximate, the uncertain places of the product will be one more than the number of places in the finite factor.

Ex. 1.

$$\begin{array}{r} 245.118- \\ .3529+ \\ \hline \end{array}$$

$$2206062$$

$$490236$$

$$1225590$$

$$735354$$

$$\hline 86.5021422$$

Ex. 2.

$$\begin{array}{r} .210526+ \\ 2.875 \\ \hline \end{array}$$

$$1052630$$

$$1473682$$

$$1684208$$

$$421052$$

$$\hline .605262250$$

In Ex. 1. the integral part of the product, *viz.* 86, is certain, and all the decimal places on the right are uncertain. In Ex. 2. only four places on the left, *viz.* .6052, are certain, and all the other places uncertain.

The reason of Rule 1. is plain. For, if in Ex. 1. we make the longest factor the multiplier, and the total product will be the same either way, it is obvious, that in this case we shall have six particular products, in each of which the right-hand figure will be uncertain, and consequently we shall have six uncertain places in the total product toward the right, and also one uncertain place more on account of the uncertain carriage from the column in which the right-hand figure of the last particular product stands.

The reason of Rule 2. is also obvious. For, in Ex. 2. by making the finite factor the multiplier, we have four particular products,

ducts, in each of which the right-hand figure is uncertain; and so we have four uncertain places in the total product, and one uncertain place more arising from the uncertain carriage.

The carriage in some cases may affect several columns on the left, and thereby render so many more figures uncertain.

The surest way therefore to determine the certain places in the product of approximates, is by a second operation, giving the approximates contrary signs; for then, so far as the two products agree, the figures are certain. The second operations of the two former examples follow.

<i>Ex. 1.</i>	<i>Ex. 2.</i>
$\begin{array}{r} 245.117 + \\ + 353 - \\ \hline \end{array}$	$\begin{array}{r} .210527 - \\ 2.875 \\ \hline \end{array}$
$\begin{array}{r} 735351 \\ 1225585 \\ 735351 \\ \hline \end{array}$	$\begin{array}{r} 1052635 \\ 1473689 \\ 1684216 \\ 421054 \\ \hline \end{array}$
86.526301	$.605265125$

In *Ex. 1.* 86.5 is certain, and all the other figures uncertain. In *Ex. 2.* .60526 are the only certain places.

Because the multiplication of decimals that consist of many places, proves, in the way hitherto practised, a tedious operation, I shall here explain a method whereby decimals of this sort, whether finite or approximates, may be multiplied expeditiously, and at the same time have the decimal place in the product limited to any number proposed. This may be effected by the following

R U L E.

Under the multiplicand place the multiplier inverted, so that its units place may stand under that place of the multiplicand to which you propose to limit the product; then multiply the right-hand figure of the multiplier into that figure of the multiplicand which stands directly over it, taking in the carriage from the right, and go on to multiply it into all the other figures on the left. Proceed in like manner with every other figure of the multiplier, placing the right-hand figures of all the particular products directly under other. The total product will be approximate, and the right-hand figure uncertain.

To make this rule more easily understood, the reader may look back to the multiplication of integers; where it was observed, that instead of beginning with the right-hand figure of the multiplier, we may begin with the left, and still have a just product, provided the right-hand figure of every particular product be placed directly

ly under the multiplying figure. Now, the working an example, both in this manner, and also by the rule, and comparing the steps and results of the two operations, will throw a light upon the matter, and unfold the reason of the rule. Which take as follows.

Multiply 18.634375 into 9.875, and limit the product to four decimal places.

By the rule.

18.634375
578.9

1677093

149075

13044

931

184.0143 +

By the other method.

18.634375

9.875

A

167709375

149075000

130440625

93171875

184.014453125

B

In working by the rule, I invert the multiplier, and place 9, the units figure, under 3, the fourth place of decimals, because the product is limited to four decimal places; then I multiply, saying, $9 \times 3 = 27$, and 6 carried from the right makes 33, &c. In multiplying by 8, I say, $8 \times 4 = 32$, and 3 of carriage makes 35; so I set 5 under 3; and proceed in like manner to multiply the figures on the left. The right-hand figure of the product is defective, as wanting the carriage from the columns cut off on the right by the line A B. The figures expressing the sum of the columns so cut off, are so many uncertain places of the product, when the factors are approximate, and on that account to be rejected as useless. The figures, moreover, on the right of the line A B, show how far the operation is contracted, or how much labour is saved in working by the rule.

In working by this rule, some teach to carry from the right hand 1 for every product betwixt 5 and 15, and 2 for every product betwixt 15 and 25, &c.; but rather than adopt a method so irregular, and at the same time inaccurate, it is better to allow the product one decimal place more than properly you have occasion for.

If there be no units in the multiplier, in this case set the right-hand figure of the inverted multiplier under that figure of the multiplicand, below which it would have stood had there been units.

Ex. Multiply .825 by .825, limiting the product to three decimal places.

By

By the rule.

$$\begin{array}{r}
 .825 \\
 528. \\
 \hline
 660 \\
 16 \\
 4 \\
 \hline
 .680
 \end{array}$$

The common way at large.

$$\begin{array}{r}
 .825 \\
 825 \\
 \hline
 4125 \\
 1650 \\
 6600 \\
 \hline
 .680625
 \end{array}$$

The decimal places of the factors may either be retained at full length, or turned into approximates before you begin to multiply.

Ex. Multiply 25.849013625 by 42.97235, limiting the product to two decimal places.

$$\begin{array}{r}
 25.849013625 \\
 53279.24 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 103396 \\
 5169 \\
 2326 \\
 180 \\
 5 \\
 \hline
 \end{array}$$

$$1110.76 +$$

I shall next turn the decimals of the former example into approximates, and then the operation will be as follows.

By the rule.

$$\begin{array}{r}
 25.85- \\
 + 79.24 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 103400 \\
 5170 \\
 2326 \\
 180 \\
 \hline
 \end{array}$$

$$\text{Prod. } 1110.76 +$$

Common way at large.

$$\begin{array}{r}
 25.85- \\
 42.97 + \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 18095 \\
 23265 \\
 5170 \\
 10340 \\
 \hline
 \end{array}$$

$$1110.7745$$

It remains to be observed, that the want of carriage from the right hand may sometimes affect more columns on the left than one, and thereby occasion more uncertain figures in the product than that on the right hand. The best security on this head is, never to limit the product to fewer than four or five decimal places.

To conclude, when decimals to be multiplied are long, you may frequently perform the operation more easily in vulgar fractions, and then reduce the product to a decimal.

RULE

RULE II. If the multiplier be finite, and the multiplicand repeat, in multiplying carry at 9 on the right hand; and before you add, prolong the repetends of the particular products, till their right-hand figures stand directly under one another; and in adding carry at 9 on the right hand.

The product repeats, as in Ex. 1. 2. &c.; or turns out finite, as in Ex. 6.

Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.	Ex. 5.
3	.16	27.083	354.26	4.06
4	7	.5	.08	5.2
1.3	1.16	13.5416	28.3413	813
				20333
				21.146

Ex. 6.	Or thus:	Ex. 7.	Ex. 8.
6.43	6.43	27.883	538.24
123	123	8.75	.345
1930	1930	139416	269124
12866	77200	1951833	2152977
64333		22306666	16147333
	791.30		
791.30		243.97916	185.69433

Note, If the multiplier have ciphers on the right, instead of annexing ciphers to the product, reiterate its right-hand figure so many times as there are ciphers.

Ex. 1.	Ex. 2.	Ex. 3.
79.6	874.3	84.7
50	900	28000
3983.3	786900.0	6782
		16985
		2373777.7

RULE III. If the multiplier be finite, and the multiplicand circulate, to the product of the right-hand figure of the circle add the carriage from the left, then proceed as in multiplication of integers; but before you add the particular products, make them conterminous, and then add as in addition of circulates.

The product commonly circulates; and then its circle is similar to the circle of the multiplicand, as in Ex. 1. 2. 7. and 8.; but the product

product sometimes repeats, as in Ex. 5.; or it may turn out finite, as in Ex. 6.

In Ex. 1. it is obvious, that in multiplying .481, by 7, the carriage from the left would be 3; so I say, $7 \times 1 = 7$, and 3 of carriage make 10, &c. The product circulates, and its circle .370, is similar to .481, the circle of the multiplicand.

Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.
.481,	7.518,	7.518,	7.518,
7	.5	.05	.005
3.370,	3.7,592,	.37,592,	.037,592,

In Ex. 2. 3. and 4. the products are mixt circulates, the three figures on the right being the circle, and the figures on the left the finite parts.

Ex. 5.	Ex. 6.
8.962,	4.7,857142,
24	8.05
35,851,	239,285714,
179,259,	38285,714285,
215.711	38.524,999999
	1
	38.525

In Ex. 5. because the circle of the multiplicand consists of three places, the circle of every particular product will likewise consist of three places; and so, before I add, I make the circle of the second particular product conterminous with that of the first, by transferring its left-hand figure 9 to the right under 1; and in adding I take in the carriage from the left-hand column of the circles, and the product is a mixt repeater, equal to $215\frac{1}{3}$.

In Ex. 6. I transfer 85, in the second particular product, from the left to the right of the circle, which makes it conterminous with the circle above it; and the circle of the product being a series of nines, I add 1 to the finite part, and the total product is finite.

The reason of adding the carriage from the left to the product of the right-hand figure of the circle, and of making the circles of the particular products conterminous, is the same here as in addition of circulates.

Ex. 7.

$$\begin{array}{r}
 \text{Ex. 7.} \\
 46.02439, \\
 \quad 7.42 \\
 \hline
 92,04878, \\
 1840,97560, \\
 32217,07317, \\
 \hline
 341.50,09756,
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 8.} \\
 86.53,571428, \\
 \quad 58.75 \\
 \hline
 43267,857142, \\
 605749,999999, \\
 6922857,142857, \\
 43267857,142857, \\
 \hline
 5083.9732,142857,
 \end{array}$$

Note. If the multiplier have ciphers on the right, first point the product in the manner you would do were the ciphers subjoined to it; and then, instead of annexing the ciphers, transfer so many figures from left to right as serve to complete the circle.

$$\begin{array}{r}
 \text{Ex. 1.} \\
 .72, \\
 \quad 20 \\
 \hline
 14.54,
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 2.} \\
 3.962, \\
 \quad 480 \\
 \hline
 31,703, \\
 158,518, \\
 \hline
 1902.22
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 3.} \\
 4.3,571428, \\
 \quad 600 \\
 \hline
 2614.285714,
 \end{array}$$

In Ex. 2. the product repeats, and so there is no occasion to transfer any figures from left to right.

RULE IV. If the multiplier be interminate, reduce it to a vulgar fraction, as directed in reduction of decimals, Prob. V.; then multiply the given multiplicand by the numerator, (working as in integers, if the multiplicand be finite; or as directed in Rule 2. if it repeat; or as prescribed in Rule 3. if it circulate; and divide the product by the denominator.

Here there are six cases; for the multiplier may repeat or circulate, and may multiply a finite, a repeating, or circulating multiplicand.

CASE I. When a repeating multiplier multiplies a finite multiplicand.

EXAMPLE I.

Multiply 638.25 by $.4 = \frac{2}{5}$.

638.25

638.25

4

9)2553.00(283.6 product sought.

18...

75

72

33

27

60

54

*6

EXAMPLE II.

Multiply 872.35 by .07 = $\frac{7}{100} = \frac{7}{9 \times 10}$

Here I divide first by 9, and then I divide that quot by 10, which is done by moving the decimal point one place to the left.

872.35

7

9|0)6106.45(

67.8494 prod.

EXAMPLE III.

Multiply 7989 by 47.8 = $\frac{478}{10}$

The numerator of the vulgar fraction is found by subtracting the finite part, as follows:

7989

428

47.8

47

428

63912

15978

31956

9)3419292(

379921.2 prod.

EXAMPLE IV.

Multiply 87.34 by 2.67 = $\frac{267}{100} = \frac{267}{9 \times 10}$

241

9 × 10

The

The numerator is found as before.

$$\begin{array}{r} 2.67 \\ 26 \\ \hline 241 \end{array} \qquad \begin{array}{r} 87.34 \\ 241 \\ \hline 8734 \\ 34936 \\ 17468 \end{array}$$

I divide first by 9, and then by 10,
as in Ex. 2.

$$9|0)21048.94($$

233.8777 product.

E X A M P L E V.

Multiply 680.748 by $3.21\bar{6} = \frac{2895}{9 \times 100} =$

$$\begin{array}{r} 3.21\bar{6} \\ 321 \\ \hline 2895 \text{ num.} \end{array} \qquad \begin{array}{r} 680.748 \\ 2895 \\ \hline 3403740 \\ 6126732 \\ 5445984 \\ 1361496 \end{array}$$

After dividing by 9, I divide
by 100, which is done by mo-
ving the decimal point two pla-
ces to the left.

$$9|00)1970765.460($$

$$8 \quad 2189.73940$$

Note 1. When the multiplier is .3 or .6, it will be shorter and
easier to work by the following directions:

1. When the multiplier is .3, take $\frac{1}{3}$ of the multiplicand.

Ex. 1.

Multiply 368 by .3

$$3)368($$

Prod. 122.6

Ex. 2.

Multiply 47.32 by .03

$$3)47.32($$

1.5776 prod.

2. When the multiplier is .6, take $\frac{1}{3}$ of the multiplicand twice.

Ex. 1.

Multiply 12.49 by .6

$$3)12.49($$

4.163

4.163

Prod. 8.326

Ex. 2.

Multiply 12.49 by .06

$$3)12.49($$

4.163

4.163

.8326 prod.

Note

Note 2. You may turn the multiplicand into multiplier, and then work as directed in Rule 2. Example 4. done in this way follows.

Multiply 87.34 by 2.67.

$$\begin{array}{r}
 2.67 \\
 87.34 \\
 \hline
 1071 \\
 8033 \\
 187444 \\
 2142222 \\
 \hline
 \end{array}$$

Product 233.877 the same as before.

Note 3. Because every repeating figure is so many ninth parts, we may multiply by the repeating figure as by an integer, and take $\frac{1}{9}$ of the product, placing the product of the next figure of the multiplier under the finite part of the quot, as in addition, and allowing no decimal place in the product for the repeating figure. The former example done in this manner follows.

Multiply 87.34 by 2.67.

$$\begin{array}{r}
 87.34 \\
 2.67 \\
 \hline
 9)61138 \\
 \hline
 67931 \\
 52404 \\
 17468 \\
 \hline
 \end{array}$$

Product 233.877 the same as formerly.

Here I place 7 beyond the right of the multiplicand, as we do ciphers in multiplication of integers. Under 6793, the finite part of the quot, I place 52404, as in addition. I give only three decimal places to the product for those in the two factors, and one more for the repeating 7.

The product of the repeating 7 into the multiplicand may also be found by taking $\frac{1}{9}$ of the multiplicand, and multiplying the quot by 7, as in the margin.

$$\begin{array}{r}
 9)87.34 \\
 \hline
 9.704 \\
 7 \\
 \hline
 67.931
 \end{array}$$

The same ways of operation may be used in Cases 2. and 3. following, but the method prescribed in the rule is more simple and easy.

CASE

CASE II. When both factors repeat.

E X A M P L E I.

Multiply $6.8\bar{3}$ by $.7 = \frac{7}{10}$

In dividing by 9, after the dividend is exhausted, I continue the division by annexing to the remainders the repeating figure of the dividend; and the quot or product sought comes out a mixt circulate.

$$\begin{array}{r}
 6.8\bar{3} \\
 \times 7 \\
 \hline
 9)47.8\bar{3} \text{ (5.3, 148, product.} \\
 \underline{45} \\
 28 \\
 \underline{27} \\
 *13 \\
 \underline{9} \\
 43 \\
 \underline{36} \\
 73 \\
 \underline{72} \\
 *1
 \end{array}$$

E X A M P L E II.

Multiply $5.\bar{6}$ by $2.\bar{3} = \frac{21}{9}$

$$\begin{array}{r}
 2.\bar{3} \\
 \times 2 \\
 \hline
 21
 \end{array}$$

21 numerator.

$$\begin{array}{r}
 5.\bar{6} \\
 \times 21 \\
 \hline
 5\bar{6} \\
 11\bar{8}3 \\
 \hline
 9)119.0(\\
 \text{13.2 product.}
 \end{array}$$

E X A M P L E III.

Multiply $1.\bar{1}$ by $1.\bar{1} = \frac{10}{9}$

$$\begin{array}{r}
 1.\bar{1} \\
 \times 1 \\
 \hline
 1
 \end{array}$$

10 num.

$$\begin{array}{r}
 1.\bar{1} \\
 \times 10 \\
 \hline
 9)11.\bar{1}1111111(\\
 \text{1.234567901,2 prod.}
 \end{array}$$

The multiplication by 10 is performed by moving the decimal point one place to the right.

T

E X.

EXAMPLE IV.

Multiply 16.8 by 3.84 = $\frac{346}{90}$

3.84
38

346 num.

346
9 x 10
16.8
346

1013
6755
50866

After dividing by 9, I divide by 10, which is done by moving the decimal point one place to the left.

64.9,283950617,2 product.

Note. When the multiplier is .3 or .6, it will be easier to work by the directions of Note I. Case I.

Ex. 1.

Multiply 18.6 by .3

3)18.6(
6.2 product.

Ex. 2.

Multiply 18.6 by .03

3)18.6(
.62 product.

Ex. 3.

Multiply 489.8 by .6

3)489.855(
163.185,
163.185,

326.370, product.

Ex. 4.

Multiply 489.8 by .06

3)489.855(
163.185,
163.185,

32.6,370, product.

CASE III. When a repeating multiplier multiplies a circulating multiplicand.

EXAMPLE I.

Multiply 24.36, by .4 = $\frac{4}{9}$

In multiplying by 4, I take in the carriage from the left of the circle; and in dividing by 9, I continue the division by annexing to the remainders the circulating figures of the dividend.

24.36,
4

9)97.45,(10.82, prod.
9

*74
72

25
18

*74

E X.

E X A M P L E II.

Multiply 38.428571 , by $.7 = \frac{7}{10} = \frac{7}{9 \times 10}$

After dividing by 9, I divide again by 10, which is done by moving the decimal point one place to the left.

$$\begin{array}{r} 38.428571, \\ \underline{7} \\ 9|0)268.999999, \\ \hline 2.98 \quad \text{product.} \end{array}$$

E X A M P L E III.

Multiply $555.23,12195$, by $45.8 = \frac{410}{9} = \frac{41 \times 10}{9}$

$$\begin{array}{r} 45.8 \\ \underline{45} \end{array}$$

410 num.

$$\begin{array}{r} 555.23,12195, \\ \underline{410} \\ 55523,12195, \\ \underline{2220924,87804,} \end{array}$$

After multiplying by 41, I again multiply by 10; which is done by giving the product only six decimal places, instead of seven.

$$\begin{array}{r} 9)227644.7,99999, \\ \hline 25293.88 \quad \text{product.} \end{array}$$

E X A M P L E IV.

Multiply 37.53658 , by $2.47 = \frac{223}{90}$

$$\begin{array}{r} 2.47 \\ \underline{24} \end{array}$$

223 num.

$$\begin{array}{r} 37.53658, \\ \underline{223} \\ 112,60975, \\ \underline{750,73170,} \\ 7507,31707, \end{array}$$

$$\begin{array}{r} 9|0)8370.65853, \\ \hline 93.0,07317, \quad \text{product.} \end{array}$$

E X A M P L E V.

Multiply $78.53,571428$, by $54.83 = \frac{4931}{90}$

$$\begin{array}{r} 54.8\bar{3} \\ 548 \\ \hline 4935 \text{ num.} \end{array}$$

$$\begin{array}{r} 78.53,571428, \\ 4935 \\ \hline 39267,857142, \\ 235607,142857, \\ 7068214,285714, \\ 31414285,714285, \\ \hline 910)387573.74,999999,(\\ 4306.374,999999, \\ \hline \text{I} \end{array}$$

Prod. 4306.375 finite.

Note. When the multiplier is .3 or .6 it will be easier to work by the directions of Note 1. Case 1.

Ex. 1.

$$\begin{array}{l} \text{Multiply .851, by .3} \\ 3).851,851,851,(\\ \hline \text{Prod. .283950617,28 } \bar{c}c. \end{array}$$

Ex. 2.

$$\begin{array}{l} \text{Multiply .846153, by .03} \\ 3).846153,(\\ \hline \text{Prod. .0,282051,} \end{array}$$

Ex. 3.

$$\begin{array}{l} \text{Multiply .53,571428, by .6} \\ 3).53,571428,(\\ \hline .17,857142, \\ .17,857142, \\ \hline \text{Prod. .35,714285,} \end{array}$$

Ex. 4.

$$\begin{array}{l} \text{Multiply .461538, by .06} \\ 3).461538,(\\ \hline .153846, \\ .153846, \\ \hline \text{Prod. .0,307692,} \end{array}$$

In Ex. 1. the given circulate .851, = $\frac{2}{3}$, being no multiple of the divisor 3, the circle of the quot runs on to nine places. See the remarks subjoined to the specimen of pure circulates in reduction of decimals, Prob. I.

CASE IV. When a circulate multiplies a finite multiplicand.

EXAMPLE I.

Multiply 825 by .36, = $\frac{36}{100}$

$$\begin{array}{r} 825 \\ 36 \\ \hline 4950 \\ 2475 \\ \hline \end{array}$$

99)29700(300 product.
297

E X.

E X A M P L E II.

Multiply 56.874 by 4.296, = $\frac{244103.208}{999}$

4.296,	56.874
4	4292
—	—
4292 num.	113748
	511866
	113748
	227496
	—
	999)244103.208(244.3478
	1998
	—
	4430
	3996
	—
	4343
	3996
	—
	3472
	2997
	—
	4750
	3996
	—
	7548
	6993
	—
	5550
	4995
	—
	*555

Instead of dividing by 999, as above, it will be shorter and easier to divide first by 1000, which is done by moving the decimal point three places to the left, and then proceed by the method of dividing by 9, 99, 999, &c. taught in division of integers, as in the margin.

$$\begin{array}{r}
 999)244.103208 \\
 \underline{.244103} \\
 244 \\
 \hline
 244.347855
 \end{array}$$

E X A M P L E III.

Multiply 458.75 by 8.481, = $\frac{3910.5375}{999}$

T 3

8.481,

$$\begin{array}{r}
 8.481, \\
 \underline{8} \\
 \text{Num. } 8473
 \end{array}
 \qquad
 \begin{array}{r}
 458.75 \\
 8473 \\
 \hline
 137625 \\
 321125 \\
 183500 \\
 367000 \\
 \hline
 999)3886988.75(3890.87,962, \\
 2997 \dots \dots \\
 \hline
 8899 \\
 7992 \\
 \hline
 9078 \\
 8991 \\
 \hline
 8787 \\
 7992 \\
 \hline
 7955 \\
 6993 \\
 \hline
 *9620 \\
 8991 \\
 \hline
 6290 \\
 5994 \\
 \hline
 2960 \\
 1998 \\
 \hline
 *962
 \end{array}$$

But the other method of division is shorter and better ; only observe, that the circle of the product will be similar to that of the multiplier, or it will consist of so many places as there are 9's in the divisor ; and in order to bring it out complete, you must to the sum of the right-hand column add the carriage from the left of the circle. The former product, divided in this manner, follows.

Here I first divide by 1000, by moving the decimal point three places to the left ; and in adding, I add 1, the carriage from the left of the circle, to the right-hand column.

$$\begin{array}{r}
 999)3886.98,875, \\
 \quad 3,88,698, \\
 \quad \quad 388, \\
 \hline
 3890.87,962,
 \end{array}$$

If you want to bring out a repetition of the circle, it may be done

done by continuing the second partial quot to its full extent, and then proceeding in the other parts of the operation as before.

The former example, continued to a repetition of the circle, follows.

By continuing the third partial quot to its full extent, the circle will come out thrice; and by continuing the fourth, it will come out four times, &c.

$$\begin{array}{r} 3886.98,875, \\ 3.88,698,875, \\ 388,698, \\ 388, \\ \hline 3890.87,962,962, \end{array}$$

If at any time, instead of a circle, you have a series of nines, in this case 1 must be added to the finite part, and the quot or total product will be finite.

Thus, the first product in Example 1. when divided in this manner by 99, will stand as in the margin.

$$\begin{array}{r} 99)297.00 \\ 2.97 \\ .02 \\ \hline 299.99, \\ \text{I} \\ \hline 300 \end{array}$$

EXAMPLE IV.

Multiply 64.35 by .7,72, = $\frac{765}{99 \times 10}$

$$\begin{array}{r} 64.35 \\ 765 \\ \hline 32175 \\ 38610 \\ 45045 \\ \hline 99)49227.75, \\ 492.27, \\ 4.92, \\ 4, \\ \hline 49724.99, \\ \text{I} \\ \hline \end{array}$$

Prod. 49.725 finite.

Note. You may turn the multiplicand into multiplier, and then work as directed in Rule 3. The above example done in this manner follows.

T 4

.7,72,

$$\begin{array}{r}
 .7,72, \\
 64,35 \\
 \hline
 38,63, \\
 231,81, \\
 3090,90, \\
 46363,63, \\
 \hline
 49.724,99, \\
 \text{I}
 \end{array}$$

Product 49.725 the same as before.

CASE V. When a circulate multiplies a repeating multiplicand.

EXAMPLE I.

Multiply 8.02083 by $.72 = \frac{72}{100}$.

$$\begin{array}{r}
 8.02083 \\
 72 \\
 \hline
 1604166 \\
 56145833 \\
 \hline
 99)577.50000(5.83 \\
 495 \cdot \cdot \\
 \hline
 825 \\
 792 \\
 \hline
 330 \\
 297 \\
 \hline
 *33
 \end{array}$$

Or thus :

$$\begin{array}{r}
 99)5.7.75, \\
 57, \\
 \hline
 5.833,
 \end{array}$$

In the second way of division, the two places on the right-hand are considered as the places of the circle ; so I bring 1 of carriage from left to right, and the quot or total product repeats.

EXAMPLE II.

Multiply 18.783416 by 4.36, $= \frac{436}{100}$.

18.783416

$$\begin{array}{r} 18.78341\bar{6} \\ 432 \end{array}$$

$$\begin{array}{r} 37566833 \\ 563502500 \\ 7513366\bar{6}66 \end{array}$$

$$99)8114.436000(81.964$$

$$792 \dots$$

$$\begin{array}{r} 194 \\ 99 \end{array}$$

$$\begin{array}{r} 954 \\ 891 \end{array}$$

$$\begin{array}{r} 633 \\ 594 \end{array}$$

$$\begin{array}{r} 396 \\ 396 \end{array}$$

$$\text{Or thus: } 99)81.144,36$$

$$811,44$$

$$8,11$$

$$8$$

$$\begin{array}{r} 81.963,99, \\ 1 \end{array}$$

$$\text{Prod. } 81.964 \text{ finite.}$$

E X A M P L E III.

$$\text{Multiply } 198.705208\bar{3} \text{ by } .0,45, = \frac{45}{100}$$

$$\begin{array}{r} 198.705208\bar{3} \\ 45 \end{array}$$

$$\begin{array}{r} 993526041\bar{6} \\ 794820833\bar{3}3 \end{array}$$

$$99|0)8941.7343750(9.0320549,24,$$

$$891 \dots\dots\dots$$

$$\begin{array}{r} 317 \\ 297 \end{array}$$

$$\begin{array}{r} 203 \\ 198 \end{array}$$

$$\begin{array}{r} 543 \\ 495 \end{array}$$

$$\begin{array}{r} 487 \\ 396 \end{array}$$

$$\begin{array}{r} 915 \\ 891 \end{array}$$

$$\text{Carried up, } *240$$

$$\text{Brought up, } *240$$

$$198$$

$$420$$

$$396$$

$$*24$$

$$\text{Or thus:}$$

$$8.9417343,75,$$

$$894173,43,$$

$$8941,73,$$

$$89,41,$$

$$89,$$

$$9.0320549,24,$$

CASE

CASE VI. When both factors circulate.

EXAMPLE I.

Multiply .714285, by .36, = $\frac{36}{89}$

$$\begin{array}{r}
 .714285, \\
 36 \\
 \hline
 4,285714, \\
 21,428571, \\
 \hline
 99)25,714285,7 \\
 257142,8 \\
 2571,4 \\
 25,7 \\
 \hline
 \end{array}$$

Prod. .25,974025, = .259740,25

The circle of the first product is always similar to that of the multiplicand, and in the above example consists of six places; but to secure the carriage from right to left, and thereby complete the circle of the quot or total product, I transfer 7, the left-hand figure of the circle of the first product, to the right, and fill up the places under it with the figures that come in course, and from the sum of these figures on the right I carry 2, which completes the circle of the total product.

EXAMPLE II.

Multiply .32,142857,
by .6,81,
6

$$\begin{array}{r}
 \text{Multiplier} \quad 675 \\
 \hline
 160,714285, \\
 2249,999999, \\
 19285,714285, \\
 \hline
 99|0)21696,428571,4 \\
 216,964285,7 \\
 2,169642,8 \\
 21696,4 \\
 216,9 \\
 2,1 \\
 \hline
 \end{array}$$

Prod. .21915,584415, = .219,155844,15

E X.

E X A M P L E III.

$$\begin{array}{r}
 \text{Multiply } 53.571428, \\
 \text{by } 2.18, \\
 \hline
 2 \\
 \text{Multiplier } 216 \\
 \hline
 321,428571, \\
 535,714285, \\
 10714,285714, \\
 \hline
 99)11571,428571,4 \\
 115,714285,7 \\
 1,157142,8 \\
 11571,4 \\
 115,7 \\
 1,1 \\
 \hline
 \end{array}$$

$$\text{Prod. } 116.88,311688, = 1,16.8831,1688$$

From the examples hitherto adduced, it is obvious, that when one, two, or more figures, in the right of the circle are the same with the like number in the right of the finite part, or integral part, of the product, these latter figures may be made part of the circle.

When both factors are circulates, the number of places in the circle of the total product will always be equal to the number of units in the least multiple of the two numbers that denote the number of places in the circles of the multiplicand and multiplier.

If, therefore, the circle of the first product contain fewer places than there are units in the said least multiple, you must, before you divide, prolong the circle of the first product till it have the number of places required, and one or two places more to secure the carriage from the right; as is done in the following example.

E X A M P L E IV.

$$\begin{array}{r}
 \text{Multiply } 78.45, \\
 \text{by } 8.296, \\
 \hline
 8 \\
 \text{Multiplier } 8288 \\
 \hline
 627,63, \\
 6276,36, \\
 15690,90, \\
 627636,36, \\
 \hline
 999)650231,272727,27 \\
 650,231272,72 \\
 650231,27 \\
 650,23 \\
 65 \\
 \hline
 \end{array}$$

$$650.882,154882, = 650.882154,882$$

In

CASE VI. When both factors circulate.

EXAMPLE I.

Multiply .714285, by .36, = $\frac{36}{89}$

$$\begin{array}{r}
 .714285, \\
 36 \\
 \hline
 4,285714, \\
 21,428571, \\
 \hline
 99)25,714285,7 \\
 \quad 257142,8 \\
 \quad \quad 2571,4 \\
 \quad \quad \quad 25,7 \\
 \hline
 \end{array}$$

Prod. .25,974025, = .259740,25

The circle of the first product is always similar to that of the multiplicand, and in the above example consists of six places; but to secure the carriage from right to left, and thereby complete the circle of the quot or total product, I transfer 7, the left-hand figure of the circle of the first product, to the right, and fill up the places under it with the figures that come in course, and from the sum of these figures on the right I carry 2, which completes the circle of the total product.

EXAMPLE II.

Multiply .32,142857,
by .6,81,
6

Multiplier 675

$$\begin{array}{r}
 160,714285, \\
 2249,999999, \\
 19285,714285, \\
 \hline
 99)21696,428571,4 \\
 \quad 216,964285,7 \\
 \quad \quad 2,169642,8 \\
 \quad \quad \quad 21696,4 \\
 \quad \quad \quad \quad 216,9 \\
 \quad \quad \quad \quad \quad 2,1 \\
 \hline
 \end{array}$$

Prod. .21915,584415, = .219,155844,15

E X.

E X A M P L E III.

$$\begin{array}{r}
 \text{Multiply } 53.571428, \\
 \text{by } 2.18, \\
 \hline
 \text{Multiplier } 216 \\
 \hline
 321,428571, \\
 535,714285, \\
 10714,285714, \\
 \hline
 99)11571,428571,4 \\
 115,714285,7 \\
 1,157142,8 \\
 11571,4 \\
 115,7 \\
 1,1 \\
 \hline
 \end{array}$$

$$\text{Prod. } 116.88,311688, = 1,16.8831,1688$$

From the examples hitherto adduced, it is obvious, that when one, two, or more figures, in the right of the circle are the same with the like number in the right of the finite part, or integral part, of the product, these latter figures may be made part of the circle.

When both factors are circulates, the number of places in the circle of the total product will always be equal to the number of units in the least multiple of the two numbers that denote the number of places in the circles of the multiplicand and multiplier.

If, therefore, the circle of the first product contain fewer places than there are units in the said least multiple, you must, before you divide, prolong the circle of the first product till it have the number of places required, and one or two places more to secure the carriage from the right; as is done in the following example.

E X A M P L E IV.

$$\begin{array}{r}
 \text{Multiply } 78.45, \\
 \text{by } 8.296, \\
 \hline
 \text{Multiplier } 8288 \\
 \hline
 627,63, \\
 6276,36, \\
 15690,90, \\
 627636,36, \\
 \hline
 999)650231,272727,27 \\
 650,231272,72 \\
 650231,27 \\
 650,23 \\
 65 \\
 \hline
 \end{array}$$

$$650.882,154882, = 650.882154,882$$

In

In the above example, the circle of the multiplicand consists of two places, and the circle of the multiplier of three places, and the least multiple of 2 and 3 is 6; so I conclude, that the circle of the total product will consist of six places: but the circle of the first product being similar to that of the multiplicand, has only two places; so I prolong it to six places, and give it two places more in order to secure the carriage.

If the circle of the first product repeat, extend the repeating figure at pleasure; and proceed to find the circle of the total product as before.

EXAMPLE V.

Multiply 6.851,
by .72,

13,703,
479,629,

99)498,333333
4,9333333
493333
4933
49

Product 4.98316,498

If you foresee that the circle of the total product will run on to many places, to save the trouble of a tedious addition, you may proceed thus: Continue the circle of the first product at pleasure; divide it into periods, so as each period may consist of as many places as there are figures in the circle of the multiplier; then add the first period to the second, and the result to the third period, and this result again to the fourth, and so on till you bring out the circle of the total product, or as many places of it as you think proper. Place the carriage-figure from each period under the right-hand figure of the period on the left; add these carriage-figures to the results under which they stand; and their sum will be the total product complete, or so many true figures of it. But here observe, that the results, before they be added to the subsequent periods, must be increased by having these carriage-figures added to them.

E X.

E X A M P L E VI.

Multiply 6.142857,
by .56097,

$$\begin{array}{r}
 42,999999, \\
 552,857142, \\
 36857,142857, \\
 307142,857142, \\
 \hline
 99999)3.4459\overline{)5,857142,857142,85} \\
 \quad \quad \quad 34459 \quad 93031 \quad 35888 \\
 \quad \quad \quad \hline
 \quad \quad \quad 93030 \quad 35888 \quad 50173 \\
 \quad \quad \quad \quad \quad \quad 1 \quad \quad \quad 1
 \end{array}$$

Tot. pr. 3.4459 93031 35888 50174+

$$\begin{array}{r}
 \text{First pr. continued. } 7142,8\overline{)57142,857142,8571} \\
 \quad \quad \quad 50174 \quad 21602 \quad 78745 \quad 64459 \\
 \quad \quad \quad \hline
 \quad \quad \quad 21602 \quad 78744 \quad 64459 \quad 93030 \\
 \quad \quad \quad \quad \quad \quad 1 \quad \quad \quad 1
 \end{array}$$

Tot. pr. continued. 21602 78745 6,4459 93031

The total product complete is

3.445993031358885017421602787456,445993031

In the above example, the circle of the multiplicand consists of six places, and the circle of the multiplier of five places, and the least multiple of 5 and 6 is 30; and so the circle of the total product runs on to thirty places, as above. The learner may divide the first product the other way; and by comparing the steps of the two operations, the truth and reason of the rule will appear.

P R A C T I C A L Q U E S T I O N S.

1. What is the price of 868 gallons of rum, at 5s. per gallon?
Ans. 217 l.
2. What will 56086 yards of tape cost, at 3 farthings per yard?
Ans. 175 l. 5 s. 4½ d.
3. What is the price of 6548 lb. of tea, at 6 s. 8 d. per lb.?
Ans. 2182 l. 13 s. 4 d.
4. What is the price of 276 C. 2 Q. at 3 s. 8½ d. per C.? *Ans.* 51 l. 5 s. 4¼ d.
5. What is the value of .9989583 l. Sterling? *Ans.* 19 s. 11¼ d.
6. What

6. What is the value of .99982638 lb. Troy? *Ans.* 11 oz. 19 dw. 23 gr.

7. What is the price of $648\frac{1}{2}$ yards of cloth, at 13 s. 4 d. per yard? *Ans.* 432 l. 6 s. 8 d.

8. What is the price of $456\frac{1}{2}$ yards, at 6 s. 2 d. per yard? *Ans.* 140 l. 15 s. 1 d.

9. What is the price of 2000 yards ribbon, at $4\frac{1}{2}$ d. per yard? *Ans.* 35 l. 8 s. 4 d.

10. What is the value of .49944196, 428 571, C.? *Ans.* 1 Q. 27 lb. 15 oz.

11. What is the price of 15 C. 0 Q. 16 lb. at 2 l. 6 s. 8 d. per C.? *Ans.* 35 l. 6 s. 8 d.

12. What will a yearly pension of $113\frac{5}{11}$ l. amount to in 2 years 7 months, reckoning 13 months to the year? *Ans.* 288 l.

13. A capital or stock consists of 13 equal shares, whereof A has 1 share, B 3, C 4, and D 5. They propose to make a dividend of 390 l. profits, what is each man's share of the dividend?

Multiply .076023, $= \frac{1}{13}$ by 390, and the product is A's share; and this share multiplied by 3, by 4, and by 5, gives the other shares.

Ans. A's share 30
B's — 90
C's — 120
D's — 150

Proof 390

CHAP. VI.

DIVISION OF DECIMALS.

Before we enter on Division, it will be proper to observe, that there are two rules for pointing the quot, both which are general in their nature, and extend to decimals of every sort, whether terminate or interminate; but unwilling to perplex the learner with too many things at once, I shall at present lay down only one of these rules; and afterwards, when the rule now to be assigned appears to be sufficiently exemplified, I shall then bring the other rule upon the field.

I. GENERAL RULE.

The decimal places in the divisor and quot together must always be equal in number to those of the dividend.

The five following practical directions will make the application of the general rule easy.

1. When the divisor and dividend have an equal number of decimal places, the quot comes out an integer; as in Ex. 2.

2. When the decimal places of the dividend are more than those of the divisor, the number of decimal places in the quot must be equal to the excess; as in Ex. 1. 4. and 8.

3. When

3. When the decimal places of the divisor are more than those of the dividend, annex ciphers to the dividend, so as to make them equal, and the quot by Direction 1. will be integers; as in Ex. 3. 5. and 7.

4. When, after division is finished, the quot has not so many figures, as, by the general rule, it ought to have decimal places, supply that defect by prefixing ciphers; as in Ex. 6.

5. If, after the dividend is exhausted, there be a remainder, annex a cipher, or ciphers, to the remainder, and continue the division till 0 remain, or till the quot repeat or circulate, or till you think proper to limit it; as in Ex. 9. 10. 11. and 12.

We now proceed to division.

RULE I. If the divisor and dividend are both finite or approximate, work exactly as in division of integers.

$$\begin{array}{r}
 \text{Ex. 1.} \\
 .75 \overline{) 58875(.785} \\
 \underline{525} \\
 637 \\
 \underline{600} \\
 375 \\
 \underline{375} \\
 0000
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 2.} \\
 2.5 \overline{) 182.5(73} \\
 \underline{175} \\
 75 \\
 \underline{75} \\
 0000
 \end{array}$$

In Ex. 1. a decimal divides a decimal; and because the dividend has five decimal places, and the divisor only two, I give three decimal places to the quot, according to Direction 2.

In Ex. 2. a mixt number divides a mixt number, and the divisor and dividend having an equal number of decimal places, the quot comes out an integer, according to Direction 1.

The reason of the rule for pointing the quot is obvious; for multiplication gives as many decimal places to the product as are in both factors; but the dividend is the product of the divisor and quot, and so has as many decimal places as are in both; consequently the decimal places in the divisor and quot together must be equal in number to those of the dividend.

$$\begin{array}{r}
 \text{Ex. 3.} \\
 .85 \overline{) 476(} \\
 .85 \overline{) 476.00(560} \\
 \underline{425} \\
 510 \\
 \underline{510} \\
 0000
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 4.} \\
 7 \overline{) .875(.125} \\
 \underline{7} \\
 17 \\
 \underline{14} \\
 35 \\
 \underline{35} \\
 0000
 \end{array}$$

In

In Ex. 3. a decimal divides an integer ; and the dividend having no decimal place, I annex two ciphers, because there are two decimal places in the divisor, and the quot comes out an integer, according to Direction 3.

In Ex. 4. an integer divides a decimal ; and because the dividend has three decimal places, and the divisor none, I give the quot 3, by Direction 2.

Ex. 5.

$$\begin{array}{r} .375 \overline{)12.75(} \\ .375 \overline{)12.750(34} \\ \underline{1125} \\ 1500 \\ \underline{1500} \\ \hline \end{array}$$

Ex. 6.

$$\begin{array}{r} 2.5 \overline{)22875(.0915} \\ \underline{225} \\ 37 \\ \underline{25} \\ 125 \\ \underline{125} \\ \hline \end{array}$$

In Ex. 5. a decimal divides a mixt number ; and the divisor having three decimal places, and the dividend but two, I supply that defect by annexing a cipher, and the quot comes out an integer, by Direction 3.

In Ex. 6. a mixt number divides a decimal ; and because the dividend has four decimal places more than the divisor, and the quot, after the division is finished, has only three figures, I supply this defect by prefixing a cipher to it, according to Direction 4.

Ex. 7.

$$\begin{array}{r} 3.75 \overline{)180(} \\ 3.75 \overline{)180.00(48} \\ \underline{1500} \\ 3000 \\ \underline{3000} \\ \hline \end{array}$$

Ex. 8.

$$\begin{array}{r} 38 \overline{)243.2(6.4} \\ \underline{228} \\ 152 \\ \underline{152} \\ \hline \end{array}$$

In Ex. 7. a mixt number divides an integer ; and the dividend having no decimal places, I supply that defect by annexing two ciphers, the number of decimal places in the divisor, and the quot is an integer, by Direction 3.

In Ex. 8. an integer divides a mixt number ; and the divisor having no decimal place, and the dividend only one, I give one to the quot, according to Direction 2.

Ex. 9.

$$\begin{array}{r} \text{Ex. 9.} \\ .8)29(36.25 \end{array}$$

24

50

48

20

16

40

40

$$\begin{array}{r} \text{Ex. 10.} \\ .018).0024(.13 \end{array}$$

18

60

54

*6

In Ex. 9. a decimal divides an integer ; and after the dividend is exhausted, I annex a cipher to the remainder, and continue the division till 0 remain, according to Direction 5.

In Ex. 10. a decimal divides a decimal ; and after the dividend is exhausted, I annex a cipher to the remainder, and continue the division till I find the quot repeats.

$$\begin{array}{r} \text{Ex. 11.} \\ 11)8(\end{array}$$

$$11)8.0(.72,$$

77

30

22

*8

$$\begin{array}{r} \text{Ex. 12.} \\ 3.25)76.75(23.615+ \end{array}$$

$$650$$

1175

975

2000

1950

500

325

1750

1625

125

In Ex. 11. an integer divides an integer ; and the dividend being less than the divisor, I annex a cipher to it ; again, after the dividend is exhausted, I annex a cipher to the remainder, and continue the division till I find the quot circulates.

In Ex. 12. a mixt number divides a mixt number : and after the dividend is exhausted, by annexing ciphers to the remainder, I continue the division till the quot has three decimal places ; and as there is still a remainder, it might be carried further ; but three decimal

decimal places being in most cases sufficiently accurate, here I chuse to limit it; so the quot is approximate.

In division of decimals the place of the first figure of the quot may likewise be known from the first dividual, much after the same manner as in division of integers, by the following

H. GENERAL RULE.

The place of the first figure of the quot is the same with the place of that figure in the dividend which stands over the units of the first product.

Thus, in the example of integers in the margin, the figure 0, that stands over 5, the units of the product of 9×35 , is in the place of hundreds; and therefore 9, the first figure of the quot, is likewise hundreds; and so the quot is 917 integers.

$$\begin{array}{r}
 35)32095(917 \\
 \underline{315} \\
 59 \\
 \underline{35} \\
 245 \\
 \underline{245} \\
 \hline
 \end{array}$$

To illustrate the rule, I shall give decimal places to the dividend of the above example; and thereby exhibit the varieties that will occur in pointing the quot.

Variety 1.

$$\begin{array}{r}
 35)3209.5(91.7 \\
 \underline{315} \\
 59 \\
 \underline{35} \\
 245 \\
 \underline{245} \\
 \hline
 \end{array}$$

Var. 2.

$$\begin{array}{r}
 35)320.95(9.17 \\
 \underline{315} \\
 59 \\
 \underline{35} \\
 245 \\
 \underline{245} \\
 \hline
 \end{array}$$

Var. 3.

$$\begin{array}{r}
 35)32.095(.917 \\
 \underline{315} \\
 59 \\
 \underline{35} \\
 245 \\
 \underline{245} \\
 \hline
 \end{array}$$

Var. 4.

$$\begin{array}{r}
 35)3.2095(.0917 \\
 \underline{315} \\
 59 \\
 \underline{35} \\
 245 \\
 \underline{245} \\
 \hline
 \end{array}$$

Var. 5.

$$\begin{array}{r}
 35).32095(.00917 \\
 \underline{315} \\
 59 \\
 \underline{35} \\
 245 \\
 \underline{245} \\
 \hline
 \end{array}$$

Var. 6.

$$\begin{array}{r}
 35).032095(.000917 \\
 \underline{315} \\
 59 \\
 \underline{35} \\
 245 \\
 \underline{245} \\
 \hline
 \end{array}$$

In all the above varieties, the figure 0 in the dividend stands over the units of the first product; and in Var. 1. the figure 0 is in the place of tens, and accordingly 9, the first figure of the quot, is tens; in Var. 2. the figure 0 is in the place of units, and so 9 is units; in Var. 3. the figure 0 is in the place of primes, and so 9 is primes, &c.

Here observe, that 9, the first significant figure of the quot, in all the above varieties, as well as in the varieties that follow, must always be considered, in multiplying the divisor, as an integer; and, in pointing the first product, no decimal place is to be allowed for it.

We shall now keep the dividend an integer, and give decimal places to the divisor.

$$\begin{array}{r} \text{Var. 1.} \\ 3.5)32095(9170 \\ \underline{31.5} \end{array}$$

$$\begin{array}{r} 59 \\ 35 \\ \hline \end{array}$$

$$\begin{array}{r} 245 \\ 245 \\ \hline \end{array}$$

$$0.0$$

$$\begin{array}{r} \text{Var. 2.} \\ .35)32095(91700 \\ \underline{3.15} \end{array}$$

$$\begin{array}{r} 59 \\ 35 \\ \hline \end{array}$$

$$\begin{array}{r} 245 \\ 245 \\ \hline \end{array}$$

$$0.00$$

$$\begin{array}{r} \text{Var. 3.} \\ .035)32095(917000 \\ \underline{0.315} \end{array}$$

$$\begin{array}{r} 59 \\ 35 \\ \hline \end{array}$$

$$\begin{array}{r} 245 \\ 245 \\ \hline \end{array}$$

$$0.000$$

$$\begin{array}{r} \text{Var. 4.} \\ .0035)32095(9170000 \\ \underline{0.0315} \end{array}$$

$$\begin{array}{r} 59 \\ 35 \\ \hline \end{array}$$

$$\begin{array}{r} 245 \\ 245 \\ \hline \end{array}$$

$$0.0000$$

In Var. 1. I say, $9 \times 3.5 = 31.5$; and the unit 1 standing under the place of thousands, the figure 9 is also thousands; and as 0 at last remains, I annex a cipher for the decimal .5 in the divisor; then dividing, I get 0 to the quot; and because 9 stands in the place of thousands, the quot is wholly integers.

In Var. 2. the unit 3 stands under the place of ten-thousands, and so 9 is ten-thousands; and to the remainder 0 I annex .00, for the two decimal places in the divisor; then dividing, I get 00 to the quot; and because 9 stands in the place of ten-thousands, the

U 2

quot

quot continues to be wholly integers. The process is the same in Var. 3. and 4.

Lastly, we shall allow decimal places to both dividend and divisor.

$$\begin{array}{r} \text{Var. 1.} \\ 3.5)320.95(91.7 \\ \underline{31.5} \end{array}$$

$$\begin{array}{r} 59 \\ 35 \\ \hline 245 \\ 245 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Var. 2.} \\ 3.5)32.095(9.17 \\ \underline{31.5} \end{array}$$

$$\begin{array}{r} 59 \\ 35 \\ \hline 245 \\ 245 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Var. 3.} \\ .35).32095(.917 \\ \underline{3.15} \end{array}$$

$$\begin{array}{r} 59 \\ 35 \\ \hline 245 \\ 245 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Var. 4.} \\ 3.5).32095(.0917 \\ \underline{31.5} \end{array}$$

$$\begin{array}{r} 59 \\ 35 \\ \hline 245 \\ 245 \\ \hline \end{array}$$

In Var. 1. the units of the first product stand under tens of the dividend; and so 9, the first figure of the quot, is tens. In Var. 2. the units of the first product stand under units of the dividend, and so 9 is units. In Var. 3. the units of the first product stand under primes, and so 9 is primes, &c.

To divide by 10, 100, 1000, &c. is to move the decimal point one place toward the left for every cipher in the divisor.

Thus,	And thus,
10)768(76.8	10)17.28(1.728
100)768(7.68	100)17.28(.1728
1000)768(.768	1000)17.28(.01728
10000)768(.0768	10000)17.28(.001728

APPROXIMATES.

In dividing approximates, the certain places of the quot may be determined by the following

RULE.

R U L E.

Place the divisor, under the first dividual, and the number of certain figures in the quot shall be one less than the number of places from the left of the divisor to the first + or —, whether in the divisor or in the dividend.

Ex. 1.

Dividend 1110.79286078—
Divisor 42.9723+
Certain places five, where-
of three are decimals.

Ex. 2.

Dividend 1110.7929—
Divisor 25.8490136+
Certain places six, where-
of four are decimals.

Ex. 3.

Dividend 4.675
Divisor 34.823—
Certain places four,
all decimals.

Ex. 4.

Dividend 765.84632+
Divisor 237.5
Certain places seven, where-
of six are decimals.

But here it is to be observed, that the uncertain carriage may, in some cases, affect several columns on the left, and thereby render more figures of the quot uncertain than the rule prescribes. The surest way, therefore, is, to make two operations with contrary signs, and then the figures in which the two quots agree are certain.

In order to make the reason of the rule appear, it will be necessary to work an example or two.

Ex. 1.

$$\begin{array}{r}
 \text{A} \\
 42.9723+)1110.79286078-(25.849 \\
 \underline{859446} \dots \\
 2513468 \\
 \underline{2148615} \\
 3648536 \\
 \underline{3437784} \\
 2107520 \\
 \underline{1718892} \\
 3886287 \\
 \underline{3867507} \\
 18780 \\
 \text{B} \\
 \text{U } 3
 \end{array}$$

Here

Here we stop, no more places being certain. The reason is obvious; for the right-hand figure of the first product, viz. 6, is uncertain; and consequently all the figures under it, on the right of the line A B, will be so too; that is, the last remainder and new dividend are uncertain, and of course the figure that would go next to the quot.

Ex. 2.

$$\begin{array}{r}
 \text{A} \\
 25.8490136+)1110.7929-(42.9723 \\
 \underline{1033960} \\
 76832 \\
 \underline{516980} 272 \\
 25134 \\
 \underline{23264} 11224 \\
 1870 \\
 \underline{1809430} 952 \\
 607856080 \\
 \underline{516980272} \\
 908758080 \\
 \underline{775470408} \\
 133287072 \\
 \text{B}
 \end{array}$$

Here again we stop, no more places being certain: and the reason is plain; for the right-hand figure of the dividend, viz. 9, being uncertain, when the first product is subtracted from the dividend, the remainder 356 will be uncertain, and of course all the figures on the right of the line A B will be so likewise; that is, the last remainder and new dividend are uncertain, and so will the figure be that would thence arise to the quot. Hence the rule appears to be just.

From these two examples it appears, that all the figures on the right of the line A B are uncertain and useless; if therefore a way of working, without writing down these useless figures, can be found, we shall then have a method of dividing long decimals, whether finite or approximate, so as to contract the operation, and limit the product to any number of decimal places proposed. And this may be effected by observing the following

R U L E.

Write the product of the first quotient-figure under the dividend;

dend; and from the situation of the units place, consider how many figures of the dividend must be retained to give the quot the number of decimal places intended; cut off the other figures on the right, and also the figures corresponding to them on the right of the divisor; then subtract; esteem this and every following remainder a new dividual; and for each new dividual drop a figure on the right of the divisor; but in multiplying the quotient-figures into the divisor, take in the carriage from the right-hand; as in the following examples.

E X A M P L E I.

Divide 95.432756463275 by 3.4637528; and limit the quot to four decimal places.

Contracted by the rule.

$$\begin{array}{r} 3.46375 \overline{) 95.432756463275} (27.5518 \\ \underline{692750} \end{array}$$

261577

242462

19115

17318

1797

1731

66

34

32

27

(5)

In the above example, the units of the first product standing under the place of tens, the first figure of the quot is tens; and hence it is easy to foresee, that six figures of the dividend retained will give four decimal places to the quot; and accordingly I cut off all the other figures on the right of the dividend; I cut off likewise from the divisor two figures that correspond to them.

At every new dividual I drop or omit a figure on the right of the divisor, and mark the figure so dropped by setting a point under it; and in multiplying the quotient-figure 7 into the divisor, I say, 7 times 7 is 49, and 3 of carriage from the right, (arising from $7 \times 5 = 35$), makes 52; so I set down 2, and carry 5. The same method is observed in multiplying every other quotient-figure into the divisor.

The same Example at large.

$$\begin{array}{r}
 \text{A} \\
 3.4637528 \overline{) 95.432756463275} (27.55183 \\
 \underline{6.9275056 \dots \dots} \\
 261577004 \\
 \underline{242462696} \\
 191143086 \\
 \underline{173187640} \\
 179554463 \\
 \underline{173187640} \\
 63668232 \\
 \underline{34637528} \\
 290307047 \\
 \underline{277100224} \\
 132068235 \\
 \underline{103912588} \\
 29055647 \\
 \text{B}
 \end{array}$$

In working the same example at large, the line A B shows how far the operation is contracted, and how much labour is saved.

But here observe, that by the rule for approximates the certain places of the quot are no more than five, *viz.* 27.551. And therefore, in all operations of this kind, care should be taken to limit the quot to so many places certain; as is done in the following example.

EXAMPLE II.

Divide 87.0763264525 by 9.365407024; limiting the quot to four decimal places certain.

$$\begin{array}{r}
 9.365407024 \overline{) 87.0763264525} (9.2976 \\
 \underline{84.288663216} \\
 278766 \\
 \underline{187308} \\
 91458 \\
 \underline{84288} \\
 7170 \\
 \underline{6555} \\
 615 \\
 \underline{561} \\
 54
 \end{array}$$

Here

Here we put a stop to the operation; because, by the rule for approximates, the next figure of the quot would be uncertain.

I shall conclude division of finite decimals with two very useful problems.

P R O B. L

From a given multiplier to find a divisor that gives a quot equal to the product.

R U L E.

Divide an unit with ciphers annexed by the given multiplier, and the quot will be the divisor sought.

E X A M P L E.

What divisor will give a quot equal to the product of 125 into the dividend?

Given multiplier 125)1.000(.008 divisor sought.

1000

Now, if any number be divided by .008, and the same number be multiplied by 125, the quot and product will be equal.

.008)7315.000(914375 quot.

72....

11
8

35
32

30
24

60
56

40
40

7315
125

36575
14630
7315

914375 product.

The reason is plain: for an unit contains the quot .008 just 125 times; and consequently .008 dividing any number will give a quot 125 times greater than the dividend; that is, the quot will be equal to the product of the dividend multiplied by 125.

P R O B.

P R O B. II.

From a given divisor to find a multiplier that gives a product equal to the quot.

R U L E.

Divide an unit with ciphers annexed by the given divisor, and the quot will be the multiplier sought.

E X A M P L E.

What multiplier will give a product equal to the quot arising from the same number divided by .008?

Given divisor .008)1.000(125 multiplier sought.

$$\begin{array}{r} 8 \cdot \cdot \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline \end{array}$$

Now, if any number be multiplied by 125, and the same number be divided by .008, the product and quot will be equal; as appears in the example following:

$$\begin{array}{r} 785 \\ 125 \\ \hline 3925 \\ 1570 \\ 785 \\ \hline 98125 \text{ product.} \end{array}$$

$$\begin{array}{r} .008)785.000(98125 \text{ quot.} \\ 72 \\ \hline 65 \\ 64 \\ \hline 10 \\ 8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline \end{array}$$

RULE II. If a finite divisor divide a repeating dividend, work as in integers; but in continuing the division, instead of annexing ciphers to the remainder, annex the repeating figure of the dividend.

Ex. 1.

5) 33(.06

30

—

*3

Ex. 2.

4) 5.16(1.2916

4

—

11

8

—

36

36

—

6

4

—

26

24

—

*2

Ex. 3.

37) 4.19662(.11342

37

—

49

37

—

126

111

—

156

148

—

82

74

—

*8

Ex. 4.

12.5) 78.83(-----6.306

75.0

—

383

375

—

833

750

—

*83

12.5

—

31533

126133

—

630666

—

78.8333

proof.

Ex. 5.

30) 56.7(1.8,703,

3

—

26

24

—

*21

21

—

11

9

—

*2

Ex. 6.

800) 75.4(.094308

72

—

34

32

—

24

24

—

44

40

—

*4

75.444444

proof.

From

From Ex. 2. and 4. we may learn, that the quot does not always begin to repeat when the repeating figure of the dividend is taken down. And from Ex. 5. it appears, that the quot sometimes comes out a circulate.

In Ex. 5. and 6. the ciphers in the divisor on the right hand are cut off; but the place of the first quotient-figure is determined by that figure of the dividend which would have stood over the units of the first product, had the ciphers been retained.

RULE III. If a finite divisor divide a circulating dividend, work as integers; but in continuing the division, instead of annexing ciphers to the remainder, annex the circulating figures of the dividend.

EXAMPLE I.

$$7 \overline{) 3.370, (481, 481,} \\ 28 \dots$$

$$\begin{array}{r} 57 \\ 56 \\ \hline 10 \\ 7 \\ \hline *33 \\ 28 \\ \hline 57 \\ 56 \\ \hline 10 \\ 7 \\ \hline *3 \end{array}$$

EXAMPLE II.

$$.5 \overline{) 3.7, 592, (7.518, 5} \\ 3.5 \dots .5$$

$$\begin{array}{r} *25 \quad 3.7, 592, \text{ proof.} \\ 25 \\ \hline 9 \\ 5 \\ \hline 42 \\ 40 \\ \hline *25 \end{array}$$

EXAMPLE III.

$$6.5 \overline{) 25.0, 714285, (3.857142, 8} \\ 19.5 \dots \dots$$

$$\begin{array}{r} *557 \\ 520 \\ \hline 371 \\ 325 \\ \hline 464 \\ 455 \\ \hline \text{Carried up, } 92 \end{array}$$

$$\begin{array}{r} \text{Brought up, } 92 \\ 65 \\ \hline 278 \\ 260 \\ \hline 185 \\ 130 \\ \hline *557 \end{array}$$

E X.

E X A M P L E IV.

7.42)341.50,09756,(46.02439,02
29.68.....

4470

4452

*1809

1484

3257

2968

2895

2226

6696

6678

*1809

RULE IV. If the divisor be interminate, reduce it to a vulgar fraction, as taught in reduction of decimals, Prob. V.; then multiply the given dividend by the denominator, and divide the product by the numerator.

Here there are six cases; for the divisor may either repeat or circulate, and may divide a finite, a repeating, or circulating dividend.

CASE I. When a repeating divisor divides a finite dividend.

E X A M P L E I.

Divide 23.5 by $.4 = \frac{4}{10}$

9

4)211.5(52.875

20

11

8

35

32

30

28

20

20

E X.

EXAMPLE II.

Divide 759.011562 by $2.37 = \frac{237}{100}$

$$\begin{array}{r} 214 \overline{) 68311.040580} (319.21047 \\ 642 \dots\dots \end{array}$$

411

214

1971

1926

450

428

224

214

1005

856

1498

1498

The numerator of the vulgar fraction is found by subtracting the finite part, as follows:

2.37

23

214

The product of the dividend into 90, $= 9 \times 10$, may be readily found by the method of multiplying by 9,99,999, &c. taught in multiplication of integers, viz. multiply first by 10; that is, Move the decimal point one place toward the right hand; from this product subtract the given dividend, and the remainder will be the product of the dividend into 9. Again, multiply this product by 10, by moving the decimal point one place more to the right, and you have the product of the dividend into 90 complete; as in the margin.

The operation, when conducted in this manner, may be made to stand as follows:

$$\begin{array}{r} 2.37 \overline{) 7590.11562} \\ 23 \overline{) 759.011562} \\ \hline 214 \overline{) 68311.04058} (\end{array}$$

7590.11562

759.011562

68311.04058

EXAMPLE III.

Divide 758.816 by $6.87 = \frac{687}{100}$

6.87

$$\begin{array}{r}
 6.83 \overline{) 7588.16} \\
 \underline{68} \quad 758.816 \\
 615 \overline{) 68293.44} \quad (111.046, 24390, \\
 \underline{615} \dots \dots \\
 679 \\
 \underline{615} \\
 643 \\
 \underline{615} \\
 2844 \\
 \underline{2460} \\
 3840 \\
 \underline{3690} \\
 *1500 \\
 \underline{1230} \\
 2700 \\
 \underline{2460} \\
 2400 \\
 \underline{1845} \\
 5550 \\
 \underline{5535} \\
 *150
 \end{array}$$

Note 1. If the divisor be $.3$ or $.6$, it will be easier to work by the following directions.

1. If the divisor be $.3$, multiply the dividend by 3, and the product is the quot.

Ex. 1.

$$\begin{array}{r}
 .3 \overline{) 48.75} \\
 \underline{3} \\
 146.25 \text{ quot.}
 \end{array}$$

Ex. 2.

$$\begin{array}{r}
 .03 \overline{) 6.35} \\
 \underline{3} \\
 190.5 \text{ quot.}
 \end{array}$$

2. If the divisor be $.6$, multiply the dividend by 3, and divide the product by 2. Or, multiply the dividend by $1.5 = \frac{3}{2}$.

Ex.

$$\begin{array}{r}
 \text{Ex.} \\
 .\cancel{6})82.5(\\
 \underline{3} \\
 2)247.5(\\
 \underline{123.75} \text{ quot.}
 \end{array}$$

$$\begin{array}{r}
 \text{Or thus, } .\cancel{6})82.5(\\
 \underline{1.5} \\
 4125 \\
 \underline{825} \\
 123.75
 \end{array}$$

Note 2. In dividing by a repeating divisor, we may use the contracted form, and have a just quot, provided we extend the first product one place beyond the right of the dividend, and, in subtracting and multiplying, borrow and carry at 9 on the right hand.

$$\begin{array}{r}
 \text{Ex. 1.} \\
 .3)48.75(146.25 \\
 \underline{33333} \\
 15416 \\
 \underline{13333} \\
 2083 \\
 \underline{2000} \\
 83 \\
 \underline{66} \\
 16 \\
 \underline{16}
 \end{array}$$

$$\begin{array}{r}
 \text{Ex. 2.} \\
 .\cancel{6})82.5(123.75 \\
 \underline{6666} \\
 1583 \\
 \underline{1333} \\
 250 \\
 \underline{200} \\
 50 \\
 \underline{46} \\
 33 \\
 \underline{33}
 \end{array}$$

The like manner of operation may be used in Cases 2. and 3. following; but the method prescribed in the rule, or in Note 1. is more simple, and much easier.

Note 3. When the divisor is any repeating digit, instead of working by the rule, you may divide 9 by the repeating digit, and then multiply the dividend by the quot. Ex. 1. done in this manner, follows:

$$\begin{array}{r}
 .4)23.5(\\
 \underline{4)9} (\\
 \underline{2.25} \text{ multiplier.} \\
 1175 \\
 \underline{470} \\
 470
 \end{array}$$

Quot 52.875 the same as before.

CASE II. When a repeating divisor divides a repeating dividend.

E X A M P L E I.

Divide $43.2\overline{6}$ by $.3 = \frac{3}{10}$

$$\begin{array}{r} 43.2\overline{6} \\ 9 \end{array}$$

3)389.40(129.8 quot.

3...

8

6

29

27

24

24

Or rather thus:

432.66

43.26

3)389.40(129.8

E X A M P L E II.

Divide $56.03475\overline{7}$ by $4.\overline{2} = \frac{2}{5}$

4.2)560.347577

4 56.034757

38)504.312820(13.27139

38.....

124

114

103

76

271

266

52

38

148

114

342

342

X

E X.

EXAMPLE III.

Divide 25293.86 by 45.8 = $\frac{410}{8}$

$$\begin{array}{r} 45.8 \overline{) 25293.86} \\ 45 \quad 25293.86 \\ \hline \end{array}$$

$$\begin{array}{r} 410 \overline{) 227644.80} (555.23, 12195, \\ 205 \dots \end{array}$$

226

205

214

205

94

82

128

123

*50

41

90

82

80

41

390

369

210

205

*5

EXAMPLE IV.

Divide 21604.3 by 7.8 = $\frac{705}{8}$

$$\begin{array}{r} 7.8 \overline{) 21604.3} \\ 78 \quad 21604.33 \\ \hline \end{array}$$

$$\begin{array}{r} 705 \overline{) 1944390.0} (2758 \\ 1410 \dots \end{array}$$

5343

4935

4089

3525

5640

5640

Note.

Note. When the divisor is .3 or .6, it will be easier to work by the directions subjoined to Note I. in Case I.

1. When the divisor is .3, or .03

Ex. 1.

$$\begin{array}{r} .3 \overline{) 7583} \\ 3 \end{array}$$

2.2750 quot.

Ex. 2.

$$\begin{array}{r} .03 \overline{) 936.86} \\ 3 \end{array}$$

28106.0 quot.

2. When the divisor is .6, or .06

Ex. 1.

$$\begin{array}{r} .6 \overline{) 1928.3} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{) 5785.0} \\ 3 \end{array}$$

2892.5 quot.

Ex. 2.

$$\begin{array}{r} .06 \overline{) 85.43} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{) 2563.0} \\ 3 \end{array}$$

1281.5 quot.

CASE III. When a repeating divisor divides a circulate.

E X A M P L E I.

Divide 92.518, by $.4 = \frac{4}{10}$

$$\begin{array}{r} 92.518, \\ 9 \end{array}$$

832.866

Or rather thus:

$$\begin{array}{r} 925.185, \\ 92.518, \end{array}$$

$$\begin{array}{r} 4 \overline{) 832.866} (208.16 \\ 8 \dots \end{array}$$

32

32

6

4

26

24

*2

E X A M P L E II.

Divide 363.5,740, by $48.3 = \frac{483}{10}$

X 2

48.3

$$\begin{array}{r} 48.3 \overline{) 3635.7,407,} \\ 48 \quad 363.5,740, \\ \hline \end{array}$$

$$\begin{array}{r} 435 \overline{) 3272.1866(7.52} \\ 3045 \quad \dots \\ \hline \end{array}$$

$$2271$$

$$2175$$

$$966$$

$$870$$

$$*96$$

EXAMPLE III.

Divide 93.0,07317, by $2.47 = \frac{223}{90}$

$$\begin{array}{r} 2.47 \overline{) 930.0,73170,} \\ 24 \quad 93.0,07317, \\ \hline \end{array}$$

$$\begin{array}{r} 223 \overline{) 8370.65853,(37.53658,} \\ 669 \quad \dots\dots \\ \hline \end{array}$$

$$1680$$

$$1561$$

$$*1196$$

$$1115$$

$$815$$

$$669$$

$$1468$$

$$1338$$

$$1305$$

$$1115$$

$$1903$$

$$1784$$

$$*119$$

Note. When the divisor is .3, or .6, it will be easier to work by the directions of Note I. in Case I.

Ex. I,

Ex. 1.

3)7.814,(

3

23.444 quot.

Ex. 2.

3)4.592,(

3

2)13.777(

6.888 quot.

CASE IV. When a circulate divides a finite dividend.

E X A M P L E I.

Divide 9 by .45, = $\frac{45}{9}$

In order to multiply the dividend 9 by 99, I first multiply it by 100, which is done by annexing two ciphers; and from this product I subtract the dividend.

900
9
—
45)891(19.8
45
—
441
405
—
360
360
—

E X A M P L E II.

Divide 81.964 by 4.36, = $\frac{436}{81.964}$

4.36,)8196.4

4 81.964

432)8114.436(18.783416

432

3794

3456

3384

3024

3603

3456

1476

1296

Carried up, 1800

X 3

Brought up, 1800

1728

720

432

2880

2592

*288

E X.

EXAMPLE III.

Divide 72 by .0,148, $= \frac{148}{9990}$

Here the denominator of the divisor being 9990, I multiply the dividend by 999, and that product by 10.

$$\begin{array}{r}
 72000 \\
 72 \\
 \hline
 148)719280(4860 \\
 592 \\
 \hline
 1272 \\
 1184 \\
 \hline
 888 \\
 888 \\
 \hline
 0
 \end{array}$$

CASE V. When a circulate divides a repeating dividend.

EXAMPLE I.

Divide 5.8z by .72, $= \frac{72}{99}$

In order to multiply the dividend by 99, I move the decimal point two places to the right, and then subtract the given dividend.

$$\begin{array}{r}
 583.33 \\
 5.8z \\
 \hline
 72)577.50(8.0208z \\
 576 \\
 \hline
 150 \\
 144 \\
 \hline
 600 \\
 576 \\
 \hline
 240 \\
 216 \\
 \hline
 *24
 \end{array}$$

EXAMPLE II.

Divide 25.38z by 19.39, $= \frac{1939}{99}$

$$\begin{array}{r}
 19.39,) 2538.866 \\
 19 \quad 25.38z \\
 \hline
 192|0)2513.28|0(1.309 \\
 192 \\
 \hline
 593 \\
 576 \\
 \hline
 1728 \\
 1728 \\
 \hline
 \end{array}$$

E X A M P L E III.

Divide 25293.86 by 555.23, 12195, = $\frac{5552356672}{9999900}$

$$\begin{array}{r}
 555.23, 12195,) 252938666.66 \\
 \underline{55523} \qquad 25293.86 \\
 5552256672) 252936137280(45.8 \\
 \underline{22209026688} \\
 30845870400 \\
 \underline{27761283360} \\
 30845870400 \\
 \underline{27761283360} \\
 *3084587040
 \end{array}$$

CASE VI. When a circulate divides a circulate.

E X A M P L E I.

Divide .962, by .18, = $\frac{18}{99}$

$$\begin{array}{r}
 96.296, \\
 .962, \\
 \hline
 18)95.333(5.296, \\
 \underline{90} \dots \\
 *53 \\
 \underline{36} \\
 173 \\
 \underline{162} \\
 113 \\
 \underline{108} \\
 *5
 \end{array}$$

E X A M P L E II.

Divide 534.126, by 4.36, = $\frac{412}{99}$

X 4

4.36,

$$\begin{array}{r}
 4.36,) 53412.612, \\
 \underline{4 \quad 534.126,} \\
 432) 52878.486, (122.4, 039, 0 \\
 \underline{432 \dots \dots}
 \end{array}$$

$$\begin{array}{r}
 967 \\
 \underline{864}
 \end{array}$$

$$\begin{array}{r}
 1038 \\
 \underline{864}
 \end{array}$$

$$\begin{array}{r}
 1744 \\
 \underline{1728}
 \end{array}$$

$$\begin{array}{r}
 *1686 \\
 \underline{1296}
 \end{array}$$

$$\begin{array}{r}
 3904 \\
 \underline{3888}
 \end{array}$$

$$*16$$

To the remainders I annex the circulating figures of the dividend.

EXAMPLE III.

Divide 340.90, by 53.571428, = $\frac{53571375}{999999}$

$$\begin{array}{r}
 53.571428,) 340909090.90, \\
 \underline{53 \quad 340.90,}
 \end{array}$$

$$\begin{array}{r}
 53571375) 340908750.00 (6.36, \\
 \underline{321428250}
 \end{array}$$

$$\begin{array}{r}
 *194805000 \\
 \underline{160714125}
 \end{array}$$

$$\begin{array}{r}
 340908750 \\
 \underline{321428250}
 \end{array}$$

$$*19480500$$

When the circle of the quot is likely to run on to many places, you may stop the operation, and complete the quot by a vulgar fraction; as in the following example.

EXAMPLE IV.

Divide 34.56097, by 3.592 = $\frac{3589}{999}$

3.592,

$$\begin{array}{r}
 3.592,) 34560.97560, \\
 \underline{3 } 34.56097, \\
 3589 34526.41463, (9.6200653 \frac{272446341}{99999} \\
 \underline{32301 }
 \end{array}$$

22254

21534

7201 Or, 9.6200653 $\frac{272446341}{99999}$
 7178

23463

21534

19294

17945

13491

10767

(2724)

The quot would run on to 49 figures of a finite part, and then a circle of 95 places; but I limit it at seven places of decimals, and then complete it by a vulgar fraction; as follows, *viz.*

I complete the partial remainder 2724 by annexing to it the circle of the dividend, and placing both, by way of numerator, over the divisor 3589.

The numerator of this complex fraction, being a mixt number, I reduce it to an improper fraction, by multiplying 2724 by the denominator 99999, and adding the numerator 46341 to the product; as in the margin: and then, instead of the mixt number, the numerator of the complex fraction will be $\frac{272446341}{99999}$. Or rather work thus: Esteem 2724.46341, a circulate; and then you find the numerator of the vulgar fraction by subtracting the finite part.

272400000

2724

272397276

46341

272443617

I next divide this fractional numerator by the denominator; which is done by multiplying 3589 by 99999, as in the margin; and now the simple vulgar fraction to be annexed to the partial quot is $\frac{272443617}{358896411}$.

358900000

3589

358896411

If the quot thus completed be multiplied by the divisor, it will produce the dividend.

PRACTICAL QUESTIONS.

1. If L. 86, 11s. be equally divided among 6 men, what is each man's share? *Ans.* L. 14 : 8 : 6.

2. Divide

2. Divide L. 684 : 12 : 8 among 5 men, so that 4 men may have equal shares, and the 5th only half a share. *Ans.* The divisor is 4.5; the dividend is 684.63; and 4.5)684.63(152.1,407,4 = L. 152 : 2 : 9 : 3 $\frac{1}{2}$ to four men each, and L. 76 : 1 : 4 : 3 $\frac{1}{2}$ to the fifth man.

3. Divide L. 486 : 16 : 8 among 4 men, so as 3 men may have equal shares, and the 4th only $\frac{1}{4}$ of a share. *Ans.* The divisor is 3.75; the dividend is 486.83; and 3.75)486.83(129.821.

4. Divide L. 8469, 3s. among 88 men, so as 87 of them may have equal shares, and one of them only $\frac{1}{3}$ share. *Ans.* The divisor is 87.3; the dividend is 8469.15; and 87.3)8469.15(96.975 l.

5. Divide L. 21604 : 6 : 8 among 9 men, so as 7 may have equal shares, 1 half a share, and another one third share. *Ans.* $\frac{1}{2} = .5$; and $\frac{1}{3} = .3$; so the divisor is 7.83; the dividend is 21604.3, and 7.83)21604.3(2758 l.

6. Divide L. 1751, 9s. among A, B, and C; so as A may have $\frac{3}{4}$, B $\frac{3}{8}$, and C $\frac{3}{5}$.

$$\begin{aligned} \text{Ans. } \frac{3}{4} &= .75 \\ \frac{3}{8} &= .375 \\ \frac{3}{5} &= .6 \end{aligned}$$

Divisor 1.725. Dividend 1751.45

$$\text{And } 1.725)1751.45(1015.3 + \left\{ \begin{array}{l} .75 = 761.5 \text{ A's share.} \\ .375 = 380.75 \text{ B's share.} \\ .6 = 609.2 \text{ C's share.} \end{array} \right.$$

1751.45 proof.

7. If, in a year, or 365 days 5 hours and 49 minutes, the sun complete a revolution, or 360 degrees, what is his motion *per* day? *Ans.* .9856 +, equal to 59 min. 8 sec. nearly.

8. If 72 C. 2 Q. 4 lb. cost L. 157, 4s. what will 168 C. 8 lb. cost at that rate? *Ans.* 364.24579 +, equal to L. 364 : 4 : 11 nearly.

9. If 25 men do a piece of work in 8 weeks 4 days, how many men will do the same work in 3 weeks and 3 days? *Ans.* 62.5 men; that is, 62 men working daily, and half the daily work of another man.

10. In L. 133 : 1 : 4 $\frac{1}{2}$, how many dollars, at 37 $\frac{1}{4}$ d. *per* dollar? *Ans.* 846 dollars.

11. In L. 1066 : 13 : 4 Flemish, how many pounds Sterling, at 35 $\frac{1}{2}$ s. *per* pound Sterling? *Ans.* L. 600 Sterling.

12. If 8 $\frac{3}{4}$ cost L. 19 : 14 : 4, what will 1 cost? *Ans.* L. 2, 7 s. 8 d.

C H A P. VII.

DECIMAL PRACTICE.

The price of goods or merchandise may be cast up decimally by any of the methods following.

METHOD I.

Find the decimal of the rate, *viz.* the value of one yard, one pound, one piece, &c.; and this decimal of the rate multiplied into the number or quantity of the goods gives the price.

EXAMPLE I.

At 17s. *per* yard, what cost 936 yards?

The decimal of the rate may be had readily from the tables.

$$\begin{array}{r} 936 \\ .85 \\ \hline 4680 \\ 7488 \\ \hline \end{array} \quad \begin{array}{l} L. \quad s. \\ 795.60 = 795 \quad 12 \end{array}$$

EXAMPLE II.

At 3s. 4d. what cost 346?

The decimal of the rate is

$$.16 = \frac{1}{6} \frac{2}{3}$$

$$\begin{array}{r} 346 \\ 15 \\ \hline 1730 \\ 346 \\ \hline \end{array} \quad \begin{array}{l} L. \quad s. \quad d. \\ 90)5190(57.6 = 57 \quad 13 \quad 4 \\ 45 \cdot \\ \hline 69 \\ 63 \\ \hline 60 \\ 54 \\ \hline *6 \end{array}$$

EXAMPLE III.

At 6s. 8d. what cost 439?

The decimal of the rate

$$.3$$

$$\begin{array}{r} L. \quad s. \quad d. \\ 3)439(146.3 = 146 \quad 6 \quad 8 \end{array}$$

E X.

EXAMPLE IV.

At 13s. 4d. what cost 766?

The decimal of the rate is .6,
so I take $\frac{1}{3}$ twice.

$$\begin{array}{r} 3)766(\\ \underline{255} \\ 255 \\ \underline{8} \\ 510.6 = 510 \text{ L. } 13 \text{ s. } 4 \text{ d.} \end{array}$$

EXAMPLE V.

At L. 4 : 16 : 8, what cost 75?

Here I multiply the rate by the
number of goods, as being shortest.

$$\begin{array}{r} 4.83 \\ 75 \\ \hline 2416 \\ 33833 \\ \hline 362.50 = 362 \text{ L. } 10 \text{ s.} \end{array}$$

EXAMPLE VI.

At L. 3 : 10 : 10, what cost $38\frac{1}{2}$?

$$\begin{array}{r} 3.5416 \\ 38.5 \\ \hline 177083 \\ 2833333 \\ 10625000 \\ \hline 136.35416 = 136 \text{ L. } 7 \text{ s. } 1 \text{ d.} \end{array}$$

EXAMPLE VII.

At 14s. 8d. what cost $48\frac{2}{3}$?

The decimal of the rate is .73 =
 $\frac{66}{90} = \frac{11}{15}$

$$\begin{array}{r} 48.6 \\ 22 \\ \hline 973 \\ 973 \\ \hline 310)1070.6(\\ \underline{3568} \\ 35.68 = 35 \text{ L. } 13 \text{ s. } 9 \frac{1}{2} \text{ d.} \end{array}$$

EXAMPLE VIII.

At L. 2 : 17 : 9, what cost 16 C. 2 Q. 8 lb.?

$$16.571428,$$

16.571428,
5788.2 multiplier inverted.

331428
132571
13257
1159
82

47.849

1
L. s.
47.85 = 47 17

METHOD II.

When the rate consists of pence and farthings, find how often it is contained in one pound Sterling, divide the given number of goods by this number, or by its component parts, or work by aliquot parts, and the result will be the price sought.

We confine this method to such rates as consist of pence and farthings, because when the rate consists of shillings, pence, and farthings, or of pounds, shillings, pence, &c. it is shorter and easier to work by Method I.

To make the practice ready and easy, it will be proper to have at hand a table of rates and divisors, such as the following one.

Table of Rates and Divisors.

Rates.	0 Farth.	1 Farth.	2 Farth.	3 Farth.
d.	Divif.	Divif.	Divif.	Divif.
0		3,8,40.	8,60.	8,40.
1	6,40.	8,6,4.	4,40.	4,30,—8.
2	4,30.	4,30,+8.	4,30,+4.	80,—12.
3	80.	80,+12.	80,+6.	80,+4.
4	60.	60+4 of 4.	60+8.	60+8+2 of 8.
5	6,8.	40,—8.	40—12.	40—4 of 6.
6	40.	40+4 of 6.	40+12.	40+8.
7	40+6.	40+6+4 of 6.	40+4.	40+4+6 of 4.
8	30.	30+4 of 8.	30+2 of 8.	80,x3,—12 of 80.
9	80,x3.	80,x3,+12 of 80.	80,x3,+6 of 80.	80,x3,+4 of 80.
10	8,3.	30,+4,+8 of 4.	20,—8.	40,+2,+2,+6.
11	20,—12.	40,x2,—8 of 40.	40,x2,—12 of 40.	8,3,+5,—8 of 5.

In

In the above table the pence stand in the left-hand column; and the farthings on the head, and the divisors in the angle of meeting; which are to be understood and read as follows:

3,8,40. Divide the given number of goods by 3, divide the quot by 8, and again divide this last quot by 40.

4,30,—8. Divide the number of goods by 4, divide the quot by 30, and from this last quot subtract one 8th of itself.

80+12. To an 80th add a 12th of that 80th.

4,30,+4. To a 30th of a 4th add a 4th of that 30th.

60,+4 of 4. Divide by 60, divide again the quot by 4, and to the first quot add a 4th of the second quot.

80×3,—12 of 80. Divide by 80, multiply the quot by 3, and from the product subtract a 12th of the first quot,

EXAMPLE I.

At 1 f. per yard, what cost 432 yards?

$$\begin{array}{r} \text{One 3d} = 144 \\ \text{One 8th of that} = 18 \\ \text{One 40th of that} = .45 = 9\text{s.} \end{array}$$

EXAMPLE II.

At 3 f. what cost 728.5?

$$\begin{array}{r} \text{One 8th} = 91.0625 \quad L. \quad s. \quad d. \quad f. \\ \text{One 40th of that} = 2.2765625 = 2 \quad 5 \quad 6 \quad 1\frac{1}{2} \end{array}$$

EXAMPLE III.

At 1 d. 3 f. what cost 8472.3?

$$\begin{array}{r} \text{One 4th} = 2118.083 \\ \text{A 30th of that} = 70.6027777 \\ \text{Subtract one 8th} = 8.8253472 \\ \hline \text{Ans. } 61.7774308 = 61 \quad 15 \quad 6 \quad 2\frac{1}{2} \end{array}$$

EXAMPLE IV.

At 2 d. 2 f. what cost 748.25?

$$\begin{array}{r} \text{One 4th} = 187.0625 \\ \text{A 30th of that} = 6.23541666 \\ \text{Add one 4th} = 1.55885416 \\ \hline \text{Ans. } 7.79427083 = 7 \quad 15 \quad 10 \quad 2\frac{1}{2} \end{array}$$

EXAMPLE V.

At 4 d. 1 f. what cost 1832.6?

$$\begin{array}{r}
 \text{One 60th} = 30.544444 \\
 \text{One 4th of that} = 7.6361 \\
 \text{To one 60th add } \left. \begin{array}{l} \\ \end{array} \right\} = 1.909027 \\
 \text{one 4th of that} \left. \begin{array}{l} \\ \end{array} \right\} = 1.909027 \\
 \text{Ans. } 32.453472 = 32 \text{ L. } 9 \text{ s. } 0 \text{ d. } 3 \frac{1}{4} \text{ f.}
 \end{array}$$

EXAMPLE VI.

At 7 d. 1 f. what cost 2854.63,?

$$\begin{array}{r}
 \text{One 40th} = 71.365909,09, \\
 \text{One 6th of that} = 11.894318,18, \\
 \text{One 4th of one 6th} = 2.973579,54, \\
 \text{Ans. } 86.233806,81, = 86 \text{ L. } 4 \text{ s. } 8 \text{ d. } 0 \frac{5}{8} \text{ f.}
 \end{array}$$

EXAMPLE VII.

At 8 d. 3 f. what cost 794.518,?

$$\begin{array}{r}
 \text{One 80th} = 9.93,148, \\
 \text{Multiply by } 3 \\
 \text{Subtract one 12th of one 80th} = .82762 + \\
 \text{Ans. } 28.96682 = 28 \text{ L. } 19 \text{ s. } 4 \text{ d.}
 \end{array}$$

EXAMPLE VIII.

At 11 d. 3 f. what cost 854.75?

$$\begin{array}{r}
 \text{One 8th} = 106.84375 \\
 \text{One 3d of that} = 35.614583 \\
 \text{Add one 5th of one 3d} = 7.122916 \\
 \text{Subtract one 8th of one 5th} = 42.7375 \\
 \text{Ans. } 41.847135416 = 41 \text{ L. } 16 \text{ s. } 11 \text{ d. } 1 \frac{1}{4} \text{ f.}
 \end{array}$$

METHOD III.

The third method is by decimal tables of rates suited to the nine digits; such as those composed and published by the Reverend Mr George Brown in 1718, under the title of *Arithmetica Infinita*,

finita, and recommended by Dr John Keill, professor of astronomy in the university of Oxford.

These tables are still extant, and extend from 1 farthing to 20s.; a short specimen of which, with their construction, and the manner of using them, I shall here subjoin.

Decimal Table of Rates, 1 l. the integer.

	Rate.		Rate.		Rate.		Rate.	
	s.	d.	s.	d.	s.	d.	s.	d.
N.	11	5	11	5 $\frac{1}{4}$	11	5 $\frac{1}{2}$	11	5 $\frac{3}{4}$
1	0.57083		0.571875		0.572916		0.5739583	
2	1.1416		1.14375		1.14583		1.147916	
3	1.7125		1.715625		1.71875		1.721875	
4	2.283		2.2875		2.2916		2.29583	
5	2.85416		2.859375		2.864583		2.8697916	
6	3.425		3.43125		3.4375		3.44375	
7	3.99583		4.003125		4.010416		4.0177083	
8	4.56		4.575		4.583		4.5916	
9	5.1375		5.146875		5.15625		5.165625	

In the left-hand column stand the nine digits; and on the right of 1 are the decimals of the respective rates on the head. Thus, .57083 is the decimal of 11s. 5d. one pound being the integer; and .571875 is the decimal of 11s. 5 $\frac{1}{4}$ d. &c. Those decimals opposite to 1 being multiplied through the nine digits, make up or compose the rest of the table.

The superior excellency of tables thus constructed is, that we multiply or divide by 10, 100, 1000, &c. by moving the decimal point so many places to the right or left as there are ciphers in the multiplier or divisor.

Hence the price or value of any number of yards, or other things, denoted by a single digit, or by any of its decuples, may be readily found. Thus, the price of 7, 70, 700, 7000, 70000, 700000 yards, at 11s. 5d. *per* yard, is found as follows:

Yards.	L.	L.	s.	d.
7 =	3.99583 =	3	19	11
70 =	39.9583 =	39	19	2
700 =	399.583 =	399	11	8
7000 =	3995.83 =	3995	16	8
70000 =	39958.3 =	39958	6	8
700000 =	399583.3 =	399583	6	8

Now,

Now, every number may be resolved into decuples of the several digits of which it is composed; find therefore the price of each decuple by itself, as already taught, and their sum will be the price of the whole.

EXAMPLE I.

Required the price of 7956 yards, at 11 s. $5\frac{1}{4}$ d. per yard.

Yards.	L.			
7000	=	4017.708333		
900	=	516.5625		
50	=	28.697916		
6	=	3.44375		
			L.	s. d.
		4566.412500	=	4566 8 3

EXAMPLE II.

How much money will one spend in a year, or 365 days, at the rate of 11 s. $5\frac{1}{2}$ d. per day?

Days.	L.			
300	=	171.875		
60	=	34.375		
5	=	2.864583		
			L.	s. d.
		209.114583	=	209 2 $3\frac{1}{2}$

EXAMPLE III.

If a pensioner's daily pay or wages be 11 s. $5\frac{1}{4}$ d. what will that amount to in 154 days?

Days.	L.			
100	=	57.1875		
50	=	28.59375		
4	=	2.2875		
			L.	s. d.
		88.06875	=	88 1 $4\frac{1}{2}$

Tables of this sort may be framed for a great variety of useful purposes, and are easily constructed.

Thus, suppose a table wanted for showing the daily income of any annuity, or yearly pension; in this case, divide 1 by 365, and the quot is the income of 1 l. annuity for one day; and by multiplying this quot through the nine digits, the table is constructed, as follows.

Y

The

TABLE.

1.	0,02739726,
2.	0,05479452,
3.	0,08219178,
4.	0,10958904,
5.	0,13698630,
6.	0,16438356,
7.	0,19178082,
8.	0,21917808,
9.	0,24657534,

The use of the table will best appear by examples ; which take as follows :

EXAMPLE I.

If one has a yearly pension of 375 l. what is his daily income ?

$$\begin{array}{r}
 L. \\
 300 = .8219 \\
 70 = .1917 \\
 5 = .0136 \\
 \hline
 1.0272 = 1 \text{ } 0 \text{ } 6\frac{1}{2}
 \end{array}$$

EXAMPLE II.

The yearly rent of a gentleman's estate is 968 l. 10 s. what can he afford to spend *per day* ?

$$\begin{array}{r}
 L. \\
 900 = 2.4657 \\
 60 = .1643 \\
 8 = .0219 \\
 0.5 = .0013 \\
 \hline
 2.6532 = 2 \text{ } 13 \text{ } 0\frac{3}{4}
 \end{array}$$

If the income for any number of days be required, find the income for one day as above ; and multiply the decimal answer by the given number of days. Or, multiply the yearly pension by the given number of days, and use the product as the yearly pension. Thus, in Ex. 2. if the gentleman's income for 64 days be demanded, you may either multiply 2.6532 by 64 ; or multiply 968.5 by 64 ; and then work for the product as follows :

$$\begin{array}{r}
 968.5 \\
 64 \\
 \hline
 38740 \\
 58110 \\
 \hline
 61984.0
 \end{array}
 \qquad
 \begin{array}{r}
 60000 = 164.3835 \\
 1000 = 2.7397 \\
 900 = 2.4657 \\
 80 = .2191 \\
 4 = .0109 \\
 \hline
 169.8189 = 169 \text{ } 16 \text{ } 4\frac{1}{2}
 \end{array}$$

The

The decimals in the table being circles of eight figures, I have used them as approximates, by confining the operations to four decimal places; which, in affairs of this kind, is sufficiently accurate.

If the annual interest of any principal sum be considered as the yearly pension, the interest of the same principal for any number of days may be found by the table, as taught above.

The interest of any principal sum for a year is easily found, as being always the hundredth part of the product of the principal multiplied by the rate *per cent*.

EXAMPLE I.

Required the interest for 26 days of 685 l. principal, at 5 *per cent*.

L.		
685		
5		
34.25	annual interest.	
26		
20550		
6850		
890.50		

800	=	2.19178
90	=	.24657
0.5	=	.00136
2.43971	=	L. s. d.
		2 8 9½

EXAMPLE II.

Required the interest for 80 days of 12000 l. at 4½ *per cent*.

12000		
4½		
48		
3		
510.00	annual interest.	
80		
40800		

40000	=	109.58904
800	=	2.19178
111.78082	=	L. s. d.
		111 15 7½

DUODECIMALS.

Decimal practice may be used with great advantage in the multiplication and division of duodecimals, where the integer is divided into twelve equal parts, called *primes*, and each prime into twelve seconds, each second into twelve thirds, &c.

For the ready conversion of primes, seconds, thirds, &c. into decimals of the integer, the following table is constructed.

Decimal table of primes, seconds, &c.

N.	Primes. '	Seconds. "	Thirds. '''	Fourths. ''''
1	.083	.00694	.000578,703,	.00004822
2	.16	.0138	.001157,407,	.00009645
3	.25	.02083	.001736,111,	.00014467
4	.3	.027	.002314,814,	.00019290
5	.416	.03472	.002893,518,	.00024112
6	.5	.0416	.003472,222,	.00028935
7	.583	.04861	.004050,925,	.00033757
8	.6	.05	.004629,629,	.00038580
9	.75	.0625	.005208,333,	.00043402
10	.83	.0694	.005787,037,	.00048225
11	.916	.07638	.006365,740,	.00053047

In the column of fourths, the decimals run on to eight places of a finite part, and nine figures of a circle; but the finite part by itself, which alone is inserted in the table, will be found sufficient; and in the column of thirds too, the circle of three figures may in most cases be neglected.

I. MULTIPLICATION.

EXAMPLE I.

What is the product of 24 7 by 18 5

$$\begin{array}{r} 24 \quad 7 = 24.583 \\ 18 \quad 5 = 18.416 = 18.416 \end{array}$$

24.583

$$\begin{array}{r}
 24.583 \\
 16575 \\
 \hline
 122916 \\
 1720833 \\
 12291666 \\
 147500000 \\
 245833333 \\
 \hline
 9|00) 407468.750 \\
 \hline
 452.74308 \\
 12 \\
 \hline
 8.91666 \\
 12 \\
 \hline
 11.000
 \end{array}$$

Or thus:

$$\begin{array}{r} 24.58333 \\ \phi 14.81 \\ \hline 2458333 \\ 1966666 \\ 98333 \\ 2458 \\ \hline 452.5790 \\ \frac{1}{3} = 819 \\ \frac{1}{3} = 819 \\ \hline 452.7428 \\ 12 \\ \hline 8.9136 \\ 12 \\ \hline 10.9632 \end{array}$$

Ans. 452 8 II

In working by the inverted method, for the repeating ϕ in the multiplier, I take $\frac{2}{3}$ of the multiplicand. The result wants very little of the true answer.

EXAMPLE II.

Multiply 18 6 by 2 4, and 2 3 continually.

$$\begin{array}{r}
 18 \quad 6 = 18.5 \quad 2.8 \\
 2 \quad 4 = 2.3 \quad 18.5 \\
 2 \quad 3 = 2.25 \\
 \hline
 118 \\
 1846 \\
 2333 \\
 \hline
 43.16 \\
 2.25 \\
 \hline
 21583 \\
 86383 \\
 863333 \\
 \hline
 97.1250 \\
 12 \\
 \hline
 1.500 \\
 12 \\
 \hline
 6.0
 \end{array}$$

Ans. 97 1 6

Y 3

EX.

EXAMPLE III.

What is the square of $6\ 9\ 4 = 6.\overset{'}{9}\overset{''}{4} = 6.\overset{'}{9}\overset{''}{4} = \frac{694}{100}?$

$$\begin{array}{r} 6.\overset{'}{9}\overset{''}{4} \\ 6\overset{'}{9}\overset{''}{4} \\ \hline 6\overset{'}{9}\overset{''}{4} \\ 40\overset{'}{9}\overset{''}{6} \\ \hline 9)413.4(\\ \hline 45.938271604, \end{array}$$

$$\begin{array}{r} 45.938271604, \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 11.259,259,259, \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 3.111 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 1.2 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 1.2 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 1.2 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 4.0 \\ \hline \end{array}$$

Ans. 45 11 3 1 4

EXAMPLE IV.

What is the product of $36\ 9\ 9$ by $4\ 8$?

$$\begin{array}{r} 36\ 9\ 9 = 36.8125 \\ 4\ 8 = .38 = \frac{38}{100} \end{array}$$

$$\begin{array}{r} 36.8125 \\ 35 \\ \hline \end{array}$$

$$\begin{array}{r} 1840625 \\ 1104375 \\ \hline \end{array}$$

$$\begin{array}{r} 9)1288.4375(\\ \hline \end{array}$$

$$\begin{array}{r} 14.315972 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 3.791066 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 9.5000 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 6.0 \\ \hline \end{array}$$

Ans. 14 3 9 6

II. DIVISION.

EXAMPLE I.

Divide 452 8 11 = 452.74308
by 18 5 = 18.416 = $\frac{16575}{900}$

18.416)4527.43085
1841 452.74308

16575)407468.750(24.583

33150 12

75968 7.000

66300

96687

82875

Ans. 24 7

138125

132600

55250

49725

*5525

EXAMPLE II.

Divide 97 1 6 = 97.125
by 2 3 = 2.25 and the quot
by 18 6 = 18.5

18.5)

2.25)97.125(43.16(2.3

900 370 12

712

616 4.0

675

555

375

*61

225

1500

1350

Ans. 2 4

*150

EXAMPLE III.

Divide 14 3 9 6 = 14.315972
by 36 9 9 = 36.8125

36.8125)14.315972 (.38
1104375 12

3272222 4.06
2945000 12

*327222 8.0 *Ans.* 4 8

EXAMPLE IV.

Divide 65 9 11 0 = 65.82638
by 8 1 10 11 = 8.159142

8.159142)65.826388 (8.067808
..... 65273136

553252
489548

63704
57113

Ans. 8 0 9 8 2

6591
6527

64
64

EXAMPLE V.

Divide 15 0 0 0 = 15.000000
by 3 6 11 9 = 3.581596

3.581596)15.000000 (4.188079
..... 14326384

673616
358159

315457
286527

28930
28652

Carried up, 278

Brought up, 278
250

28

27

1

Ans. 4 2 3 1

PRACTICAL QUESTIONS.

1. In a log of timber, whose length is 6 feet 3 inches, breadth 3 feet 8 inches, and thickness 2 feet 3 inches, how many cubic feet, and what the value, at 6s. 9d. *per* cubic foot? *Ans.* 51 cubic feet 6 primes 9 seconds, and its value L. 17 : 8 : 0 : 2 $\frac{1}{4}$.

2. The area of a rectangular floor is 919 square feet 8 primes 11 seconds, and the length of it is 38 feet 9 inches 6 lines; what is the breadth? *Ans.* 23 feet 8 inches 6 lines.

3. The solid content of a parallelopiped is 86 cubic feet 7 primes 9 seconds 6 thirds, its length is 17 feet 6 inches, and its breadth 1 foot 11 inches, what is the thickness? *Ans.* 2 feet 7 inches.

SEXAGESIMALS.

Decimal practice might likewise be used to good purpose in the arithmetic of sexagesimals, as it would shorten and facilitate the operations.

Sexagesimals, strictly speaking, are degrees, minutes, seconds, thirds, &c. where each degree is divided into 60 minutes, and each minute into 60 seconds, &c.; but under this title is also usually comprehended the division of a sign into 30 degrees. They are commonly marked as under.

Signs. deg. min. sec. thirds.

° ' " "

7 24 36 54 48 &c.

Sexagesimals properly belong to astronomy, being used in computations of motion and time, where the degree of motion, and hour of time, are equally divided into 60 minutes. The preference of the decimal method to that of the sexagesimal, will appear from the following example of addition done both ways.

<i>Sexagesimally.</i>				<i>=</i>		<i>Decimally.</i>
<i>Signs.</i>	°	'	"	<i>S.</i>		
10	20	47	17	=	10.69293,518,	
7	18	50	40	=	7.62814,814,	
9	25	30	20	=	9.85018,518,	
11	10	40	50	=	11.35601,851,	
3	15	49	7	=	3.52728,703,	

From the above example it is obvious, that even in addition the decimal operation is more simple and easy than the sexagesimal, especially if care be taken to use no more decimal places than what are absolutely necessary.

But

But in multiplication and division the advantage of the decimal method is still greater; for in the sexagesimal way the operation is extremely tedious; whereas, by working decimally, it is performed in the same manner, and with the same ease, as in duodecimals.

VULGAR FRACTIONS.

Decimal practice may sometimes be profitably used in the arithmetic of vulgar fractions, the operation being shorter and easier in the decimal than in the vulgar way. This I shall illustrate by a few examples.

I. ADDITION.

Ex. 1. What is the sum of $\frac{1}{4} + \frac{2}{3}$ l.?

$$\begin{array}{r} \frac{1}{4} = .25 \\ \frac{2}{3} = .666 \\ \hline \end{array} \quad \begin{array}{r} s. \\ d. \end{array}$$

$$.916 = 18 \text{ } 4$$

Ex. 2. What is the sum of $14\frac{7}{8} + 18\frac{2}{3} + \frac{3}{4}$ of $\frac{5}{8}$ C.?

$$\begin{array}{r} 14\frac{7}{8} = 14.875 \\ 18\frac{2}{3} = 18.666 \\ \frac{3}{4} \text{ of } \frac{5}{8} = \frac{15}{32} = .46875 \\ \hline \end{array} \quad \begin{array}{r} C. \\ \text{ } \end{array}$$

$$34.1066 = 34 \text{ } 0 \text{ } 18\frac{2}{3}$$

Ex. 3. What is the sum of $35\frac{1}{2} + 24\frac{1}{2} + 12\frac{3}{4}$ lb. Troy?

$$\begin{array}{r} 35\frac{1}{2} = 35.5 \\ 24\frac{1}{2} = 24.5 \\ 12\frac{3}{4} = 12.75 \\ \hline \end{array} \quad \begin{array}{r} lb. \\ oz. \\ dw. \end{array}$$

$$72.75 = 72 \text{ } 1 \text{ } 5\frac{1}{4}$$

II. SUBTRACTION.

Ex. 1. From $\frac{3}{4}$ subtract $\frac{1}{5}$ l.

$$\begin{array}{r} L. \\ \frac{3}{4} = .75 \\ \frac{1}{5} = .233 \\ \hline \end{array} \quad \begin{array}{r} s. \\ d. \end{array}$$

$$.416 = 8 \text{ } 4$$

Ex. 2.

Ex. 2. From $\frac{2}{3}$ of $\frac{7}{8}$ subtract $\frac{1}{2}$ of $\frac{1}{4}$ lb. Troy.

$$\begin{array}{r} \text{lb.} \\ \frac{2}{3} \text{ of } \frac{7}{8} = \frac{7}{12} = .58\overline{3} \\ \frac{1}{2} \text{ of } \frac{1}{4} = \frac{1}{8} = .125 \\ \hline .458\overline{3} = 5 \text{ oz. dw.} \\ \phantom{.458\overline{3}} = 10 \end{array}$$

Ex. 3. From $36\frac{1}{2}$ subtract $22\frac{5}{8}$ days.

$$\begin{array}{r} \text{Days.} \\ 36\frac{1}{2} = 36.33 \\ 22\frac{5}{8} = 22.8\overline{3} \\ \hline 13.50 = 13 \text{ D. h.} \\ = 12 \end{array}$$

III. MULTIPLICATION.

Ex. 1. Multiply $19\frac{7}{12}$ by $22\frac{1}{3}$ feet.

$$\begin{array}{r} \text{F.} \\ 19\frac{7}{12} = 19.58\overline{3} \\ 22\frac{1}{3} = 22.3\overline{3} \\ \hline 1958\overline{3} \\ 39166\overline{6} \\ \hline 9)3936.250 \\ \hline 437.36\overline{x} = 437 \text{ Sq.f. Sq. in.} \\ \phantom{437.36\overline{x}} = 52 \end{array}$$

Ex. 2. How many solid yards of digging in a cellar that is $8\frac{2}{3}$ yards long, $5\frac{1}{2}$ yards wide, and $2\frac{1}{4}$ yards deep?

$$\begin{array}{r} \text{Yds.} \\ 8\frac{2}{3} = 8.6\overline{6} \\ 5\frac{1}{2} = 5.5 \\ 2\frac{1}{4} = 2.25 \\ \hline 43\overline{3} \\ 433\overline{3} \\ \hline 47.66\overline{6} \\ 2.25 \\ \hline 238\overline{3} \\ 953\overline{3} \\ 9533\overline{3} \\ \hline 107.250 = 107\frac{1}{4} \text{ solid yards.} \end{array}$$

Ex 3.

Ex. 3. If a cistern be 12 feet long, 7 feet wide, and $3\frac{1}{2}$ feet deep, how many wine-gallons will it hold, the gallons in a solid foot being 7.480519,?

F.	
12	7.480519,
7	294
—	
84	29,922077,
3.5	673,246753,
—	1496,103896,
420	— <i>W. gal.</i>
252	2199.272727, = 2199 $\frac{3}{11}$
—	
294.0	

IV. DIVISION.

Ex. 1. Divide $6\frac{7}{8}$ l. by $\frac{4}{11}$ l.

$$\frac{4}{11} = .36, \text{ and } 6\frac{7}{8} = 6.3\frac{7}{8}$$

$$.36 \overline{) 638.88}$$

$$00 \quad 6.3\frac{7}{8}$$

$$\begin{array}{r} \text{L. s. d.} \\ 36 \overline{) 632.50} (17.5694 = 17 \text{ II } 4\frac{2}{3} \end{array}$$

$$02 \quad 36 \cdot 00$$

$$272$$

$$252$$

$$205$$

$$180$$

$$250$$

$$216$$

$$340$$

$$324$$

$$160$$

$$144$$

$$*16$$

Ex. 2. If the solid content of a cellar be $107\frac{1}{4}$ yards, its breadth $5\frac{1}{2}$ yards, and its depth $2\frac{1}{4}$ yards, what is its length?

$$\begin{array}{r}
 2.25 \text{ Tds.} \\
 5.5)107.25(19.50(8.6=8\frac{3}{5} \\
 \underline{55} \quad \underline{1800} \\
 522 \quad 1500 \\
 \underline{495} \quad \underline{1350} \\
 275 \quad *150 \\
 \underline{275}
 \end{array}$$

Ex. 3. Divide $43\frac{7}{11}$ by $\frac{4}{5}$ of $\frac{3}{4}$.

$$43\frac{7}{11}=43.63, \text{ and } \frac{4}{5} \text{ of } \frac{3}{4}=\frac{3}{5}=.6$$

$$.6)436.36, \\ 5 \quad 43.63,$$

$$48)3927.27, (81.81, =81\frac{9}{11} \\ 384 \dots$$

$$\begin{array}{r}
 87 \\
 48
 \end{array}$$

$$*392$$

RULE OF THREE DIRECT.

Decimal practice is frequently the shortest and easiest method of operation in the rule of three.

EXAMPLE I.

If C. 3 : 1 : 14 of raisins cost L. 10 : 2 : 6, what will 6 C. 3 Q. cost at that rate?

	C.	q.	lb.	L.	s.	d.	C.	q.
Vulgar state	3	1	14	10	2	6	6	3
Decimal state	3.375			10.125			6.75	

$$6.75$$

$$50625$$

$$70875$$

$$60750$$

$$\begin{array}{r}
 3.375)68.34375(20.25=20 \quad 5 \\
 \underline{6750} \dots
 \end{array}$$

$$8437$$

$$6750$$

$$16875$$

$$16875$$

E X-

EXAMPLE II.

If a wedge of gold, weighing 14 lb. 3 oz. 8 dw. cost L. 514, 4 s. what is that *per* ounce?

	lb.	oz.	dw.	L.	s.	oz.
Vulgar state	14	3	8	: 514	4	: : 1
Decimal state	14.283			: 514.2		: : .083

514.2

166

3333

8333

41666

14.283)42.8500(

1428 42.850

L.

12855)38565.0(3 Ans.

38565

EXAMPLE III.

If 1 yard cost 8 s. 4 d. what will 3482 $\frac{1}{2}$ cost?

	Y.	s.	d.	Y.
Vulgar state	1	: 8	4	: : 3482 $\frac{1}{2}$
Decimal state	1	: 416		: : 3482.3

41

375

375

174116

2437683

10447000

9|00)1305875.0

L.

s.

d.

1450.972 = 1450 19 5 $\frac{1}{2}$

EXAMPLE IV.

If 15 hogheads 3 firkins and 8 gallons of beer cost L. 24 : 13 : 4, what is that *per* hoghead, and *per* gallon?

Hbd.

	Hhd.	f.	g.	L.	s.	d.	Hhd.
Vulgar state	15	3	8	: 24	13	4	: : 1
Decimal state	15.6,481,			: 24.0666			: : 1
	156			24.			
	156325)			246426.0		54)	
				156325		108	

991016	496
781625	486
1193916	103
1094275	54
996416	497
937950	486

584666	11
468975	

Ans.	L.	s.	d.	
1.57637 = 1	11	6	$\frac{1}{2}$	per hhd.
.02919 =		7		per gallon.
				1156916
				1094275
				62641

RULE OF THREE INVERSE.

EXAMPLE I.

If I borrow L. 64 for 8 months, what sum lent for 12 months, or a year, will requite the favour?

	$\mathcal{L}$.	L.	$\mathcal{L}$.
Vulgar state	1	: 64	: : $\frac{8}{12}$
Decimal state	1	: 64	: : 0

	L.	s.	d.
9)384(42.0 = 42	13	4	
36.			

24
18

Or thus:

60
54

3)64(21.3
21.3

*6

The same 42.0 as before.

EXAMPLE II.

If 40 poles or roods in length and 4 in breadth make an acre, what

what must be the length to make an acre when the breadth is 5 poles 1 yard?

	P.	Y.	P.	P.
Vulgar state	5	1	: 40	:: 4
Decimal state	5.18,	:	40	:: 4

$$\begin{array}{r} 5.18,) 160.00(\\ \underline{5 } 160 \end{array}$$

$$\begin{array}{r} 513) 15840(30.877 = 30 \text{ } 4 \text{ } 2 \text{ } 5 \\ \underline{1539} \end{array}$$

4500

4104

3960

3591

3690

3591

99

COMPOUND RULE OF THREE.

EXAMPLE I.

What is the interest of L. 75 : 10 : 4 for 8 months, at the rate of 5 per cent. per annum?

	M.	L.	L.	L.	s.	d.	M.
Vulgar state	12	×	100	:	5	:	75 10 4
Decimal state	12	×	100	:	5	:	75.516

8

1200 : 5 :: 604.133

5

12|00)3020.666(

L. s. d.

2.5172 = 2 10 4

The simple separate operations of the same example follow.

L.	L.	L.
100	:	5
:	:	75.516
		5

1|00)377583(

3.77583

M.	L.	M.
12	:	3.77583
:	:	8

12)30.20666(

2.5172 Ans.

E X-

EXAMPLE II.

If $\frac{3}{4}$ acre of grafs be cut down by 2 men in $\frac{2}{3}$ day, how many acres shall be cut down by 6 men in $3\frac{1}{3}$ days?

$$\begin{array}{l} \text{D. M. A. M. D.} \\ \text{Vulgar state } \frac{3}{4} \times 2 : \frac{2}{3} :: 6 \times 3\frac{1}{3} \\ \text{Decimal state } .75 \times 2 : .75 :: 6 \times 3.3\bar{3} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 1.8 \\ 1 \end{array}) \begin{array}{r} 15.00 \\ 15. \end{array} \quad (\begin{array}{r} 20.0 \\ 20. \end{array} \\ \hline .12) 135.0 \quad (11.25 = 11 \quad \text{Acres. Rood.} \\ \hline 12 \\ \hline 15 \\ 12 \\ \hline 30 \\ 24 \\ \hline 60 \\ 60 \\ \hline \end{array}$$

EXAMPLE III.

If 120 men in 20 days drink 42 hogfheads, 3 firkins and 4 gallons of beer, how much will 80 men drink in 50 days at that rate?

$$\begin{array}{l} \text{D. M. H. f. g. M. D.} \\ \text{Vulgar state } 20 \times 120 : 42 \ 3 \ 4 :: 80 \times 50 \\ \text{Decimal state } 20 \times 120 : 42.5, 740, :: 80 \times 50 \end{array}$$

$$\begin{array}{r} \begin{array}{r} 24 \overline{) 00} \end{array} \begin{array}{r} 170296.296, \\ 168 \dots \end{array} \quad (70.95679 = 70 \quad \text{H. f. g. p.} \\ \hline 229 \\ 216 \\ \hline 136 \\ 120 \\ \hline 162 \\ 144 \\ \hline 189 \\ 168 \\ \hline 216 \\ 216 \\ \hline \end{array}$$

EXAMPLE IV.

If 4 men cast a ditch $8\frac{1}{2}$ feet long, $3\frac{1}{2}$ deep, and $2\frac{1}{2}$ broad, in $9\frac{1}{2}$ days; in how many days will 8 men cast a ditch $25\frac{1}{2}$ feet long, $6\frac{1}{2}$ deep, and $4\frac{1}{2}$ broad?

Vulgar state $b. \quad d. \quad l. \quad M. \quad D. \quad M. \quad l. \quad d. \quad b.$
 $2\frac{1}{2} \times 3\frac{1}{2} \times 8\frac{1}{2} \times 8 : 9\frac{1}{2} :: 4 \times 25\frac{1}{2} \times 6\frac{1}{2} \times 4\frac{1}{2}$

Decimal state $2.25 \times 3.5 \times 8.5 \times 8 : 9.5 :: 4 \times 25.5 \times 6.5 \times 4.5$

2.25	3060	4.5
166	560	333
666	280	2666
6666	28560	30.00
7.500		25.5
8.5		765.0
375		4
600		3060
63.75		
8		

510.00) 28560 (56 days. *Ans.*

2550

3060

3060

PART

P A R T III.

PRACTICAL ARITHMETIC; or, Arithmetic Vulgar and Decimal, reduced to PRACTICE, in a great variety of useful rules.

C H A P. I.

F E L L O W S H I P.

FELLOWSHIP, called also *Company*, or *Partnership*, is when two or more persons join their stocks, and trade together, dividing the gain or loss proportionally among the partners.

Fellowship is either without or with time, called also *single* or *double*.

I. *Fellowship without time.*

Questions in fellowship without time are wrought by the following proportion :

As the total stock,
To the total gain or loss ;
So each man's particular stock,
To his share of the gain or loss.

Quest. 1. A and B make a joint stock : A puts in 12 l. and B 8 l. ; they gain 5 l. : What is each man's share ?

	<i>L.</i>	<i>Stock. Gain. Stock.</i>
A's stock	12	A. If 20 : 5 :: 12
B's stock	8	5
Total stock	20	20)60
		A's gain 3 l.

Stock. Gain. Stock.
B. If 20 : 5 :: 8

	<i>L.</i>
8	
20)40	A's gain 3
B's gain 2 l.	B's gain 2
	Total gain 5 proof.

Quest. 2. A, B, and C, make a joint stock : A puts in 78 l. B 117 l. and C 234 l. ; they gain 265 l. : What is each man's share ?

Z 2

L.

Stock of $\begin{cases} A & 78 \\ B & 117 \\ C & 234 \end{cases}$

Total stock 429

Stock. Gain. Stock. *Stock. Gain. Stock.* *Stock. Gain. Stock.*
A. If 429 : 265 :: 78 B. If 429 : 265 :: 117 C. If 429 : 265 :: 234

78	117	234
2120	1855	1060
1855	265	795
429)20670(48 l.	429)31005(72 l.	429)62010(144 l.
1716	3003	429..
3510	975	1911
3432	858	1716
78	117	1950
20	20	1716
429)1560(3 s.	429)2340(5 s.	234
1287	2145	20
273	195	429)4680(10 s.
12	12	429
429)3276(7 d.	429)2340(5 d.	390
3003	2145	12
273	195	429)4680(10 d.
4	4	429
429)1092(2 f.	429)780(1 f.	390
858	429	4
(234)	(351)	429)1560(3 f.
		1287
		(273)

	L.	s.	d.	f.	Rem.
Anf. Gain of $\begin{cases} A \\ B \\ C \end{cases}$	48	3	7	2	234
	72	5	5	1	351
	144	10	10	3	273
Proof	265	1	0	0	858

Note 1.

Note 1. When in any question there happen to be remainders, they must be reduced equally low, so as to be all of one name; and then their sum will be either equal to the divisor, or exactly double, triple, &c. of it: and accordingly 1, 2, 3, &c. carried from the sum of the remainders, and added to the particular gains, will make up the total gain; or the divisor will always divide the sum of the remainders exactly, and the quot added to the particular gains will give the total gain.

Note 2. When the partners have equal shares of stock or capital, their shares of gain, loss, or neat proceeds, is found readily by dividing the total gain, loss, &c. by the number of partners; thus,

Suppose A, B, and C, were equally concerned in a voyage to Virginia, and that the neat proceeds turned out to 575 l. each partner's share will be L. 191 : 13 : 4, cast up as under :

$$\begin{array}{r} \text{L.} \quad \text{s.} \quad \text{d.} \\ 3 \overline{) 575} \quad 191 \quad 13 \quad 4 \end{array}$$

Note 3. When the fractions denoting the partners shares of stock or capital have all the same denominator, their shares of gain, loss, &c. is quickly found, by dividing the total gain, &c. by the denominator, and multiplying the quot by the respective numerators; thus,

Suppose A, B, C, and D, buy a ship for a sum of money, whereof A, B, and C, pay $\frac{1}{6}$ each, and D $\frac{1}{2}$ or $\frac{3}{6}$; and that her freight on a voyage amounts to 260 l. the partners shares are found as follows :

$$\begin{array}{r} 6 \overline{) 260} \\ \text{L. } 43 : 6 : 8 \text{ to A.} \\ \quad 43 : 6 : 8 \text{ to B.} \\ \quad 43 : 6 : 8 \text{ to C.} \\ \quad 130 : 0 : 0 \text{ to D.} \end{array}$$

Proof 260

Note 4. The operation may sometimes be shortened or facilitated by first finding the gain, loss, or neat proceeds, *per cent.* or *per pound*, and then working by the rules of practice; thus,

Suppose A, B, and C, join in an adventure to Barbadoes, which, with all charges, amounted to 2000 l.; whereof A paid 400 l. B 600 l. and C 1000 l.; they have returns in goods, which being disposed of, the neat proceeds amounted to L. 4333 : 6 : 8 : What should each partner get?

$$\begin{array}{r} \text{If } 2000 : 4333 \quad 6 \quad 8 :: 100 \\ 200 : 4333 \quad 6 \quad 8 :: 1 \\ 10 : 216 \frac{1}{2} \quad 13 \quad 4 :: 1 \end{array}$$

13 1/2

12

40

Z 3

216

216	13	4	per cent.		
				4	
400 =	866	13	4	A gets	866 13 4
200 =	433	6	8		
600 =	1300			B gets	1300
1000 =	2166	13	4	C gets	2166 13 4
				Proof	4333 6 8

Again, suppose A, B, and C, join in an adventure to Jamaica, and ship off, on their joint credit, goods to the value of 3000l.; and, to complete the cargo, the partners, from their own warehouses, ship off such goods as they had proper for the voyage, viz. A, goods to the value of 100l. B 200l. C 300l.; the neat proceeds of their returns in sugar and rum amounted to 6930l. What share of this belongs to each partner?

$3600 : 6930 :: 1$
 $360 : 693 :: 1$
 $120 : 231 :: 1$
 $40 : 77 :: 1$

$\frac{77}{40}$ per l. A.

Shares.
 L. s.

A's stock $1100 \times 77 = 84700, \div 40 = 2117 \quad 10$
 B's stock $1200 \times 77 = 92400, \div 40 = 2310$
 C's stock $1300 \times 77 = 100100, \div 40 = 2502 \quad 10$

Proof 6930

Decimally.

Quest. 3. A, B, and C, make a joint stock of 735 l. whereof

	L.	s.	d.	L.
A puts in	277	13	4	= 277. $\frac{1}{2}$
B ———	163	6	8	= 163. $\frac{2}{3}$
C ———	294			= 294.

They gain 49l.; required each man's share.

If $735 : 49 :: \left\{ \begin{array}{l} 277.0 : 18.51 \text{ A} \\ 163.3 : 10.88 \text{ B} \\ 294. : 19.6 \text{ C} \end{array} \right.$

49.00 proof.

But the easiest way is to find the gain or loss *per l.*; and by this multiply each man's stock. This common multiplier is found by the

the rule of three, or simply by dividing the total gain or loss by the sum of the stocks; thus :

L.	L.	L.	L.	
As 735	: 49	:: 1	: .06	common multiplier.
	L.	L.	L.	L. s. d.
And	{ 277.6	× .06	= 18.51	= 18 10 2.6
	{ 163.3	× .06	= 10.88	= 10 17 9.3
	{ 294.	× .06	= 19.6	= 19 12 0
			49.00	proof. 49 00 0.0

Again, suppose A put in 15l. B 24l. and C 33l. and gain 63l.; what is each man's share of said gain?

A 15l.	
B 24	
C 33	
72)63.0	(.875 common multiplier.
	L. s. d.
15 × .875	= 13.125 = 13 2 6 A's share.
24 × .875	= 21 = 21 0 0 B's share.
33 × .875	= 28.875 = 28 17 6 C's share.
	63 proof.

MORE EXAMPLES.

Quest. 4. A, B, and C, make a joint stock: A puts in 460l. B 510l. and C 480l.; they gain 340l.: What is each partner's share?

	L.	s.	d.	f.	Rem.
Ans. Gain of	{ A 107	17	2	3	85
	{ B 119	11	8	2	110
	{ C 112	11	0	1	95
	Proof	340	00	0	0 290

Quest. 5. Three persons trade together: A puts in 230l. B 529l. and C 344l. 10s.; they gain 520l.: What is that to each?

	L.	s.	d.	Rem.
Ans. Gain of	{ A 108	7	7½	271
	{ B 249	5	7	844
	{ C 162	6	9	1092
	Proof	520	0	0 2207

Quest. 6. A, B, and C, make a joint stock of 3256l.; whereof A puts in 1026l. B 985l. and C the rest; by misfortune they lose 2000l.: What part of that must each bear?

Z 4

L.

	L.	s.	d.	Rem.
<i>Ans.</i> {	A 630	4	5	928
	B 605	0	8½	1240
	C 764	14	10	1088
	Proof 2000	0	0	3256

Quest. 7. Four partners, A, B, C, and D, built a ship, which cost 1730l.; and the freight for her first voyage amounted to 370l.; of which A's share was 74l. B's 111l. C's 148l. and D's 37l.: What was each partner's stock?

	L.
<i>Ans.</i> {	A's stock was 346
	B's - - 519
	C's - - 692
	D's - - 173

Total stock 1730 proof.

Quest. 8. A, B, and C, in company, gain 36l.: A put in 20l. B 30l. C a sum unknown; C took up 16l. of the gain: What did A and B gain? and what did C put in?

	L.
<i>Ans.</i> {	A's gain 8
	B's gain 12
	C's stock 40

Quest. 9. A, B, C, and D, in partnership, had a joint stock of 600l. and gained a certain sum; of which A, B, and C, took up 60l.; B, C, and D, 90l.; A, C, and D, 80l.; A, B, and D, 70l.: What was the stock and gain of each partner?

	Stock.	Gain.
	L.	L.
<i>Ans.</i> {	A's 60	10
	B's 120	20
	C's 180	30
	D's 240	40

II. Fellowship with time.

In fellowship with time, the gain or loss is divided among the partners, both in proportion to the stocks themselves, and also in proportion to the times of their continuance in company: for the same stock, continued a double time, procures a double share of gain; and continued a triple time, procures a triple share of gain; that is, the shares of gain or loss are as the products of the several stocks multiplied into their respective times: and accordingly questions belonging to this rule are wrought by the following proportion.

As

As the sum of the products of the several stocks into their respective times,
 To the total gain or loss;
 So the product of each man's stock into his time,
 To his share of the gain or loss,

Quest. 1. A put into company 40 l. for three months, B 75 l. for 4 months; they gain 70 l.: What share must each man have?

A $40 \times 3 = 120$ third term for A's share.

B $75 \times 4 = 300$ third term for B's share.

420 first term.

L.		L.		L.	
A. If	420 : 70 :: 120	B. If	420 : 70 :: 300		
	120		300		
42 0	840 0(20 l.	42 0	2100 0(50 l.	A's gain	20
	84		210	B's gain	50
	(0)		(0)	Tot. gain	70 pr.

Quest. 2. A put into company 560 l. for 8 months, B 279 l. for 10 months, and C 735 l. for 6 months; they gained 1000 l.: What share of the gain must each have?

A $560 \times 8 = 4480$ third term for A's share.

B $279 \times 10 = 2790$ third term for B's share.

C $735 \times 6 = 4410$ third term for C's share.

11680 first term.

	L.	s.	d.	f.	Rem.
A. If	11680 : 1000 :: 4480 : 383	11	2	3	208
B. If	11680 : 1000 :: 2790 : 238	17	4	3	80
C. If	11680 : 1000 :: 4410 : 377	11	4	1	880

Proof 1000 00 0 0 1168

Decimally.

Quest. 3. A, B, and C, enter into partnership, viz.

	L.	s.	d.	
A puts in	58	10	0	for 8 months 3 weeks.
B ———	65	15	0	for 6 months 4 days.
C ———	48	17	6	for 12 months 2 weeks.

They gain 125 l. 14 s.: What is each man's share?

L.

	L.	s.	d.	Rem.
<i>Ans.</i> {	A 630	4	5	928
	B 605	0	8½	1240
	C 764	14	10	1088
	Proof 2000	0	0	3256

Quest. 7. Four partners, A, B, C, and D, built a ship, which cost 1730l.; and the freight for her first voyage amounted to 370l.; of which A's share was 74l. B's 111l. C's 148l. and D's 37l.: What was each partner's stock?

	L.
<i>Ans.</i> {	A's stock was 346
	B's - - 519
	C's - - 692
	D's - - 173

Total stock 1730 proof.

Quest. 8. A, B, and C, in company, gain 36l.: A put in 20l. B 30l. C a sum unknown; C took up 16l. of the gain: What did A and B gain? and what did C put in?

	L.
<i>Ans.</i> {	A's gain 8
	B's gain 12
	C's stock 40

Quest. 9. A, B, C, and D, in partnership, had a joint stock of 600l. and gained a certain sum; of which A, B, and C, took up 60l.; B, C, and D, 90l.; A, C, and D, 80l.; A, B, and D, 70l.: What was the stock and gain of each partner?

	Stock.	Gain.
	L.	L.
<i>Ans.</i> {	A's 60	10
	B's 120	20
	C's 180	30
	D's 240	40

II. Fellowship with time.

In fellowship with time, the gain or loss is divided among the partners, both in proportion to the stocks themselves, and also in proportion to the times of their continuance in company: for the same stock, continued a double time, procures a double share of gain; and continued a triple time, procures a triple share of gain; that is, the shares of gain or loss are as the products of the several stocks multiplied into their respective times: and accordingly questions belonging to this rule are wrought by the following proportion.

As

As the sum of the products of the several stocks into their respective times,
To the total gain or loss;
So the product of each man's stock into his time,
To his share of the gain or loss,

Quest. 1. A put into company 40 l. for three months, B 75 l. for 4 months; they gain 70 l.: What share must each man have?

A $40 \times 3 = 120$ third term for A's share.

B $75 \times 4 = 300$ third term for B's share.

420 first term.

L.	L.	L.
A. If 420 : 70 :: 120	B. If 420 : 70 :: 300	
120	300	
42 0)840 0(20 l.	42 0)2100 0(50 l.	A's gain 20
84	210	B's gain 50
(0)	(0)	Tot. gain 70 pr.

Quest. 2. A put into company 560 l. for 8 months, B 279 l. for 10 months, and C 735 l. for 6 months; they gained 1000 l.: What share of the gain must each have?

A $560 \times 8 = 4480$ third term for A's share.

B $279 \times 10 = 2790$ third term for B's share.

C $735 \times 6 = 4410$ third term for C's share.

11680 first term.

	L.	s.	d.	f.	Rem.
A. If 11680 : 1000 :: 4480 :	383	11	2	3	208
B. If 11680 : 1000 :: 2790 :	238	17	4	3	80
C. If 11680 : 1000 :: 4410 :	377	11	4	1	880

Proof 1000 00 0 0 1168

Decimally.

Quest. 3. A, B, and C, enter into partnership, viz.

	L.	s.	d.	
A puts in 58	10	0	0	for 8 months 3 weeks.
B ——— 65	15	0	0	for 6 months 4 days.
C ——— 48	17	6	0	for 12 months 2 weeks.

They gain 125 l. 14 s.: What is each man's share?

L.

	<i>L.</i>	<i>Mon.</i>	<i>Prod.</i>
A's stock	58.5	8.75	= 511.875
B's stock	65.75	6.1428	= 403.8891
C's stock	48.875	12.5	= 610.9375

Sum of the products 1526.7016

	<i>L.</i>
As 1526.7016 : 125.7 ::	$\left\{ \begin{array}{l} 511.875 : 42.1449 \text{ A} \\ 403.8891 : 33.2539 \text{ B} \\ 610.9375 : 50.3011 \text{ C} \end{array} \right.$

Total gain nearly 125.6999 proof.

But here too the easiest method is to find the gain or loss *per l.* and then make this a common multiplier of the several products of the stocks and times. Thus,

	<i>L.</i>	<i>L.</i>
As 1526.7016 : 125.7 ::	1	.08233 common multiplier.
A 511.875	B 403.8891	C 610.9375
Multiplier 33280. inverted	33280.	33280.

409500
10237
1535
153

323111
8077
1211
121

488750
12218
1832
183

42.1425

33.2520

50.2983

The shares of the gain are nearly the same as before.

MORE EXAMPLES.

Quest. 4. A, B, and C, hire a pasture for 24 l.: A puts in 40 cows for 4 months, B 30 cows for 2 months, and C 36 cows for 5 months: What share of the rent must each pay?

	<i>L.</i>	<i>s.</i>
<i>Ans.</i> { A's share of rent	9	12
{ B's share	3	12
{ C's share	10	16

Proof 24

Quest. 5. A, B, and C, agreeing to trade together, A puts in 529 l. for 4 months, B 329 l. for 7 months, and C 900 l. for 2 months; they gain 540 l.: What is each man's share?

	<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>Rem.</i>
<i>Ans.</i> { A	183	14	8	2304
{ B	199	19	5	1332
{ C	156	5	10 $\frac{3}{4}$	2583

Proof 540 0 00 6219

Ques^t. 6. A and B enter into partnership for a year; accordingly A put in the first of January 50l.; but B could not put any money in till the first of May: What sum must B then put in to be intitled to an equal share of the gain at the year's end? *Ans.* 75l.

Quest. 7. A, B, and C, agree to trade in company for 12 months : A put in at first 364l. and at the end of 4 months he put in 40l. more; B put in at first 408l. and at the end of 7 months he took out 86l. ; C put in at first 148l. and at the end of 3 months he put in 86l. more, and at the end of other 5 months he put in 100l. more; at the year's end their gain was 1436l. : What is each man's share ?

		<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>Rem.</i>
<i>Ans.</i> {	A's gain	556	3	6	6192
	B's gain	529	16	9	5496
	C's gain	349	19	8	416

Proof 1436 o o 12104

Note. Questions in double fellowship are proper exercises for the learner, but seldom occur in real business; differences in point of time being usually adjusted by an interest account.

Q C H A P. II.
F A C T O R A G E.

FACTORS are agents or doers, residing abroad, or beyond seas, who act for their employers; and for their trouble are generally allowed, in name of commission or fees, so much *per cent.* The computations they have occasion for are such as occur in the solution of the following or like questions.

Quest. 1. A B's sales *per* the Swan amount to 675l. 18s. 8d.: What is the commission at 8 *per cent.*?

	L.	s.	d.		Decimally.	
If 100 :	8 ::	675	18	8	If 100 :	8 :: 675.93
				8		8
L.	54.07	9	4		L.	54.0746
	20					20
s.	1.49				s.	1.493
	12					12
d.	5.92				d.	5.920
	4					4
f.	3.68				f.	3.68

Quest. 2.

Quest. 2. A factor buys, and ships off to his employer, goods to the value of L. 1334 : 11 : 5½ : What is the commission at 5 per cent.?

Decimally.

L.	s.	d.	L.
If 100 : 5 :: 1334	11	5½	If 100 : 5 :: 1334.571875
		5	5
		—	—
L. 66.72	17	2½	L. 66.72859375
	20		20
	—		—
s. 14.57			s. 14.57187500
	12		12
	—		—
d. 6.86			d. 6.862500
	4		4
	—		—
f. 3.45			f. 3.4500

Quest. 3. The prime cost of a parcel of consigned goods, per invoice, is L. 37 : 8 : 1½; these the factor sells at 75 per cent. advance on the invoice: What is their value in the sale?

Decimally.

L.	s.	d.	L.
If 100 : 75 :: 37	8	1½	If 100 : 175 :: 37.40625
		20	175
		—	—
		748	18703125
		12	26184375
		—	—
		8977	3740625
		4	—
		—	L. 65.4609375
		—	20
		35910	—
		75	s. 9.2187500
		—	12
		17955	—
		25137	d. 2.62500
		—	4
		4)26932.50	—
		—	f. 2.500
		12)6733	—
		—	—
		20)5611	—
		28	1 advance.
		37	8 1½ prime cost.
		—	—
		65	9 2½ value in the sale.

Quest. 4.

Quest. 4. The neat proceeds of a sale is 620l. 12s. 6½d.: To what value ought the factor to make returns, so as to clear the debt, and have 5 per cent. commission on the returns?

			<i>Decimally.</i>		
L.	s.	d.	L.		
If 105 : 100 :: 620 12 6½			If 105 : 100 :: 620.627083		
20					
12412			105)62062.7083(591.0734		
12			525	20	
148950			956	s. 1.4680	
4			945	12	
595802			112	d. 5.616	
100			105	4	
4)			770	f. 2.464	
105)59580200(567430			735		
525			358		
12)141857	2 f.		315		
708					
630	2)0)11821	5 d.			
780	591	1 s.	433		
735			420		
			(13)		
452					
420					
320					
315					
(50)					

	L.	s.	d.
Value of returns	591	1	5½
Commission	29	11	1
Proof	620	12	6½

Quest. 5. A factor buys for his employer goods to the value of L. 7864 : 16 : 8: What will his commission come to at 2½ per cent.?

$2\frac{1}{2}$ per cent. is $\frac{1}{40}$ of 100; and therefore,

By Practice.

L. s. d. L. s. d.
4|0)786|4 16 8(196 12 5 *Ans.*

Rem. 24

20

49|6(

Rem. 16

12

20|0(

Decimally.

L.

If 100 : 2.5 :: 7864.83

2.5

3932416

15729666

L. 196.62083

20

s. 12.4166

12

d. 5.000

Quest. 6. If I grant a draught on London for 4835 l. and be allowed $1\frac{1}{8}$ per cent. in name of premium or commission, what will that amount to?

L.

1 per cent. = $\frac{1}{100}$ = 48.35

$\frac{2}{8} = \frac{1}{4}$ of that = 12.0875

$\frac{1}{8} = \frac{1}{8}$ of that = 6.04375

L. 66.48125

20

s. 9.6250

12

d. 7.500

Ans. £68-9-7½

Ans. L. 66 : 9 : 7½.

Some, instead of paying a premium, chuse to take the bill payable so many days after date or sight; and in this way, 73 days are reckoned equal to 1 per cent.

Quest. 7. A factor ships off, by his employer's order, goods to the value of L. 97 : 5 : 8. What will his commission at 2 per cent. amount to?

2 per

2 per cent. $= \frac{1}{50}$ of 100.

By Practice.

L. s. d. L. s. d.
50)917 5 8(1 18 10 $\frac{1}{4}$ Ans.

Rem. 47

20

94|5(

Rem. 45

12

54|8(

Rem. 48

4

19|2(

(42)

Decimally.

L.
If 100 : 2 :: 97.283

2

L. 1.94566

20

s. 18.913

12

d. 10.960

4

f. 3.84

MORE EXAMPLES.

Quest. 8. A factor ships off for his employer goods to the value of L. 1629 : 4 : 3 $\frac{1}{2}$: What will his commission at 5 per cent. amount to? *Ans.* L. 81 : 9 : 2 $\frac{1}{2}$.

Quest. 9. A factor owes his employer L. 262 : 11 : 7 $\frac{1}{2}$: How much ought he to remit, in order to clear the debt, and have 2 $\frac{1}{2}$ per cent. commission on the sum remitted? *Ans.* L. 256 : 3 : 6 $\frac{1}{4}$ nearly.

Quest. 10. A factor negotiates a bill of L. 76 : 10 : 9 : What commission may he charge on this head at 1 $\frac{1}{2}$ per cent.? *Ans.* L. 1, 2s. 11 $\frac{1}{2}$ d.

CHAP. III.

TARE and TRET, &c.

Gross weight is the weight both of the commodity, and of that which contains it, whether cask, hoghead, barrel, bag, sack, or wrapper.

Tare is an allowance on weighable goods, made by the King to the importer, or by the seller to the buyer, in consideration of the outside package; such as, cask, bag, box, chest, wrappers, &c.

Tret is an allowance of 4 lb. on 104 lb. on garbled goods, or such as are sold by the pound-weight, granted for break, waste, or dust mixed with the goods. *gross*

Cliff

$2\frac{1}{2}$ per cent. is $\frac{1}{40}$ of 100; and therefore,

By Practice.

Decimally.

L. s. d. L. s. d.
40)7864 16 8(196 12 5 Ans.

If 100 : 2.5 :: 7864.83
2.5

Rem. 24

20

49)60

Rem. 16

12

20)00

3932418
15729686

L. 196.62083

20

s. 12.4168

12

d. 5.000

Quest. 6. If I grant a draught on London for 4835 l. and be allowed $1\frac{1}{8}$ per cent. in name of premium or commission, what will that amount to?

L.
 1 per cent. = $\frac{1}{100}$ = 48.35
 $\frac{1}{8}$ = $\frac{1}{8}$ of that = 12.0875
 $\frac{1}{8}$ = $\frac{1}{8}$ of that = 6.04375

L. 60.48125
20

s. 9.6250
12

d. 7.500

Ans. £68-9-7½

Ans. L. 68 : 9 : 7½.

Some, instead of paying a premium, chuse to take the bill payable so many days after date or sight; and in this way, 73 days are reckoned equal to 1 per cent.

Quest. 7. A factor ships off, by his employer's order, goods to the value of L. 97 : 5 : 8. What will his commission at 2 per cent. amount to?

2 per

2 per cent. $= \frac{1}{50}$ of 100.

By Practice.

Decimally.

L.	s.	d.	L.	s.	d.	L.
510	9	7	5	8	1	18
10 $\frac{3}{4}$ Ans.						If 100 : 2 :: 97.283
						2
Rem. 47						L. 1.94566
20						20
94						s. 18.913
5						12
Rem. 45						d. 10.960
12						4
54						f. 3.84
8						
Rem. 48						
4						
19						
2						
(42)						

MORE EXAMPLES.

Quest. 8. A factor ships off for his employer goods to the value of L. 1629 : 4 : $3\frac{1}{2}$: What will his commission at 5 per cent. amount to? *Ans.* L. 81 : 9 : $2\frac{1}{2}$.

Quest. 9. A factor owes his employer L. 262 : 11 : $7\frac{3}{4}$: How much ought he to remit, in order to clear the debt, and have $2\frac{1}{2}$ per cent. commission on the sum remitted? *Ans.* L. 256 : 3 : $6\frac{1}{4}$ nearly.

Quest. 10. A factor negotiates a bill of L. 76 : 10 : 9 : What commission may he charge on this head at $1\frac{1}{2}$ per cent.? *Ans.* L. 1, 2s. 11 $\frac{1}{2}$ d.

CHAP. III.

TARE and TRET, &c.

Gross weight is the weight both of the commodity, and of that which contains it, whether cask, hoghead, barrel, bag, sack, or wrapper.

Tare is an allowance on weighable goods, made by the King to the importer, or by the seller to the buyer, in consideration of the outside package; such as, cask, bag, box, chest, wrappers, &c.

Tret is an allowance of 4 lb. on 104 lb. on garbled goods, or such as are sold by the pound-weight, granted for break, waste, or dust mixed with the goods. *gross*

Cliff

Cloff or *clough*, called also *draught*, is another small allowance on some weighable goods, usually 2 lb. on every 3 C. to turn the scale, or make the weight hold out, when the goods are re-weighed. This is claimed chiefly, or only, by the citizens of London.

Suttle or *futtle-weight* is what remains after the tare is deduced from the gross: and is so called only when tret is allowed; for when there is no tret, there is no *suttle*.

Neat weight is the weight of the goods after all given allowances are deduced. Thus, if tare, tret, and *cloff*, be allowed, the neat weight is what remains after these three allowances are deduced. If tare and tret only be allowed, the neat weight is what remains after these two allowances are deduced. If tare only be allowed, the neat weight is what remains after that allowance is deduced.

I. Tare.

Tare admits of three varieties: for sometimes the hoghead, cask, or barrel, is weighed before the goods are packed, and the tare inserted in the invoice along with the gross weight; or the tare is so much on the whole. Again, the allowance for tare is sometimes so much *per* frail, *per* firkin, *per* hoghead, *per* chest, *per* bag, &c. Lastly, the allowance for tare is sometimes so much *per* C.

1. When the tare is inserted in the invoice, or is so much on the whole, the neat weight is found by subtracting the tare from the gross.

E X A M P L E I.

Suppose a factor in Jamaica buys for the use of his employer in Britain, the three following hogheads of sugar, their neat weight in pounds, that being the usual form, is computed as under.

N ^o	C.	q.	lb.	Tare in pounds.
1.	16	1	14	114
2.	14	3	7	110
3.	15	2	21	116

46	3	14	—	340
46				
46				
4698				

5250 gross.
340 tare.

lb. 4910 neat.

E X A M P L E II.

A grocer buys 54 chests of spice, each chest weighing 4 C. 2 Q. 14 lb. gross; tare on the whole, 21 C. 3 Q. 18 lb.; Required the neat weight.

G.

C. 2 lb.

4 2 14

9

$$54 = 9 \times 6.$$

41 2 14

6

249 3 0 total gross.

21 3 118 tare.

Ans. 227 3 10 neat.

2. When the tare is so much *per* frail, *per* firkin, *per* chest, *per* bag, &c. multiply the allowance for tare by the number of frails, firkins, chests, or bags; subtract the product from the gross; and the remainder is the neat weight.

Or, from the gross weight of 1 frail, firkin, hoghead, &c. subtract its tare; the remainder is the neat weight of 1 frail, firkin, &c.; which, multiplied by the number of frails, &c. gives the total neat weight.

EXAMPLE I.

What is the neat weight of 7 frails of raisins, each weighing 3 C. 3 Q. 10 lb. gross, tare at 24 lb. *per* frail?

C. 2 lb.

3 3 10

7 frails.

lb.

24

Frails 7

26 3 14 gross.

1 2 0 tare.

$$168 = 1 \frac{1}{2}$$

Ans. 25 1 14 neat.

Or,

C. 2 lb.

3 3 10

gross of 1 frail.

24 tare.

3 2 14 neat of 1 frail.

7

25 1 14 neat of 7 frails.

EXAMPLE II.

What is the neat weight of 14 bags of cotton, each weighing 2 C. 2 Q. 7 lb. tare, at 9 lb. *per* bag?

A a

C.

	C. $\mathcal{Q}$. lb.	Or,	C. $\mathcal{Q}$. lb.
	2 2 7		20 2 7
14 = 7 x 2	7		gross of 1 bag.
			9 tare.
Bags.	17 3 21		20 1 26
14	2		neat of 1 bag.
9			7
	35 3 14		17 11 14
126	35		2
	35		
	3598		34 3 00
	4018		neat of 14 bags.
	126		34
			34
			3484
	lb. 3892		neat.

Note. The tare for several sorts of goods is ascertained in the book of rates. Thus, for every bale of silk from Smyrna or Cyprus, weighing 300 lb. or upwards, the tare is fixed at 16 lb.; for every bale weighing 200 lb. or upwards to 300, the tare is 14 lb.; and for every bale under 200 lb. the tare is 12 lb.

Accordingly the neat weight of the five following bales of silk from Smyrna, will be made out from the book of rates, as under.

N ^o	lb.	Tare.
1.	346	16
2.	300	16
3.	284	14
4.	200	14
5.	158	12
Gross	1288	72
Tare	72	
Neat	1216	

3. When the tare is so much *per C.* work by one or other of the two rules following.

R U L E I.

If the tare be 14 or 16 *per C.* take accordingly $\frac{1}{8}$ or $\frac{1}{7}$ of the gross for the tare; which, subtracted from the gross, leaves the neat weight.

EXAMPLE I.

<i>lb.</i>		Tare 14 <i>per C.</i>	
		<i>C. ℥. lb.</i>	
<i>lb.</i>		493	2 24 gross.
14	$\frac{1}{8}$	61	2 24 tare.
		432	0 0 neat.

EXAMPLE II.

<i>lb.</i>		Tare 16 <i>per C.</i>	
		<i>C. ℥. lb.</i>	
<i>lb.</i>		119	3 0 gross.
16	$\frac{1}{7}$	17	0 12 tare.
		102	2 16 neat.

R U L E II.

If the tare be any other than 14 or 16 *per C.* first work as above for 14 or 16, and then take aliquot parts of the quot.

EXAMPLE I.

<i>lb.</i>		Tare 7 <i>per C.</i>	
		<i>C. ℥. lb.</i>	
<i>lb.</i>		57	2 24 gross.
14	$\frac{1}{8}$	7	0 24
7	$\frac{1}{2}$ of $\frac{1}{8}$	3	2 12 tare.
		54	0 12 neat.

EXAMPLE II.

<i>lb.</i>		Tare 17 <i>per C.</i>	
		<i>C. ℥. lb.</i>	
<i>lb.</i>		573	2 13 gross.
14	$\frac{1}{8}$	71	2 22 $\frac{1}{2}$
2	$\frac{1}{7}$ of $\frac{1}{8}$	10	0 27
1	$\frac{1}{2}$ of $\frac{1}{7}$ of $\frac{1}{8}$	5	0 13 $\frac{1}{2}$
		87	0 7 tare.
		486	2 6 neat.

EXAMPLE III.

<i>lb.</i>		Tare 21 <i>per C.</i>	
		<i>C. ℥. lb.</i>	
<i>lb.</i>		44	2 12 gross.
14	$\frac{1}{8}$	5	2 8 $\frac{1}{2}$
7	$\frac{1}{2}$ of $\frac{1}{8}$	2	3 4 $\frac{1}{4}$
		8	1 12 $\frac{3}{4}$ tare.
		36	0 27 $\frac{1}{4}$ neat.

In the examples following, the first part of the compound fraction only is expressed, the other part or parts, for brevity's sake, being omitted.

EXAMPLE IV.

Tare 6lb. per C.

		C.	Q.	lb.	
		170	2	14	grofs.
lb.					
16	$\frac{1}{4}$	24	1	14	
8	$\frac{1}{2}$	12	0	21	
2	$\frac{1}{4}$	3	0	$5\frac{1}{4}$	
6		9	0	$15\frac{1}{4}$	tare.
		161	1	$26\frac{1}{4}$	neat.

EXAMPLE V.

Tare 10lb. per C.

		C.	Q.	lb.	
		550	1	0	grofs.
lb.					
16	$\frac{1}{4}$	78	2	12	
8	$\frac{1}{2}$	39	1	6	
2	$\frac{1}{4}$	9	3	$8\frac{1}{4}$	
10		49	0	$14\frac{1}{2}$	tare.
		501	0	$13\frac{1}{2}$	neat.

EXAMPLE VI.

Tare 12lb. per C.

		C.	Q.	lb.	
		237	1	14	grofs.
lb.					
16	$\frac{1}{4}$	33	3	18	
8	$\frac{1}{2}$	16	3	23	
4	$\frac{1}{2}$	8	1	$25\frac{1}{2}$	
12		25	1	$20\frac{1}{2}$	tare.
		211	3	$21\frac{1}{2}$	neat.

Note 1. That 14 and 16 pounds per C. may be considered as the standards of tare, in regard tare at any other rate may easily be computed from them, as is done in the above examples.

Note 2. It is not usual in computing tare to take notice of any thing lower, or less, than $\frac{1}{4}$ lb.; and accordingly, in Ex. 2. some small fractions are neglected or thrown away.

Note 3. Though the method of computing tare, explained above, be the more usual way; yet some chuse to multiply the pounds gros by the rate, or allowance of tare, and dividing the product by 112, the quot gives the tare in pounds. Ex. 1. wrought in this manner follows.

C.	Q.	lb.		lb.
57	2	24	grofs.	Tare 7 per C.
57				
57				
5780				
6464				
7	lb.	C.	Q.	lb.
112	45248	(404 = 3	2	12 as before.
448				
448				
448				

Others

Others multiply the C's by the rate of tare, and for the quarter and pounds take proportional parts, thus.

Tare 7 lb. per C.

C. $\frac{2}{4}$ lb.

57 2 24 gros.

7

Tare of 57 C. is

399

— of 2 Q. is

$3\frac{1}{2}$

— of 16 lb. is

1

— of 8 lb. is

$0\frac{1}{2}$

C. $\frac{2}{4}$ lb.

Total tare

404 = 3 2 12 as before.

Note 4. The tare for oil in uncertain casks is 18 lb. per C. but pays duty by the gallon, and the number of gallons is computed from the neat weight, by allowing $7\frac{1}{2}$ lb. to a gallon; that is, multiply the neat pounds by 2, the product divided by 15, quotes gallons; or, divide the product by 3, and that quot by 5.

E X A M P L E.

Tare 18 lb. per C.

C. $\frac{2}{4}$ lb.

128 2 0 oil gros.

1536

56

14392 lb. gros.

lb.

16

$\frac{1}{4}$

2056

2

$\frac{1}{8}$

257

2313 lb. tare.

12079 lb. neat.

2

24158 half-pounds.

$\frac{1}{15}$

1610 $\frac{1}{2}$ gallons.

In working decimally, the tare may be found by multiplying the gros weight by the number of pounds tare in one C. or by the decimal of the tare; and this product subtracted from the gros weight leaves the neat. But it is easier and more usual to work by one or other of the three methods following.

1. Multiply the gros weight by the neat pounds in one C. and the product is the pounds neat.

A a 3

Tare

Tare 17 lb. per C.

C. 2 lb.

Ex. 573 2 13 gros.

C.

573.616 gros.

95 lb. neat in 1 C.

2868080

5162544

C. 2 lb.

112)54493.520(486 2 5 $\frac{1}{2}$

448

969

896

733

672

(61)

2. Multiply the gross weight by the decimal of the neat pounds in 1 C. and the product is neat C's.

lb. C.
95 = .8482

C.
Ex. 573.616 gros.
2848. multiplier inverted.

4588928

229446

45889

1147

486.5410 neat weight nearly,

3. Work by aliquot parts, as in Practice.

Tare 17 lb. per C.

C.

Ex. 573.616 gros.

lb.

14 = $\frac{1}{8}$ of the gros = 71.7022 = $\frac{1}{4}$ of the former = 10.2431 = $\frac{1}{2}$ of the former = 5.121

17

87.066 tare.

486.550 neat weight nearly.

In

In working by aliquot parts, it will be convenient to have at hand a table of divisors, such as the following; which is constructed in the same manner as the table of rates and divisors in decimal practice.

<i>lb.</i> <i>Tare.</i>	<i>Divisors.</i>	<i>lb.</i> <i>Tare.</i>	<i>Divisors.</i>	<i>lb.</i> <i>Tare.</i>	<i>Divisors.</i>
1	2, 7, 8	10	2, 7, + 4	19	7, + 4, - 20
2	7, 8	11	8, 7, - 12	20	7, + 4
3	7, 8, + 2	12	8, - 7	21	8, + 2
4	4, 7	13	8, - 7, + 12	22	7, + 4, + 2
5	4, 7, + 4	14	8	23	7, + 2, - 8
6	4, 7, + 2	15	8, + 7, - 2	24	7, + 2
7	2, 8	16	7	25	7, + 2, + 8
8	2, 7	17	8, + 7, + 2	26	7, + 2, + 4
9	2, 7, + 8	18	7, + 8	27	7, + 2, + 4, + 2

II. *Tret.*

Tret, being always 4 lb. per 104 lb. futtle, is found by taking $\frac{1}{16}$ of the futtle-weight.

EXAMPLE I.

Tret 4 lb. per 104 lb.

C. 2 lb.
 $\frac{1}{16}$ | 428 I 19 futtle.
 16 I 25 $\frac{1}{2}$ tret.
 —
 411 3 21 $\frac{1}{2}$ neat.

C. 2 lb. C. 2 lb.
 26)428 I 19(16 I 25 $\frac{1}{2}$

26

—
 168

156

—
 12 rem.

4

—
 26)49(1

26

—
 23 rem.

28

—
 Carried up, 193

Brought up, 193

47

—
 26)663(25 $\frac{1}{2}$

52

—
 143

130

—
 (13)

A a 4

E X

EXAMPLE II.

Tare 22 lb. per C.

Tret 4 lb. per 104 lb.

C. 2 lb.

lb.					
		836	2	17	gross.
16	$\frac{1}{4}$	119	2	$2\frac{1}{4}$	
4	$\frac{1}{4}$	29	3	$14\frac{1}{2}$	
2	$\frac{1}{2}$	14	3	$21\frac{1}{2}$	
—		—	—	—	
22		164	1	10	tare.
		—	—	—	
	$\frac{1}{32}$	672	1	7	futtle.
		25	3	12	tret.
		—	—	—	
		646	1	23	neat.

Note. The neat weight may also be had by annexing two ciphers to the futtle pounds, and then dividing by 104. Ex. 1. wrought in this manner follows :

Tret 4 lb. per 104 lb.

C. 2 lb.

428 1 19 futtle.

5136

47

104)4798300(46137 $\frac{1}{2}$ = 411 3 21 $\frac{1}{2}$ as before,
416.....

638

624

143

104

390

312

780

728

(52)

In working decimally, multiply the futtle by $.0385 = \frac{4}{104}$, and the product is the tret ; which deduced from the futtle, leaves the neat.

But

But it is shorter to multiply the futtle by $.9615 = \frac{100}{104}$, and the product is the neat. Ex. 2, done in this manner follows :

Tare 22 lb. per C.

Tret 4 lb. per 104 lb.

C. 2 lb. C.
836 2 17 = 836.6517 gros.

7 + 4 + 2 { 119.5216
29.8804
14.9402

164.3422 tare.

672.3095 futtle.

5169 multiplier inverted.

60507855

4033857

67230

33615

646.42557 neat weight nearly.

III. Cloff.

When tret is allowed, the remainder, after deducing the tare from the gros, is called *puttle* ; and when cloff is allowed, the remainder of the *puttle*-weight, after deducing the tret, is again called *puttle*, or rather *subputtle*.

Cloff, being always 2 lb. on every 3 C. subputtle, is found by taking $\frac{1}{15}$ of the subputtle weight.

Or, divide the subputtle C.'s by 3, the quot is cloff in double pounds ; these divided by 14 quote quarters ; and the remainder is double pounds.

EXAMPLE I.

Cloff 2 lb. per 3 C.

C. 2 lb.
4033 2 14 subputtle.
24 0 1 cloff.

4009 2 13 neat.

Method 1.

Method 1.

168)4033(24 C.

336.

673

672

1

4

168)6(0 Q.

28

168)182(1 lb.

168

(14)

Method 2.

3)4033(1

4) C. Q. lb.

14)1344)96(24 0 1

126.

84

84

Note. The 1 C. remaining in the first division, and the 2 Q. in the subsuttle, afford the 1 lb. cloff.

E X A M P L E II.

C. Q. lb.

32 3 20 gros.

4 0 13 tare,

28 3 7 futtle.

1 0 12 tret.

27 2 23 subsuttle.

18 $\frac{1}{4}$ cloff.27 2 4 $\frac{3}{4}$ neat.

lb.

Tare 14 per C.

Tret 4 per 104.

Cloff 2 per 3 C.

lb.

Note. The 27 C. afford 18 cloff.

And the 2 Q. afford 0 $\frac{1}{4}$ 18 $\frac{1}{4}$

The cloff may be found decimally by multiplying the subsuttle by .00595 = $\frac{1}{168}$.

Any operation in tare and tret may be proved by working the same question by the rule of three.

P R A C T I C A L Q U E S T I O N S.

1. What is the neat weight of 346 C. 3 Q. 12 lb. gros, tare 12 lb. per C.? *Ans.* 309 C. 2 Q. 21 $\frac{1}{2}$ lb. neat.

2. What is the neat weight of 139 C. 1 Q. 22 lb. gros, tare 8 lb. per C. tret 4 lb. per 104, and cloff 2 lb. per 3 C.? *Ans.* 123 C. 3 Q. 1 $\frac{3}{4}$ lb. neat.

3. The neat weight being 216 C. tare 14 lb. per C. what is the tare and gros? *Ans.* The tare is 30 C. 3 Q. 12 lb. and the gros 246 C. 3 Q. 12 lb.

4. The neat weight being 97 C. 3 Q. 2 lb. tret 4 lb. per 104, and tare 16 lb. per C. what is the tret, futtle, tare, and gros? *Ans.*

Ans. The tret is 3 C. 3 Q. 18 lb. the futtle 101 C. 2 Q. 20 lb. the tare 16 C. 3 Q. 22 lb. and the gross 118 C. 2 Q. 14 lb.

5. The gross weight being 44 C. 3 Q. 18 lb. the tare on the whole 1 C. 1 Q. 2 lb. and the tret 4 lb. *per* 104, what is the neat weight, and what the price, at 2 l. 16 s. *per* C. neat? *Ans.* The neat weight is 41 C. 3 Q. 24 lb. and the price is 117 l. 10 s.

6. Six hogheads contain in all 55 C. and 26 lb. gross; now tare being allowed at 30 lb. *per* hoghead, tret at 4 lb. *per* 104, and cloff at 2 lb. *per* 3 C. what will be the neat weight, and what the price thereof, at 9 d. *per* lb. neat? *Ans.* Neat weight 51 C. 1 Q. 0 $\frac{1}{4}$ lb. price 215 l. 5 s. 6 $\frac{1}{4}$ d.

C H A P. IV.

B A R T E R.

BARTER, or truck, is the exchanging of one commodity for another; in doing of which the price of one of the commodities, and an equivalent quantity of the other, must be found either by practice, or by the rule of three.

Quest. 1. How many pounds of cotton, at 9 d. *per* lb. must be given in barter for 13 C. 3 Q. 14 lb. of pepper, at 2 l. 16 s. *per* C.?

First, Find the price or value of the commodity whose quantity is given, as follows :

	C.	Q.	lb.	L.	s.
	13	3	14 at 2	16	
2 l.			26		
16 s.			10	8	
2 Q.			1	8	
1 Q.				14	
14 lb.				7	
	L. 38 17				

Secondly, Find how much cotton, at 9 d. *per* lb. 38 l. 17 s. will purchase, as under :

d. lb. L. s.
If 9 : 1 :: 38 17

20

—

777

12

—

9)9324(

C. Q.

Ans. 1036 lb. = 9 1

If

If the above question be wrought decimally, the operation may stand as follows :

$$\begin{array}{r}
 \begin{array}{c} C. \quad L. \quad C. \\ \text{If } 1 : 2.8 :: 13.875 \\ \quad \quad \quad 2.8 \\ \hline 111000 \\ 27750 \\ \hline 113750 \\ 113750 \\ \hline 0 \\ 375 \dots \end{array} \\
 \begin{array}{c} 1350 \\ 1125 \\ \hline 2250 \\ 2250 \\ \hline 0 \end{array}
 \end{array}$$

Ans. 1036 = 9

Quest. 2. A merchant receives 126 yards of cloth in barter for 189 gallons brandy, at 6s. 8d. per gallon, what was the cloth valued at per yard?

$$\begin{array}{c}
 \begin{array}{c} \text{Gall.} \quad s. \quad d. \\ 189 \text{ at } 6 \quad 8 \\ \hline L. 63 \end{array} \\
 \begin{array}{c} \text{yds.} \quad L. \quad \text{yd.} \quad s. \\ \text{If } 126 : 63 :: 1 : 10 \end{array}
 \end{array}$$

Ans.

The decimal operation of the same question follows :

$$\begin{array}{r}
 G. \quad L. \quad 3)G. \\
 \text{If } 1 : .3 :: 189 \\
 \quad \quad \quad L. \quad s. \\
 126)63.0(.5 = 10 \text{ Ans.} \\
 630 \\
 \hline 0
 \end{array}$$

Quest. 3. A and B barter; A gets 20 C. of cheese, at 21s. 6d. per C.; B gets eight pieces of cloth, at 3l. 14s. per piece: What is the balance, and to whom due?

$$\begin{array}{r}
 \begin{array}{c} L. \quad s. \\ \text{B's 8 pieces cloth, at 3l. 14s. come to} \quad 29 \quad 12 \\ \text{A's 20 C. cheese, at 21s. 6d. amount to} \quad 21 \quad 10 \\ \hline \text{Balance due to B} \quad \quad \quad 8 \quad 2 \end{array}
 \end{array}$$

Quest. 4. A and B barter; A hath ginger worth 1l. 17s. 4d. per C. ready money, but in barter he will have 2l. 16s. per C.; B hath nutmegs worth L. 5, 12s. per C. ready money: How must B rate his nutmegs in barter, to be on an equal footing with A? Say,

L.

L. s. d. L. s. L. s. L. s.
 If 17 4 : 2 16 :: 5 12 : 8 8 *Ans.*

The value or price of the goods received and delivered in barter being always equal, it is obvious, that the product of the quantities received and delivered, multiplied into their respective rates, will be equal.

Hence arises a rule which may be used with advantage in working several questions; namely, Multiply the given quantity and rate of the one commodity, and the product divided by the rate of the other commodity quotes the quantity sought; or divided by the quantity quotes the rate.

Quest. 5. How many yards of linen, at 4s. per yard, should I have in barter for 120 yards of velvet at 15s. 6d.?

Yards. Sixpences. Sixp. Yards.
 120 $\times$ 31 = 3720, and 8)3720(465 *Ans.*

Quest. 6. If I receive 140 yards of broad cloth in barter, for 28 yards of velvet, at 16s. 8d. per yard, what was the broad cloth valued at per yard?

Yards. Groats. s. d.
 28 $\times$ 50 = 1400, and 140)1400(10 groats, or 3 4 *Ans.*

Quest. 7. How many hats at 7s. napkins at 4s. and caps at 1s. of each an equal number, may I have in barter for 280 yards of linen, at 3s. per yard?

280 $\times$ 3 = 840, and 7 + 4 + 1 = 12)840(70 of each sort. *Ans.*

Quest. 8. A has 41 C. hops, at 30s. per C. for which B gives him 20l. in money, and the rest in prunes, at 5d. per lb.: How many prunes did B give A beside the 20l.? *Ans.* 17 C. 3 Q. 4 lb.

Quest. 9. A has 608 yards broad cloth, at 14s. per yard; for which B gives him 125l. 12s. and 85 C. 2 Q. 24 lb. bees wax: How was the wax rated per C? *Ans.* At 3l. 10s. per C.

Quest. 10. A hath 120 yards druggets, worth 6s. per yard, but in barter he will have 8s. per yard; B hath shalloon worth 4s. per yard: How ought B to rate his shalloon in barter? and how many yards ought he to give A for his druggets? *Ans.* The shalloon is to be rated at 5s. 4d. per yard; and 180 yards to be given for the druggets.

Quest. 11. A hath 324 yards linen, at 4s. 6d. per yard; for which B gives him hats, at 7s. per piece, and stockings, at 6s. 6d. per pair: How many hats and pairs of stockings, of each an equal number, must B give A for his linen? *Ans.* 108 hats, and as many pairs of stockings.

Quest.

Quest. 12. A has 100 pieces silk, worth 3l. *per* piece; but bar-
ters them at 4l. *per* piece, and takes their value from B in wool,
at 7l. 10s. *per* C. which was only worth 6l. *per* C.: How much
wool must B give A for the silk? Whether was A or B the gain-
er? and how much? *Ans.* B gives A 53 $\frac{1}{2}$ C. wool, and A gains
20l.

C H A P. V.

L O S S and G A I N.

Questions relating to loss and gain, are likewise solved by prac-
tice, or the rule of three, as in the following examples.

Quest. 1. A merchant bought 436 yards of broad cloth, at 8s.
6d. *per* yard, which he sold at 10s. 4d. *per* yard: How much did
he gain on the whole?

Find both the buying price and selling price by practice or the
rule of three, and their difference is the gain.

<i>Yds.</i>	<i>s.</i>	<i>d.</i>		<i>Yds.</i>	<i>s.</i>	<i>d.</i>
436	at	8 6		436	at	10 4
6s.	130	16		6s.	130	16 0
2s. 6d.	54	10		1s.	21	16 0
	185	6	buying price.	3s. 4d.	72	13 4
					225	5 4 selling price.
					185	6 0 buying price.
						39 19 4 gain.

Or, Find the gain *per* yard, by subtracting the buying rate from
the selling rate; and then find the gain on 436 yards, by practice
or by the rule of three.

<i>s.</i>	<i>d.</i>		<i>Yds.</i>	
10	4	buying rate.	436	at 1s. 10d.
8	6	selling rate.	218	6d.
			145	4d.
		1 10 gain <i>per</i> yard.	79	9 4
				L. 39 19 4 gain.

Yds. d. Yds.
 Or, If 1 : 22 :: 436

$$\begin{array}{r} 22 \\ \hline 872 \\ 872 \\ \hline 12)9592 \\ \hline d. \\ 2|0)79|9 \quad 4 \end{array}$$

Gain *L. s. d.* 39 19 4
Yds. L. Yds. L. s. d.
 Or, Decimally, If 1 : .0918 :: 436 : 39 19 4

Quest. 2. A draper bought 124 yards of holland for 3*l.*: How must he sell it *per* yard to gain 10*l.* 6*s.* 8*d.* on the whole?

To the buying price add the intended gain, and then find the rate *per* yard, by the rule of three.

L. s. d. Yds. L. s. d. Yds. s. d.
 31 0 0 buying price. If 124 : 41 6 8 :: 1 : 6 8 *Ans.*
 10 6 8 gain.

Yds. L. Yds. s. d.
 41 6 8 selling price. Or, decimally, If 124 : 41.3 :: 1 : 6 8

Quest. 3. A grocer bought 3 C. 1 Q. 14 lb. of cloves, at 2*s.* 4*d.* *per* lb. which he sold for 52*l.* 14*s.*: How much did he gain on the whole?

Find the buying price, subtract it from the selling price, and the remainder is the gain, *viz.* 8*l.* 12*s.*

Quest. 4. A parcel of goods was bought for 60*l.* and sold for 75*l.*: What was gained *per cent.*? that is, How many pounds, shillings, or pence, would have been gained in the sale upon 100*l.* 100*s.* or 100*d.* laid out in the purchase? *Ans.* 25 *per cent.* For,

L. L.
 As 60 : 75 :: 100 : 125. Or, As 60 : 15 :: 100 : 25.

Quest. 5. If a halfpenny in the shilling be a merchant's profit, what does he gain *per cent.*? *Ans.* 4*l.* 3*s.* 4*d.* *per* 100*l.* or 4½ *per cent.*

Because ½*d.* *per* shilling is 10*d.* *per* *l.* you may work thus: Or, Because ½*d.* is ¼*s.* and 24=4×6, you may work also thus:

L. 100

$$\begin{array}{r} 10 \\ \hline 12)1000 \text{ pence.} \\ \hline \end{array}$$

$$\begin{array}{r} 4)100 \text{ l.} \\ \hline 6)25 \end{array}$$

L. 4 3 4 per 100 l.

$$\begin{array}{r} 2|0)8|3 \quad 4 \\ \hline \end{array}$$

L. 4 3 4
 Hence

$$\begin{array}{r} \text{L.} \\ \text{If } 100 : 115 :: 425 \\ \hline 115 \\ 2125 \\ 425 \\ 425 \\ \hline \text{L. } 488 | 75 \\ 20 \\ \hline \text{s. } 15 | 00 \end{array}$$

Or thus :
 425 at 15 per cent.

$$\begin{array}{r} 4 \\ 400 \quad 60 \\ 25 \quad 3 \quad 15 \\ \hline 63 \quad 15 \text{ profit.} \\ 425 \quad \text{prime cost.} \\ \hline \text{L. } 488 \quad 15 \text{ price.} \end{array}$$

$$\begin{array}{r} \text{L.} \quad \text{s.} \\ \text{Ans. } 488 \quad 15 \end{array}$$

Quest. 9. The prime cost of a parcel of goods sent to Jamaica is L. 37 : 8 : $1\frac{1}{2}$; and being sold at 75 per cent. advance on the invoice, what does that amount to? *Ans.* L. 65 : 9 : $2\frac{1}{2}$. Say,

$$\begin{array}{r} \text{L.} \quad \text{s.} \quad \text{d.} \\ \text{If } 100 : 75 :: 37 \quad 8 \quad 1\frac{1}{2} \\ \hline 20 \\ 748 \\ 12 \\ \hline 8977 \\ 4 \\ \hline 35910 \\ 75 \\ \hline 17955 \\ 25137 \\ \hline 4)26932.50 \\ \hline 12)6733 \\ \hline 20)561 \quad 1 \end{array}$$

Advance L. 28 1 1
 Prime cost 37 8 $1\frac{1}{2}$
 Amount 65 9 $2\frac{1}{2}$

Or, If 100 : 175 :: 37 8 $1\frac{1}{2}$

$$\begin{array}{r} \text{L.} \quad \text{s.} \quad \text{d.} \\ 20 \\ 748 \\ 12 \\ \hline 8977 \\ 4 \\ \hline 35910 \\ 175 \\ \hline 17955 \\ 25137 \\ 3591 \\ \hline 4)62842.50 \\ \hline 12)15710 \quad 2 \\ \hline 20)1309 \quad 2 \\ \hline \text{Amount L. } 65 \quad 9 \quad 2\frac{1}{2} \end{array}$$

The 50 cut off is $\frac{50}{100}$ f. = $\frac{1}{2}$ f. more.

Quest. 10. If 86 pieces of cloth cost in all 129 l. how must they be sold *per* piece to gain 15 *per cent.*? *Ans.* L. 1 : 14 : 6. Say,

$$\begin{array}{r} \text{L.} \quad \text{L.} \quad \text{s.} \\ \text{If } 100 : 115 :: 129 : 148 \quad 7 \\ \text{P.} \quad \text{L.} \quad \text{s.} \quad \text{P.} \quad \text{L.} \quad \text{s.} \quad \text{d.} \\ \text{And, If } 86 : 148 \quad 7 :: 1 : 1 \quad 14 \quad 6 \end{array}$$

Quest. 11. A grocer bought 4 C. 1 Q. pepper for L. 15 : 17 : 4; which happening to be damaged, he is willing to lose $12\frac{1}{2}$ *per cent.*: How must he sell it *per* lb.? *Ans.* At 7 d. *per* lb.

From 100 l. subtract 12 l. 10 s. and there remains 87 l. 10 s. Then say,

$$\begin{array}{r} \text{L.} \quad \text{L.} \quad \text{s.} \quad \text{L.} \quad \text{s.} \quad \text{d.} \quad \text{L.} \quad \text{s.} \quad \text{d.} \\ \text{If } 100 : 87 \quad 10 :: 15 \quad 17 \quad 4 : 13 \quad 17 \quad 8 \text{ the selling price of the whole.} \end{array}$$

$$\begin{array}{r} \text{C.} \quad \text{Q.} \quad \text{L.} \quad \text{s.} \quad \text{d.} \quad \text{lb.} \quad \text{d.} \\ \text{And, If } 4 \quad 1 : 13 \quad 17 \quad 8 :: 1 : 7 \text{ the rate } \textit{per} \text{ lb.} \end{array}$$

Quest. 12. If a merchant, in selling goods at 8 d. *per* lb. gain 12 *per cent.* what will he gain *per cent.* in selling the same goods at 9 d. *per* lb.? *Ans.* 26 *per cent.*

$$\text{If } 8 : 112 :: 9 : 126$$

Quest. 13. Sold a puncheon of rum for 30 l. and gained 20 *per cent.*: What was the prime cost? *Ans.* 25 l.

$$\begin{array}{r} \text{L.} \quad \text{L.} \\ \text{If } 120 : 100 :: 30 : 25 \end{array}$$

Quest. 14. A plumber sold 10 fodder of lead, each fodder containing $19\frac{1}{2}$ C. for 204 l. 15 s. and gained $12\frac{1}{2}$ *per cent.*: How much did it cost him in all? and how much *per* C.? *Ans.* 182 l. in all: and 18 s. 8 d. *per* C.

Quest. 15. A merchant sells 8 tuns of wine for 440 l. and loses 12 *per cent.*: How much did it cost *per* tun? and how does he sell it *per* gallon? *Ans.* It cost 62 l. 10 s. *per* tun; and he sells it at 4 s. 4 d. $1\frac{1}{4}$ f. *per* gallon.

Quest. 16. If a merchant sell cloth at 11 s. 6 d. *per* yard, and thereby gain 15 *per cent.* what would he gain *per cent.* by selling the same cloth at 12 s. *per* yard? *Ans.* 20 *per cent.*

Quest. 17. A merchant selling corn at 8 s. *per* bushel, gained 10 *per cent.*; but the market falling, he was obliged to sell the rest of the same corn at 7 s. *per* bushel: What did he gain or lose *per cent.* on this last sale? *Ans.* He lost $3\frac{1}{4}$ *per cent.*

Quest. 18. A draper bought 28 pieces of stuffs, at 4 l. *per* piece; and sells 10 of them at 4 l. 6 s. *per* piece, and 8 of them at 4 l. 8 s.

8 s. per piece : At what rate must he sell the rest, to gain 10 per cent. on the whole? *Ans.* At 4 l. 10 s. per piece.

C H A P. VI.

MISCELLANIES.

I. *Brokage.*

BROKERS are agents employed by merchants in transacting mercantile affairs; such as, buying, selling, negotiating bills, &c. They are generally men that have been bred merchants, understand trade and exchange, with the proper seasons of doing business. They must too be men of character; and no decayed merchant is allowed to officiate as broker without a special licence. The payment or fee for their service is called *brokage*, being a scanty allowance of a few shillings *per cent.* or on the 100 l.; and is readily cast up by practice, as in the following examples:

Find the brokage at 1 per cent. by dividing the given sum by 100, and then take aliquot parts of the quot.

1. What is the brokage of L. 985 : 15 : 6, at 6 s. per cent.?

L. 9	85	15	6		L. s. d.	Or,	L. s. d.
	20				6 s. = $\frac{3}{10}$ l.		9 17 $1\frac{3}{4}$
							3
s. 17	15				10)29 11 $5\frac{1}{4}$	5 s. = $\frac{1}{4}$	2 9 $3\frac{1}{4}$
	12					1 s. = $\frac{1}{7}$	9 $10\frac{1}{4}$
d. 1	86				<i>Ans.</i> 2 19 $1\frac{1}{2}$		2 19 $1\frac{1}{2}$ <i>Ans.</i>
	4						
f. 3	44						

2. What is the brokage of 439 l. 19 s. at 7 s. 6 d. per cent.?

L. 9	39	19			7 s. 6 d. = $\frac{3}{8}$ l.
	20				
					L. s. d.
s. 7	99				4 7 $11\frac{3}{4}$
	12				3
d. 11	88				8)13 3 $11\frac{1}{4}$
	4				
f. 3	52				1 12 $11\frac{3}{4}$ <i>Ans.</i>

3. What is the brokage of 345 l. 16 s. at 4 s. 9 d. per cent.?

L. 3	45	16				L. s. d.
	20					3 9 1 $\frac{3}{4}$
s. 9	16		s. d.			13 9 $\frac{3}{4}$
	12		4 0 =	$\frac{1}{2}$		1 8 $\frac{1}{2}$
d. 1	92		6 =	$\frac{1}{8}$		10 $\frac{1}{4}$
	4		3 =	$\frac{1}{2}$		16 4 $\frac{1}{4}$ Ans.
f. 3	68					

MORE EXAMPLES.

4. What is the brokage of L. 834 : 18 : 4, at 5 s. 6 d. *per cent.* ?
Ans. L. 2 : 5 : 10 $\frac{1}{4}$.
5. What is the brokage of L. 476 : 12 : 9, at 6 s. 8 d. *per cent.* ?
Ans. L. 1 : 11 : 9 $\frac{1}{4}$.
6. What is the brokage of L. 954, 17 s. at 4 s. 9 d. *per cent.* ?
Ans. L. 2 : 5 : 4.

II. Bankruptcy.

When a person in trade fails, stops payment, or turns insolvent, he is called *bankrupt* ; and the computations used in settling affairs among the creditors are such as occur in the following questions :

1. A bankrupt's effects amount to 811 l. 10 s. and he owes,

	L.	s.	d.
To A	220	16	6
To B	312	0	0
To C	117	12	6
To D	106	12	6
To E	200	6	0
To F	124	12	6

In all 1082 0 0

How much can he afford to pay *per pound* ? and what must each creditor have ?

Ans. 15 s. *per pound* : and the creditors get as follows :

	L.	s.	d.
A	165	12	4 $\frac{1}{2}$
B	234	0	0
C	88	4	4 $\frac{1}{2}$
D	79	19	4 $\frac{1}{2}$
E	150	4	6
F	93	9	4 $\frac{1}{2}$

Proof 811 10 0

First find the dividend for 1 l. by saying,

L. L. s. d.
If 1082 : 811 10 :: 1

20
1082)16230(15 s.
1082
5410
5410

Then find the dividend to each creditor by practice, as follows :

L. s. d.
A 220 16 6, at 15 s.
s.
10 = $\frac{1}{2}$ 110 8 3
5 = $\frac{1}{2}$ 55 4 $1\frac{1}{2}$
165 12 $4\frac{1}{2}$

L.
B 312, at 15 s.
s.
10 = $\frac{1}{2}$ 156
5 = $\frac{1}{2}$ 78
234

Work in like manner for each of the other creditors.

Or you may work by decimal practice, *viz.* turn the dividend for 1 l. into a decimal, multiply it through the nine digits ; and you have a table constructed, from which may be collected the dividend for each creditor, by moving the decimal point, and taking aliquot parts, as follows :

TABLE.

L. s. d.
A 220 16 6
L.
200 00 0 = 150.
20 00 0 = 15.
10 00 = .375
5 00 = .1875
1 00 = .0375
6 = .01875
165.61875

1	.75
2	1.50
3	2.25
4	3.00
5	3.75
6	4.50
7	5.25
8	6.00
9	6.75

L.
B 312
300 = 225.
10 = 7.5
2 = 1.5
234

Or, Multiply 312 by .75

312
.75
1560
2184
234.00

B b 3

2. A

2. A stops payment, and agrees with his creditors at 17 s. 6 d. *per pound*: What will B receive, to whom he owes 400 l.?

$$\begin{array}{r}
 \text{L.} \\
 400, \text{ at } 17 \text{ s. } 6 \text{ d.} \\
 \hline
 \begin{array}{r}
 \text{s.} \quad \text{d.} \\
 10 \quad 0 = 200 \\
 5 \quad 0 = 100 \\
 2 \quad 6 = 50 \\
 \hline
 \end{array}
 \end{array}$$

350 *Anf.*

3. A compounds with his creditors for 13 s. 4 d. *per pound*: What may B expect, to whom he owes 415 l. 9 s. 6 d.?

$$\begin{array}{r}
 \text{L.} \quad \text{s.} \quad \text{d.} \\
 415 \quad 9 \quad 6, \text{ at } 13 \text{ s. } 4 \text{ d.} \\
 \hline
 \begin{array}{r}
 \text{s.} \quad \text{d.} \\
 6 \quad 8 = \frac{1}{3} = 138 \quad 9 \quad 10 \\
 6 \quad 8 = \frac{1}{3} = 138 \quad 9 \quad 10 \\
 \hline
 \end{array}
 \end{array}$$

276 19 8 *Anf.*

MORE EXAMPLES.

4. A agrees with his creditors at 5 s. 5 d. *per pound*: What will B get, to whom he was owing 1000 l. ? *Anf.* L. 270 : 16 : 8.

5. A bankrupt's effects afford 11 s. 7½ d. *per pound*: What will B get, to whom there is owing 300 l. ? *Anf.* L. 174 : 7 : 6.

6. A bankrupt settles with his creditors at 6 s. 8 d. *per pound*, and clears with B, by paying him 317 l. 6 s. 8 d.: What was the original debt ? *Anf.* 952 l.

III. Insurance.

Insurance is a security given by the insurers or underwriters, to indemnify the insured from such losses as are mentioned in the policy of insurance, in consideration of a sum of money called *premium*; which varies according to the risk, and is generally so much *per cent.*

It is an usual article in the policy, that in case of loss, the insurer is to be allowed a small discount or abatement, commonly 2 *per cent.*; that is, he is to pay but 98 l. for every 100 l. insured; and the 98 l. is called the *short recovery*.

A merchant sometimes insures, not only the full value of his cargo, but even the premium and discount; that, in case of loss, he may be entitled to a sum from the underwriters, or insurance office, exactly equal to the value of the said cargo; and this is called *covering his outset*.

In case of damage or loss, an estimate thereof is made out, and the

the underwriters pay in proportion to their several subscriptions; and this is called *an average*.

Most of the computations relative to insurance, fall under one or other of the cases following.

Case I. Given the rate *per cent.* to find the premium.

Work by practice or aliquot parts.

Ex. What premium must be paid for insuring 854 l. at $7\frac{1}{2}$ per cent.? and for insuring 2860 l. at $13\frac{1}{2}$ per cent.?

<i>Ex. 1.</i>	
	854
<i>p.c.</i>	
$5 = \frac{1}{20}$	42.7
$2\frac{1}{2} = \frac{1}{4}$	21.35
$7\frac{1}{2}$	64.05 = 64 1 <i>Ans.</i>

<i>Ex. 2.</i>	
	2860
<i>p.c.</i>	
$10 = \frac{1}{10}$	286
$2 = \frac{1}{5}$	57.2
$1 = \frac{1}{2}$	28.6
$\frac{1}{2} = \frac{1}{2}$	14.3
$13\frac{1}{2}$	386.1 = 386 2 <i>Ans.</i>

Case II. Given the discount *per cent.* to find the short recovery.

Work by aliquot parts.

Ex. What is the short recovery of L. 2000 insured, when discount is allowed at 2 per cent.? and at $2\frac{1}{2}$ per cent.?

<i>p.c.</i>	
$2 = \frac{1}{50}$	2000
	40 discount.
	1960 short recovery.

<i>p.c.</i>	
$2\frac{1}{2} = \frac{1}{40}$	2000
	50 discount.
	1950 short recovery.

Case III. Given the rate *per cent.* of premium and discount, to find what sum must be insured to cover the outset in a single voyage.

Subtract the sum of the premium, and discount from 100, and then say, As the remainder to 100, so the outset to the answer.

Ex. 1. If the premium be 10 per cent. and the discount 2 per cent. what sum must be insured to cover L. 100 outset?

10 premium.	100
2 discount.	12
—	—
12 sum.	As 88 : 100 :: 100 : 113 12 8 <i>Ans.</i>

Proof.

	<i>L. s. d.</i>
Sum insured.	113 12 8
Deduct 2 per cent.	2 5 5 discount.
	—
Insurer pays	111 7 3 in case of loss.
Deduct 10 per cent.	11 7 3 premium.
	—
Outset covered	100 or made good.
	B b 4

Ex.

Ex. 2. If the premium be $13\frac{1}{2}$ per cent. and the discount 2 per cent. what sum must be insured in order to cover 800l. outset?

$13\frac{1}{2}$ premium,	100			
2 discount,	15.5			
$15\frac{1}{2}$ sum,	As 84.5 : 100 :: 800 :	946 14 10 $\frac{1}{4}$	Ans.	

Case IV. Given the rate per cent. of premium and discount, to find the premium out and home, and what sum must be insured to cover the infet.

First, Find the premium on the voyage out, as taught above, and for the voyage home work thus: From the short recovery deduct the premium; then say, As the remainder to the premium, so the sum of the outset and premium of the voyage out, to the premium of the voyage home; which, added to the premium of the voyage out, gives the total premium. Lastly, Find what sum insured will cover the infet by Case 3.

Ex. If the premium be 10 per cent. and the discount 2 per cent. what premium must be paid for insuring 100l. out and home? and what sum must be insured to cover the infet?

By Case 3. Ex. 1. the premium of the voyage out is L. 11 : 7 : 3.

	L.	s.	d.	
98	11	7	3	premium out.
10	100	0	0	outset.
As 88 : 10 ::	111	7	3	: 12 13 1 premium home.
				11 7 3 premium out.
	24	00	4	total premium.

Lastly, by Case 3. find what sum must be insured to cover L. 111 : 7 : 3, viz. say,

	L.	s.	d.	
As 88 : 100 ::	111	7	3	
			10	
	1113	12	6	
			10	
	88)	11136	5 0	(126 10 11 Ans.

In former times the policies of insurance were generally conceived in such vague terms, that sometimes it was no easy matter to settle an average when a loss happened. The affair was usually brought before a court, critical questions were started, difficulties and clouds of dust were raised, that tended to inveigle the cause, and

and render it a subject of endless altercation: but the case is otherwise now; for scarce any misfortune can happen but what is expressly specified in the policies used at present. I shall conclude insurance with one example of a general average on ship, goods, and freight.

A ship, by stress of weather, lost her masts, was forced ashore, and the damage estimated at 36l.

	L.	Now, If 2580l. pays 36l. then,
A's goods, - - -	1200	
B's goods, - - -	480	
Insured by Messrs Adair, 1680		L. s. d.
A's own risk, - - -	60	1680 pays - - - 23 8 10
B's own risk, - - -	40	60 pays - - - 0 16 9
		40 pays - - - 0 11 2
Value of the goods, 1780		600 the ship, pays 8 7 5½
Value of the ship, - 600		200 freight, pays 2 15 9¾
Amount of freight, 200		
	2580	Proof 36 0 0

IV. Of purchasing Stocks.

The British government, ever since the Revolution in 1688, have been in use to raise supplies, for carrying on war, or answering other exigencies of the state, by borrowing money, and appropriating the public taxes for payment of the interest.

These national debts, contracted by borrowing from the bank of England, from trading companies, and from private persons, or by selling annuities, with the provisions made by parliament for paying the interest due thereon, or for cancelling the debts, are commonly called the *public funds* or *stocks*; and come under various designations; such as, *Bank Stock*, *India Stock*, *India Annuities*, *India Bonds*, *South-Sea Stock*, *South-Sea Annuities*, *Three per cent. Reduced Annuities*, *Three per cent. Consolidated Annuities*, *Three and a half per cent. Annuities*, *Four per cent. Annuities*, *Long Annuities*, *Exchequer Bills*, *Navy and Victualling Bills*, &c. &c.

When the parliament has voted the supplies, a subscription is opened, and generally a few men of fortune subscribe for the whole; and as the subscriptions are all transferable, they afterwards sell off these stocks in small shares, and at high rates, and the profits thence arising are so much clear gain.

The annuities and growing interest have hitherto been always punctually paid; and as the stocks now amount to many millions Sterling, not only monied corporations and men of fortune in Britain, but many foreigners, have come to be concerned in them. The British stocks are now become a considerable branch of commerce,

merce, carried on in Exchange alley; and commissions are sent up from all the parts of Europe to make purchases in them.

Stocks are bought or sold at so much *per cent.*; and the price, or sum to be paid, may be cast up by Practice, or Decimally, as in the following examples:

1. What must be paid for 403l. 18s. bank annuities, at 94 *per cent.*?

<i>By Practice.</i>		<i>Decimally.</i>
L. s.	L.	
403 18, at 94 <i>per cent.</i>	As 100 : 94 :: 403.9	
20 = $\frac{1}{5}$	80 15 7	94
5 = $\frac{1}{4}$	20 3 10 $\frac{1}{4}$	16156
1 = $\frac{1}{4}$	4 0 9 $\frac{1}{4}$	36351
6	24 4 8 sub.	
	379 13 4 <i>Anf.</i>	<i>Anf.</i> 379.666

2. What sum will purchase 875l. 10s. bank stock, at 131 $\frac{1}{4}$ *per cent.*?

<i>By Practice.</i>		<i>Decimally.</i>
L. s.	L.	
100 =	875 10 at 131 $\frac{1}{4}$ <i>per cent.</i>	As 100 : 131.25 :: 875.5
25 = $\frac{1}{4}$	218 17 6	875.5
5 = $\frac{1}{5}$	43 15 6	
1 $\frac{1}{4}$ = $\frac{1}{4}$	10 18 10 $\frac{1}{2}$	65625
131 $\frac{1}{4}$	1149 1 10 $\frac{1}{2}$ <i>Anf.</i>	65625
		91875
		105000
		1149.09375 <i>Anf.</i>

3. What sum will purchase 432l. 15s. South-sea stock, at 113 $\frac{5}{8}$ *per cent.*?

<i>By Practice.</i>		<i>Decimally.</i>
L. s. d.	L.	
100 =	432 15 0	As 100 : 113.625 :: 432.75
10 = $\frac{1}{10}$	43 5 6	Mult. 57.234 inverted.
2 = $\frac{1}{5}$	8 13 1	
1 = $\frac{1}{2}$	4 6 6 $\frac{1}{2}$	454500
$\frac{1}{2}$ = $\frac{1}{2}$	2 3 3 $\frac{1}{4}$	34087
$\frac{1}{8}$ = $\frac{1}{4}$	0 10 9 $\frac{1}{4}$	2272
113 $\frac{5}{8}$	491 14 2 $\frac{1}{2}$ <i>Anf.</i>	795
		56
		491.710 <i>Anf.</i>

If the stock be any just number of hundreds, multiply the rate *per cent.* by the number of hundreds, and the product is the price, as in the following example :

4. What must I pay for 800l. three *per cent.* annuities, at 124 $\frac{7}{8}$ *per cent.*?

$$\begin{array}{r}
 \text{L. } s. \text{ } d. \\
 124 \ 17 \ 6 \\
 \ 8 \\
 \hline
 999 \ 0 \ 0 \text{ } Ans.
 \end{array}$$

V. *An easy way of finding the value of the short hundred or five score.*

R U L E I.

If the rate, or price of 1, be shillings, multiply the rate by 5, and the product is the answer in pounds.

The answer, multiplied by 2, 3, 4, &c. or by 10, gives the price of 200, 300, 400, &c. or of 1000.

E X A M P L E S.

1. What cost 100 ells, at 16s. *per ell*?

$$\begin{array}{r}
 16 \text{ s.} \\
 5 \\
 \hline
 \text{L. } 80 \text{ } Ans. \\
 3 \\
 \hline
 240 \text{ price of 300 ells.}
 \end{array}$$

2. What cost 100 yards, at 35s. *per yard*?

$$\begin{array}{r}
 35 \text{ s.} \\
 5 \\
 \hline
 \text{L. } 175 \text{ } Ans. \\
 10 \\
 \hline
 1750 \text{ price of 1000 yards.}
 \end{array}$$

3. What cost 100 lb. at 3l. 14s. *per lb.*?

$$\begin{array}{r}
 \text{L. } s. \\
 3 \ 14 \\
 20 \\
 \hline
 74 \\
 5 \\
 \hline
 \text{L. } 370 \text{ } Ans.
 \end{array}$$

4. What cost 100 gallons, at 2l. 17s. *per gallon*?

$$\begin{array}{r}
 \text{L. } s. \\
 2 \ 17 \\
 20 \\
 \hline
 57 \\
 5 \\
 \hline
 \text{L. } 285 \text{ } Ans.
 \end{array}$$

R U L E II.

If the rate be pence, multiply the rate by 5, divide the product by 12, the quot will be pounds, and every unit of the remainder is. 8 d.

E X-

EXAMPLES.

1. What will 100 yards cost, at 9d. per yard?

$$\begin{array}{r} 9d. \\ 5 \\ \hline 12)45(3 \quad 15 \text{ Ans.} \\ 36 \\ \hline (9) \end{array}$$

2. What will 100 ells cost, at 11d. per ell?

$$\begin{array}{r} 11d. \\ 5 \\ \hline 12)55(4 \quad 11 \quad 8 \text{ Ans.} \\ 48 \\ \hline (7) \end{array}$$

RULE III.

If the rate be shillings and pence, multiply the rate by 5; the product under shillings will be pounds, and every unit of the product under pence will be 1s. 8d.

EXAMPLES.

1. What will 100 ells cost, at 4s. 5d. per ell?

$$\begin{array}{r} s. \quad d. \\ 4 \quad 5 \\ 5 \\ \hline 22 \quad 1 \end{array}$$

Ans. L. 22 1 8

2. What will 100 yards cost, at 7s. 6d. per yard.

$$\begin{array}{r} s. \quad d. \\ 7 \quad 6 \\ 5 \\ \hline 37 \quad 6 \end{array}$$

Ans. L. 37 10

RULE IV.

If the rate be farthings, multiply the rate by 5, and every unit of the product will be 5d.

EXAMPLES.

1. What will 100 apples cost, at 1f. each?

$$\begin{array}{r} 1f. \\ 5 \\ \hline \end{array} \quad \begin{array}{r} d. \quad s. \quad d. \\ 25 = 2 \quad 1 \end{array}$$

Ans. $5 \times 5 = 25 = 2 \quad 1$

2. What will 100 oranges cost, at 3f. each?

$$\begin{array}{r} 3f. \\ 5 \\ \hline \end{array} \quad \begin{array}{r} d. \quad s. \quad d. \\ 15 \times 5 = 75 = 6 \quad 3 \end{array}$$

Ans. $15 \times 5 = 75 = 6 \quad 3$

If the value of 100 be given, and the rate, or price of 1, be required, reverse the operation.

VI. *An easy way of finding the value of the long hundred, or six score.*

RULE.

Esteem the pence in the rate so many pounds; and for the farthings,

things, take such a part of a pound as they are of a penny, and the half of this amount is the answer.

EXAMPLES.

1. What will 120 ells cost, at $7\frac{1}{2}$ d. per ell?

$$\begin{array}{r} L. \quad s. \\ 2) 7 \quad 10 \\ \hline \end{array}$$

L. 3 15 *Ans.*

2. What will 120 yards cost, at $13\frac{1}{4}$ d. per yard?

$$\begin{array}{r} L. \quad s. \\ 2) 13 \quad 5 \\ \hline \end{array}$$

L. 6 12 6 *Ans.*

3. What will 120 gallons cost, at $16\frac{3}{4}$ d. per gallon?

$$\begin{array}{r} L. \quad s. \\ 2) 16 \quad 15 \\ \hline \end{array}$$

L. 8 7 6 *Ans.*

4. What will 120 lb. cost, at 17s. $3\frac{1}{2}$ d. per lb.

$$\begin{array}{r} s. \quad d. \\ 17 \quad 3\frac{1}{2} \\ 12 \\ \hline \end{array}$$

$$2) 207 \quad 10$$

L. 103 15 *Ans.*

If the price of 120 be given, and the rate be required, reverse the operation.

VII. *A short way of finding the value of 1 C. or 112 lb.*

R U L E.

Esteem every penny in the rate 9s. 4 d.; esteem three farthings 7s.; esteem a halfpenny 4s. 8 d.; and a farthing 2s. 4 d.

EXAMPLES.

1. What will 1 C. cost, at $7\frac{1}{2}$ d. per lb.?

$$\begin{array}{r} L. \quad s. \quad d. \\ 1 d. = 0 \quad 9 \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 7 d. = 3 \quad 5 \quad 4 \\ \frac{1}{2} d. = 0 \quad 4 \quad 8 \\ \hline \end{array}$$

L. 3 10 0 *Ans.*

2. What will 1 C. cost, at $9\frac{3}{4}$ d. per lb.?

$$\begin{array}{r} L. \quad s. \quad d. \\ 1 d. = 0 \quad 9 \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 9 d. = 4 \quad 4 \quad 0 \\ \frac{3}{4} d. = 0 \quad 7 \quad 0 \\ \hline \end{array}$$

L. 4 11 0 *Ans.*

VIII. *A short way of finding the value of the great gross.*

R U L E.

Multiply the rate or price of 1 dozen, in pence, by 3, divide the product by 5, and the quot will be the answer in pounds.

E X-

EXAMPLES.

1. What will 1 great grofs
cost, at 7d. *per* dozen?

$$\begin{array}{r} 7d. \\ 3 \\ \hline 5)21 \end{array}$$

L. 4 4 *Ans.*

2. What will 1 great grofs
cost, at 3s. 6d. *per* dozen?

$$\begin{array}{r} 42d. \\ 3 \\ \hline 5)126 \end{array}$$

L. 25 4 *Ans.*

In the same manner may the value of the small grofs be found from the price of 1 given.

If the value of the grofs be given, and the rate be required, reverse the operation; that is, multiply by 5, and divide the product by 3, and the quot is the answer in pence.

IX. *The price of goods cast up by inverting the question.*

In many cases, the value or price of goods may be found with great ease, by inverting the question, *viz.* esteem the given quantity to be the rate, and the rate to be the quantity.

EXAMPLES.

1. What cost 15 yards, at 7d. *per* yard?

Invert the question, by saying, What cost 7 yards, at 15d. or 1s. 3d.; and then work as follows:

$$\begin{array}{r} s. \quad d. \\ 1 \quad 3 \\ 7 \\ \hline \end{array}$$

s. 8 9 *Ans.*

2. What cost 53 ells, at 9d. *per* ell; that is, 9 ells at 53d.?

$$\begin{array}{r} s. \quad d. \\ 4 \quad 5 \\ 9 \\ \hline \end{array}$$

L. 1 19 9 *Ans.*

3. What cost $58\frac{1}{2}$ gallons, at 11d. *per* gallon? or 11 gallons, at $58\frac{1}{2}$ d.?

$$\begin{array}{r} s. \quad d. \\ 4 \quad 10\frac{1}{2} \\ 11 \\ \hline \end{array}$$

L. 2 13 $7\frac{1}{2}$ *Ans.*

4. What cost $42\frac{1}{4}$ hogheads, at 28s. *per* hoghead? or 28 hogheads at $42\frac{1}{4}$ s.?

$$\begin{array}{r} L. \quad s. \quad d. \\ 2 \quad 2 \quad 3 \\ 28=7 \times 4 \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 14 \quad 15 \quad 9 \\ 4 \\ \hline \end{array}$$

59 3 0 *Ans.*

C H A P. VII.

E X C H A N G E.

EXCHANGE is the receiving or paying money in one country for the like sum in another.

The par in exchange is when the sum received and paid away are of the same intrinsic value, or contain an equal quantity of pure gold or silver.

The course of exchange is the current price betwixt two places, which is always fluctuating and unsettled, being sometimes above and sometimes below par, according to the circumstances of trade.

When the course of exchange rises above par, the country where it rises may conclude for certain, that the balance of trade runs against them. The truth of this will appear, if we suppose Britain to import from any foreign place goods to the value of 100,000*l.* at par, and export only to the value of 80,000*l.*; in this case bills on the said foreign place will be scarce in Britain, and consequently will rise in value; and after the 80,000*l.* is paid, bills must be procured from other places at a high rate to pay the remainder, so that perhaps 120,000*l.* may be paid for bills to discharge a debt of 100,000*l.*

Though the course of exchange be in a perpetual flux, and rises or falls according to the circumstances of trade, yet the exchanges of London, Holland, Hamburgh, and Venice, in a great measure regulate those of all other places in Europe.

I. *Exchange with Holland.*

M O N E Y - T A B L E.

		Par in Sterling.	s.	d.
8 Pennings, or 2 duytes,	} make	1 groat or penny	= 0	0.54
2 Groats, or 16 pennings,		1 stiver	= 0	1.09
6 Stivers, or 12 pence,		1 schilling	= 0	6.56
20 Schillings,		1 pound Flemish	= 10	11.18
20 Stivers, or 40 pence,		1 guilder or florin	= 1	9.86
6 Guilders, or florins,		1 pound Flemish	= 10	11.18
2½ Guilders, or florins,	}	1 rixdollar	= 4	6.66

In Holland there are two sorts of money, bank and current. The bank is reckoned good security; demands on the bank are readily answered; and hence bank money is generally rated from 3 to 6 *per cent.* better than the current. The difference between the bank and current money is called the *agio*.

Bills on Holland are always drawn in bank money; and if accounts be sent over from Holland to Britain in current money, the

British

British merchant pays these accounts by bills, and in this case has the benefit of the agio.

P R O B. I.

To reduce bank money to current money.

R U L E.

As 100 to 100 + agio, so the given guilders to the answer.

E X A M P L E.

What will 2210 guilders in bank money amount to in Holland currency, the agio being $3\frac{1}{8}$ per cent.?

Guild.			Or, by practice.	
As 100	: $103\frac{1}{8}$	:: 2210	50)2210	
8	8	825	44.2	= 2 per cent.
—	—	—	22.1	= 1 per cent.
800	825	11050	2.7625	= $\frac{1}{8}$ per cent.
		4420		
		17680	2279.0625	
		—		
		8 00)18232 50	(2279	1 4 cur.
		16...	20	
		—	—	
		22	10 00(	
		16	8	
		—	—	
		63	2	
		56	16	
		—	—	
		72	32	
		72	32	
		—	—	

If the agio only be required, make the agio the middle term, thus :

As 100 : $3\frac{1}{8}$:: 2210 : 69 1 4 agio. Or, work by practice, as above.

P R O B. II.

To reduce current money to bank money.

R U L E.

As 100 + agio to 100, so the given guilders to the answer.

E X.

E X A M P L E.

What will 2279 guilders 1 stiver 4 pennings, Holland currency, amount to in bank money, the agio being $3\frac{1}{8}$ per cent.?

	Guild.	Guild.	Guild.	st.	pen.
As	$103\frac{1}{8}$	:	100	::	2279 1 4
	8		8		20
	825		800		45581
	20				16
	16500				273490
	16				45581
	990				729300
	165				800
	8)264 000				8)583440 000
	3)33				3)72930 Guild.
	11				11)24310(2210 bank.

In Amsterdam, Rotterdam, Middleburgh, &c. books and accounts are kept by some in guilders, stivers, and pennings, and by others in pounds, schillings, and pence, Flemish.

Britain gives 1 l. Sterling for an uncertain number of schillings and pence Flemish. The par is 1 l. Sterling for 36.59 s. Flemish; that is, 1 l. 16 s. 7.08 d. Flemish.

When the Flemish rate rises above par, Britain gains and Holland loses by the exchange, and the contrary.

Sterling money is changed into Flemish, by saying,

As 1 l. Sterling to the given rate,
So is the given Sterling to the Flemish sought.

Or, the Flemish money may be cast up by practice.

Dutch money, whether pounds, schillings, pence, Flemish, or guilders, stivers, pennings, may be changed into Sterling, by saying,

As the given rate to 1 l. Sterling,
So the given Dutch to the Sterling sought.

E X A M P L E S.

1. A merchant in Britain draws on Amsterdam for 782 l. Sterling: How many pounds Flemish, and how many guilders will that amount to, exchange at 34 s. 8 d. per pound Sterling?

C c

L.

			<i>Decimally.</i>		
<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>L.</i>	<i>s.</i>	<i>L.</i>
If 1 :	34	8 ::	782	If 1 :	34 8 :: 782
	12				782
	416				693
	782				27783
	832				242666
	3328				
	2912				210)2710 9.3
12)325312					
	<i>d.</i>				<i>L.</i> 1355 9 4 <i>Flem.</i>
210) 2710 9	4				
<i>L.</i> 1355	9	4 <i>Flem.</i>			

<i>By Practice.</i>			<i>Or thus :</i>		
<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>L.</i>	<i>s.</i>	<i>d.</i>
	782			782	
10 s. = $\frac{1}{2}$	391		14 s. = $\frac{7}{10}$	547	8
4 s. = $\frac{1}{5}$	156	8	8 d. = $\frac{1}{10}$	26	1 4
8 d. = $\frac{1}{8}$	26	1 4			
	1355	9 4 <i>Flem.</i>		1355	9 4 <i>Flem.</i>

Multiply the Flemish pounds and shillings by 6, and the product will be guilders and stivers; and if there be any pence, multiply them by 8 for pennings; or, divide the Flemish pence by 40, and the quot will be guilders, and the half of the remainder, if there be any, will be stivers, and 1 penny odd will be half a stiver, or 8 pennings, as follows:

<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>Flem. pence.</i>
1355	9	4	
	6		410)32531 2(32 rem.
<i>Guild.</i> 8132	16	<i>stiv.</i>	<i>Guild.</i> 8132 16 <i>stiv.</i>

2. Change 591 l. 5 s. Flemish into Sterling money, exchange at 37 s. 6 d. Flemish *per* l. Sterling?

Flem.

Flem.		Ster.	Flem.	Decimally.	
s. d.	L.		L. s.	L.	L.
If 37 6 : 1 ::	591	5	If 1.875 : 1 ::	591.25	
2	20		5) .375	5) 118.25	
5) 75	11825		5) .075	5) 23.65	
5) 15	2		.015	.015	4.73(315.8
3	5) 23650				45
	5) 4730				23
	3) 946				15
	315 $\frac{1}{2}$				80
					75
					50
					45
					*5

3. Change 1693 guilders 6 stivers into Sterling money, exchange at 34s. 5d. per pound Sterling.

Flem.		Ster.	Decimally.	
s. d.	L.	Guild. stiv.		
If 34 5 : 1 ::	1693	6	34.416	
12	20		.3	
413	33866		10.3250 : 1 ::	1693.3
	2		10.325) 1693.300	(164 L. Ster. Ans.
			10325	
	413) 67732	(164 L. Ster.	66080	
	413		61950	
	2643		41300	
	2478		41300	
	1652			
	1652			

In working decimally, because 10s. Flemish is equal to 3 guilders, we might say, As 10 : 3 :: 34.416 to the guilders in the rate; but it is the same in effect, and shorter, to multiply by .3.

When the rate is 33s. 4d. Sterling money is easily converted into guilders, or guilders into Sterling, the guilder in this case being equal to 2s. Sterling.

4. A merchant in London remits to Rotterdam 1000 l. Sterling, exchange at 35s. 4d. and some time afterwards draws for 1000 l. Sterling, exchange at 34s. 3d.: How much Flemish money has he still in the bank, and how much Sterling money ought he to have drawn for to bring home the whole remittance?

s. d.		s. d.
35 4	1000 = 10 × 10 × 10	1 1
34 3		10
1 1		10 10
		10
		5 8 4
		10
		L. 54 3 4 <i>Flem. in bank.</i>

To find how much Sterling must be drawn for to bring home the remittance, say,

s. d.	s. d.	L.St.	L. s. d.	
As 34 3	: 35 4	:: 1000	: 1031 12 7	<i>Ster. Ans.</i>
12	12			
411	424			

5. When the rate of exchange is 36s. 8d. what is the value of the guilder?

s. d.		d.St.	d. Flem. in 1 guilder.
36 8			
12			
		Flem. d. 4) 440	: 240 :: 40
		11	6
			40
			11) 240 (21 $\frac{2}{11}$ d. <i>Sterl. Ans.</i>

6. When the value of the guilder is 21 $\frac{2}{11}$ d. Sterling, what is the rate of exchange?

d.St.	d.Fl.	d.St.	
If 21 $\frac{2}{11}$	: 40	:: 240	
11		11	

6) 240	2640	
--------	------	--

4) 4	44	
------	----	--

1	11	
---	----	--

40	40	
----	----	--

12) 440 (36 8	Flem.	<i>Ans.</i>
---------------	-------	-------------

7. When

7. When the current guilder is worth 22 d. Sterling, what is the value of the bank guilder, the agio being 4 per cent.?

d.
If 100 : 104 :: 22

22

—
208

208

100)2288(22.88 d. Sterling. *Anf.*

M O R E E X A M P L E S.

8. Britain draws on Holland for 1814 l. Sterling: How many guilders will that come to, exchange at 34 s. 8 d. Flemish *per* l. Sterling? *Anf.* 18865 guilders 12 stivers.

9. What may I draw for on London, if I pay in Amsterdam 3628 guilders, exchange at 35 s. 2½ d. *per* l. Sterling? *Anf.* 343 l. 9 s. 7 d.

10. What Sterling money is equivalent to 687 guilders 5 stivers, exchange at 33 s. 4 d. *per* l. Sterling? *Anf.* L. 68 : 14 : 6.

Holland exchanges with other nations as follows, *viz.* with

	<i>Flem.d.</i>		<i>Flem.d.</i>
Hamburgh, on the dollar,	= 66½	Leghorn, on the piaſtre,	= 100
France, on the crown,	= 54	Florence, on the crown,	= 120
Spain, on the ducat,	= 109¼	Naples, on the ducat,	= 74½
Portugal, on the cruſade,	= 50	Rome, on the crown,	= 136
Venice, on the ducat,	= 93	Milan, on the ducat,	= 102
Genoa, on the pezzo,	= 100	Bologna, on the dollar,	= 94¼

Exchange betwixt Britain and Antwerp, as alſo the Auſtrian Netherlands, is negotiated the ſame way as with Holland, only the par is ſomewhat different, as will be deſcribed in article 2. following.

II. *Exchange with Hamburgh.*

M O N E Y - T A B L E.

	<i>Par in Sterling.</i>	<i>s.</i>	<i>d.</i>
12 Phenningſ } 16 Schilling-lubs }	{ 1 ſchilling-lub =	0	1½
2 Marks } 3 Marks } 6¼ Marks }	{ 1 mark = 1 dollar = 1 rixdollar = 1 ducat =	1 3 4 9	6 0 6 4½

Books and accounts are kept at the bank, and by moſt people in the

the city, in marks, schilling-lubs, and phennings; but some keep them in pounds, schillings, and groots Flemish.

The agio at Hamburgh runs between 20 and 40 per cent. All bills are paid in bank-money.

Hamburgh exchanges with Britain, by giving an uncertain number of schillings and groots Flemish for the pound Sterling. The groot or penny Flemish here, as also at Antwerp, is worth $\frac{3}{8}$ of a penny Sterling; and so something better than in Holland, where it is only $\frac{3}{100}$ d. Sterling.

		<i>Flemish.</i>
6 Phennings	} make	1 groot or penny.
6 Schilling-lubs		1 schilling.
1 Schilling-lub		2 pence or groots.
1 Mark		32 pence or groots.
7½ Marks		1 pound.

The par with Hamburgh, and also with Antwerp, is 35s. 6½ d. Flemish for 1l. Sterling.

E X A M P L E S.

1. How many marks must be received at Hamburgh for 300l. Sterling, exchange at 35s. 3 d. Flemish per 1. Sterling?

<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>L.</i>	<i>Decimally.</i>
If 1	: 35	3	:: 300	<i>Flem.s. Marks. Flem.s.</i>
	12			If 20 : 7.5 :: 35.25
				4 : 1.5 :: 35.25
				1.5
423				17625
300				3525
			<i>M. fch.</i>	
32)126900			(3965 10	
9600				4)52.875
309				Marks in 1l. Sterling 13.21875
288				300
210				Marks in 300l. Sterling 3965.62500
192				16
180				3750
160				625
(20)				Schilling-lubs 10.000
16				
320				
32				
(0)				

2. How

2. How much Sterling money will a bill of 3965 marks 10 schilling-lubs amount to, exchange at 35 s. 3 d. Flemish *per* l. Sterling?

<i>Fl.s. d. L.St.</i>	<i>Mks. sch.</i>	<i>Decimally.</i>
If 35 3 : 1 ::	3965 10	4 : 1.5 ::
12	32 2	1.5
423	7930 20 d.	17625
11897		3525
423)126900(300 l. Ster.		4)52.875(13.21875
1269		
	13.21875)3965.62500(300 l. Ster.	
	3965625	

3. Hamburgh is indebted to Britain 2648 marks current money: For how many marks may Britain draw on the bank, the *agio* being 30 *per cent.*?

<i>Marks.</i>	<i>Mks. sch. phen.</i>
As 130 : 100 :: 2648	
13 : 10	10
	13)26480(2036 14 9 <i>Anf.</i>

4. What is the Sterling value of a mark-lub, when the exchange between Britain and Hamburgh is 36 s. 8 d. Flemish *per* l. Sterling?

<i>Flem.s. d.</i>	<i>d.St. d. Flem.</i>	
If 36 8 : 240 ::	32	
12	6	
440	11)192(17 $\frac{5}{11}$ d. Ster. <i>Anf.</i>	
44 : 24	11	
11 : 6		
	82	
	77	
	(5)	

Because 40 d. Flemish make a guilder of Holland, and 32 d. Flemish make a mark of Hamburgh, you may reduce marks to guilders, by saying, As 40 to 32, so the given marks to the guilders sought. Or, from the given number of marks subtract one-fifth of itself, and the remainder is the answer.

E X A M P L E.

How many guilders of Holland are equivalent to 1730 marks of Hamburg?

Marks.	Or thus:
If 40 : 32 :: 1730	1730 marks.
5 : 4 4	$\frac{1}{5} = 346$
<u> </u> Guild.	<u> </u>
5)6920(1384 Ans.	1384 guilders.

Hamburg exchanges with other countries as follows, viz. with

Holland, on the dollar,	= 66 $\frac{2}{3}$ d. Flemish.
France, on the crown,	= 51 $\frac{1}{2}$ d. Flemish.
Spain, on the ducat,	= 52 $\frac{1}{2}$ schilling-lubs.
Portugal, on the crusade,	= 48 d. Flemish.
Venice, on the ducat,	= 89 $\frac{3}{8}$ d. Flemish.
Leghorn, on the piafre,	= 96 d. Flemish.

III. *Exchange with France.*

M O N E Y - T A B L E.

	Par in Ster.	s.	d.
12 deniers	} make	1 sol	= 0 0 $\frac{1}{8}$ $\frac{2}{8}$
20 fols		1 livre	= 0 9 $\frac{3}{4}$
3 livres		1 crown	= 2 5 $\frac{1}{4}$

At Paris, Rouen, Lyons, &c. books and accounts are kept in livres, fols, and deniers; and the exchange with Britain is on the crown, or ecu, of 3 livres, or 60 fols Tournois. Britain gives for the crown an uncertain number of pence, commonly between 30 and 34, the par, as mentioned above, being 29 $\frac{1}{4}$ d.

E X A M P L E S.

1. What Sterling money must be paid in London to receive in Paris 1978 crowns 25 fols, exchange at 31 $\frac{1}{8}$ d. per crown?

Sols.

Sols.	d.	Cr.	fols.	By Practice.		
				Cr.	fols.	
If 60 : 31 $\frac{1}{8}$:: 1978 25				1978 25, at 31 $\frac{1}{8}$ d.		
		60		d.		
253				30 = $\frac{1}{8}$	247	5 0
	118705			1 $\frac{1}{2}$ = $\frac{1}{16}$	12	7 3
	253			$\frac{1}{8}$ = $\frac{1}{12}$	1	0 7 $\frac{1}{2}$
				Sols 20 = $\frac{1}{3}$	0	0 10 $\frac{1}{2}$
				5 = $\frac{1}{4}$	0	0 2 $\frac{1}{2}$
	356115					
	593525					
	237410					
					260	13 11 $\frac{1}{2}$
60)300323615						
	Rem.					
8)500539	3					
12)62567	11					
20)52113	13					
	L. 260	13	11 $\frac{1}{2}$	Ans.		

If you work decimally, say,

Cr.	d. Ster.	Cr.	d. Ster.
As 1	: 31.625 ::	1978.416	: 62567.427083

2. How many French livres will L. 121 : 18 : 6 Sterling amount to, exchange at 32 $\frac{7}{8}$ d. per crown ?

d.	Liv.	L.	s.	d.
If 32 $\frac{7}{8}$: 3 :: 121 18 6				
	8	20		
263	—	—		
	24	2438		
		12		
		—		
		29262		
		24		
		—		
		117048		
		58524		
		—		
		Liv. fols. den.		
		263)702288(2670 5 11		Ans.
		Rem. (78) = 5 fols 11 deniers.		

3. What must I pay in Britain to receive in France 3965 livres, exchange at 31 d. Sterling per crown ?

Liv.

Liv. d. Liv. L. s. d.
 As 3 : 31 :: 3956 : 170 6 $6\frac{2}{3}$

Or thus :

3)3956

8)1318 13 4

30)164 16 8 at 30 d.
 5 9 $10\frac{2}{3}$ at 1 d.

L. 170 6 $6\frac{2}{3}$ *Ans.*

4. A banker in London, with a view to make some little gain, remits to France 6000 crowns, at a time when the rate of exchange was low, *viz.* at 30 d. Sterling *per* crown; and when the exchange rose to $31\frac{1}{2}$ d. he drew for the same number of crowns: What did he gain, and how much *per cent.*?

Cr.
 8)6000

Then say,

If 30 : 1.5 :: 100
 100

20) 750 L. paid at 30 d.

37 10, at $1\frac{1}{2}$, equal to the gain.787 10 Received at $31\frac{1}{2}$ d.3)150.0 (5 *per cent.* *Ans.*

M O R E E X A M P L E S.

5. What is the value of 930 crowns 26 sols 3 deniers, at $32\frac{1}{8}$ d. *per* crown? *Ans.* 126 l. 9 s. $7\frac{1}{2}$ d.

6. How many French crowns shall I have for 102 l. 13 s. $11\frac{1}{2}$ d. at $31\frac{3}{8}$ d. *per* crown? *Ans.* 785 crowns 34 sols.

France exchanges with other places also on the crown, the par being as follows, *viz.*

1 French crown in
 Spain = $184\frac{3}{8}$ mervadies.
 Portugal = $431\frac{1}{2}$ rees.
 Milan = 62 soldi.
 Sardinia = $92\frac{1}{2}$ soldi.

100 French crowns in
 Naples = $72\frac{1}{4}$ ducats.
 Florence = 45 crowns.
 Denmark = 54 rubles.
 Sweden = 78 rixdollars.

IV. Exchange with Portugal.

M O N E Y - T A B L E.

	Par in Ster.	s.	d.	f.
1 ree	=	0	0	0.27
400 rees	} make {	1 crusade	=	2 3
1000 rees		1 millree	=	5 $7\frac{1}{2}$

In Lisbon, Oporto, &c. books and accounts are generally kept in rees and millrees; and the millrees are distinguished from the rees, by a mark set between them thus, 485 Ψ 372; that is, 485 millrees and 372 rees.

Britain, as well as other nations, exchanges with Portugal on the millree, the par, as in the table, being $67\frac{1}{2}$ d. Sterling. The course with Britain runs from 63 d. to 68 d. Sterling *per* millree.

E X A M P L E S.

1. How much Sterling money will pay a bill of 827 Ψ 160 rees, exchange at $63\frac{3}{8}$ d. Sterling *per* millree?

Rees.	d.	Rees.	d.	By Practice.
If 1000 :	$63\frac{3}{8}$:	827.160		Rees.
8		507		827.160, at $63\frac{3}{8}$ d.
8000	507	579012		206.790
		413580		10.3395
		Rem.		.861625
		8000)419370.120	2	.4308125
				218.4219375
		12) 52421	5 d.	
		20) 4368	8 s.	
		L. 218 8 $5\frac{1}{4}$	Ans.	

The rees being thousandth parts of the millrees, are annexed to the integer, and the operation proceeds exactly as in decimals.

2. How many rees of Portugal will 500 l. Sterling amount to, exchange at 5 s. $4\frac{5}{8}$ d. *per* millree?

d.	Rees.	L.
If $64\frac{5}{8}$:	1000 :	500
8	8	20
517	8000	10000
		12
		120000
		8000
	Rees.	
	517)9600000000	(1856.866 Ans.

M O R E E X A M P L E S.

3. How much Sterling money will pay a bill of 181 millrees, exchange at $64\frac{3}{4}$ d. *per* millree? *Ans.* 48 l. 12 s. $10\frac{1}{2}$ d.

4. If

4. If I pay in London 933l. 18s. 8d. how many rees shall I receive at Lisbon, exchange at 5s. 4d. *per millree*? *Ans.* 3502.250 rees.

Portugal exchanges with other places as follows, *viz.* with

	<i>Rees.</i>
Spain, on the piaftre,	= 637
Venice, on the ducat,	= 744
Genoa, on the pezzo,	= 800
Naples, on the ducat,	= 597
Sardinia, on the dollar,	= 637
Palermo, on the crown,	= 889

V. *Exchange with Spain.*

M O N E Y - T A B L E.

		<i>Par in Ster.</i>	<i>s.</i>	<i>d.</i>
34 mervadies	} make {	1 rial	= 0	5 $\frac{3}{8}$
8 rials		1 piaftre	= 3	7
375 mervadies		1 ducat	= 4	11 $\frac{1}{4}$

In Madrid, Bilboa, Cadiz, Malaga, Seville, and most of the principal places, books and accounts are kept in piaftres, called also *dollars*, rials, and mervadies; and they exchange with Britain generally on the piaftre, and sometimes on the ducat. The course runs from 35d. to 45d. Sterling for a piaftre or dollar of 8 rials.

E X A M P L E S.

1. London imports from Cadiz goods to the value of 2163 piaftres and 4 rials: How much Sterling will this amount to, exchange at 38 $\frac{1}{8}$ d. Sterling *per* piaftre?

<i>Piaft. Rials.</i>		
2163 4, at $38\frac{1}{8}$ d.		
<i>d.</i>		<i>d.</i>
24 = $\frac{1}{10}$	216 6	<i>Rials.</i> $38\frac{1}{8}$ each.
12 = $\frac{1}{2}$	108 3	
2 = $\frac{1}{6}$	18 0 6	
$\frac{2}{8}$ = $\frac{1}{8}$	2 5 $0\frac{3}{4}$	
$\frac{1}{8}$ = $\frac{1}{2}$	1 2 $6\frac{3}{8}$	
	345 17 $1\frac{1}{8}$	4 = $19\frac{3}{8}$
	1 7 $7\frac{1}{16}$	

L. 345 18 8 $\frac{5}{16}$ *Ans.*

2. London remits to Cadiz 345l. 18s. 8 $\frac{5}{16}$ d.: How much Spanish

Spanish money will this amount to, exchange at $38\frac{1}{2}$ d. Sterling per piaftre ?

d.	Piaft.	L.	s.	d.	
If $38\frac{1}{2}$	: 1	:: 345	18	$8\frac{1}{8}$	(614) 1328389 (2163 piaftres.
		20			1228...
307					
2		6918		1003	
		12		614	
614					
		83024		3898	
		16		3684	
		498149		2149	
		83024		1842	
Carried up,	1328389			307	
				8	
	Piaft. Rials.			(614) 2456 (4 rials.	
Ans. 2163	4			2456	

M O R E E X A M P L E S.

3. Spain remits to Britain 1768 dollars, or piaftres, 7 rials, at $40\frac{1}{2}$ d. Sterling per piaftre : How much Sterling will this amount to? *Ans.* L. 301 : 5 : $2\frac{1}{8}$.

4. If I pay in Britain L. 1000 : 16 : 10, how much Spanish money may I draw for, exchange at $41\frac{1}{2}$ d. Sterling per piaftre ? *Ans.* 5788 piaftres.

The filver money in Spain is of two forts, viz. old and new plate : the old plate is 25 per cent. better than the new ; and fo the piaftre of exchange confifts of 8 rials old plate, or of 10 rials new plate.

The copper money in Spain is called vellon ; and though the piaftre vellon be of the fame value with the piaftre of exchange old plate, yet the rial vellon is little more than half a rial old plate ; for 32 rials vellon are equal in value to 17 rials old plate, or 100 rials vellon are equal to $53\frac{1}{8}$ rials old plate.

Spain exchanges with other places as follows, viz. with

Mervadies.

Venice, on the ducat,	= 318
Genoa, on the pezzo,	= $341\frac{1}{8}$
Rome, on the crown,	= 464
Florence, on the crown,	= 409
Naples, on the ducat,	= 255
Milan, on the ducat,	= 348
Bologna, on the dollar,	= $322\frac{1}{8}$
Messina, on the crown,	= 380

VI. Ex.

VI. *Exchange with Venice.*

M O N E Y - T A B L E.

$$\left. \begin{array}{l} 5\frac{1}{8} \text{ Soldi} \\ 24 \text{ Gros} \end{array} \right\} \text{ make } \left\{ \begin{array}{l} 1 \text{ gros.} \\ 1 \text{ ducat} = 50\frac{1}{4} \text{ d. Sterling.} \end{array} \right.$$

The money of Venice is of three sorts, *viz.* two of bank money, and the picoli money. One of the banks deals in banco money, and the other in banco current. The bank money is 20 *per cent.* better than the banco current, and the banco current 20 *per cent.* better than the picoli money. Exchanges are always negotiated by the ducat banco, the par being 4s. 2 $\frac{1}{4}$ d. Sterling, as in the table.

Though the ducat be commonly divided into 24 gros, yet bankers and negotiators, for facility of computation, usually divide it as follows, and keep their books and accounts accordingly.

$$\left. \begin{array}{l} 12 \text{ Deniers d'or} \\ 20 \text{ Sols d'or} \end{array} \right\} \text{ make } \left\{ \begin{array}{l} 1 \text{ fol d'or.} \\ 1 \text{ ducat} = 50\frac{1}{4} \text{ d. Sterling.} \end{array} \right.$$

The course of exchange is from 45 d. to 55 d. Sterling *per* ducat.

E X A M P L E S.

1. How much Sterling money is equal to 1459 ducats 18 sols 1 denier, bank money of Venice, exchange at 52 $\frac{3}{4}$ d. Sterling *per* ducat?

Duc. d.	Duc. sol. den.	d.	Sols.	den.	d.
If 1 : 52 $\frac{3}{4}$::	1459 18 1	52 $\frac{3}{4}$ rate.			
	52 $\frac{3}{4}$		10 = $\frac{1}{2}$		26 $\frac{3}{8}$
	2918		5 = $\frac{1}{4}$		13 $\frac{3}{8}$
	7295		2 = $\frac{1}{8}$		5 $\frac{1}{4}$
d. 75868			1 = $\frac{1}{16}$		2 $\frac{3}{8}$
$\frac{1}{4}$ = 729 $\frac{5}{8}$			1 = $\frac{1}{16}$		2 $\frac{3}{8}$
$\frac{1}{4}$ = 364 $\frac{5}{8}$					47 $\frac{5}{8}$
	76962 $\frac{3}{8}$				
	47 $\frac{5}{8}$				
	Rem.				
12) 77010	(6d.				
20) 6417	(17s.				
L. 320 = 17	6 Sterling.				

Ans.

2. How

2. How many ducats at Venice are equal to 385l. 12 s. 6 d. Sterling, exchange at 4 s. 4 d. *per* ducat?

$$\begin{array}{r}
 \text{L. Duc. L.} \\
 \text{If } .21\phi : 1 :: 385.625 \\
 \quad .21\phi) 385.625 \\
 \quad \underline{21} \quad 385625 \\
 \quad \quad \quad \text{Duc.} \\
 \quad 195) 347062.5 (1779.8 \text{ Ans.} \\
 \quad \quad \underline{195} \\
 \quad \quad 1520 \\
 \quad \quad \underline{1365} \\
 \quad \quad \quad 1556 \\
 \quad \quad \quad \underline{1365} \\
 \quad \quad \quad \quad 1912 \\
 \quad \quad \quad \quad \underline{1755} \\
 \quad \quad \quad \quad \quad 1575 \\
 \quad \quad \quad \quad \quad \underline{1560} \\
 \quad \quad \quad \quad \quad \quad (15)
 \end{array}$$

Bank money is reduced to current money, by allowing for the *agio*, as was done in exchange with Holland; *viz.* say, As 100 to 120, or as 10 to 12, or as 5 to 6, so the given bank money to the current sought. And current money is reduced to bank money by reversing the operation. And in like manner may *picoli* money be reduced to current or to bank money, and the contrary.

100 ducats banco of Venice.

In Leghorn = 93 pezzos	In Lucca = 77 crowns
In Rome = 68½ crowns	In Francfort = 139½ florins.

VII. Exchange with Genoa.

M O N E Y - T A B L E.

12 Denari } make { 1 soldi s. d.
20 Soldi } { 1 pezzo = 4 6 Sterling.

Books and accounts are generally kept in pezzos, soldi, and denari; but some keep them in lires, soldi, and denari; and 12 such denari make 1 soldi, and 20 soldi make 1 lire.

The pezzo of exchange is equal to 5½ lires; and, consequently, exchange money is 5½ times better than the lire money. The course of exchange runs from 47 d. to 58 d. Sterling *per* pezzo.

E X A M P L E.

How much Sterling money is equivalent to 3390 pezzos 16 soldi, of Genoa, exchange at $51\frac{7}{8}$ d. Sterling *per pezzo*?

$$\begin{array}{rcl} \text{Soldi} & \text{d.} & \text{Pez. soldi.} \\ \text{If } 20 : 51\frac{7}{8} :: 3390 & 16 & \end{array}$$

$$\begin{array}{r} 8 \\ \hline 160 \end{array} \quad \begin{array}{r} 415 \\ \hline 67816 \\ 415 \\ \hline \end{array}$$

$$\begin{array}{r} 339080 \\ 67816 \\ \hline 271264 \end{array}$$

$$160)28143640(175897\frac{3}{4} = 732 \text{ } 18 \text{ } 1\frac{3}{4}$$

If Sterling money be given, it may be reduced or changed into pezzos of Genoa, by reverfing the former operation.

Exchange money is reduced to lire money, by being multiplied by $5\frac{3}{4}$, as follows:

Pez. soldi.

$$\begin{array}{r} 3390 \text{ } 16 \\ \hline 5\frac{3}{4} \end{array}$$

Decimally.

$$\begin{array}{r} 3390.8 \\ \hline 5.75 \end{array}$$

$$\begin{array}{r} 16954 \text{ } 0 \\ \frac{1}{2} = 1695 \text{ } 8 \\ \frac{1}{4} = 847 \text{ } 14 \\ \hline \end{array}$$

$$\begin{array}{r} 169540 \\ 237356 \\ 169540 \\ \hline \end{array}$$

Lires 19497 2

Lires 19497.100

And lire money is reduced to exchange money by dividing it by $5\frac{3}{4}$.

Soldi of Genoa.

$$\begin{array}{lcl} \text{In Milan, 1 crown} & = & 80 \\ \text{In Naples, 1 ducat} & = & 86 \\ \text{In Leghorn, 1 piaftre} & = & 20 \\ \text{In Sicily, 1 crown} & = & 127\frac{3}{5} \end{array}$$

VIII. Exchange with Leghorn.

M O N E Y - T A B L E.

$$\begin{array}{lcl} 12 \text{ Denari} & \} & \text{make } \{ 1 \text{ foldi} \quad \text{s. } d. \\ 20 \text{ Soldi} & \} & \{ 1 \text{ piaftre} = 4 \text{ } 6 \text{ Sterling.} \end{array}$$

Books and accounts are kept in piaftres, foldi, and denari. The piaftre here confifts of 6 lires, and the lire contains 20 foldi, and the foldi 12 denari, and confequently exchange money is 6 times better than lire money. The courfe of exchange is from 47 d. to 58 d. Sterling *per piaftre*.

E X.

E X A M P L E.

What is the Sterling value of 731 piaftres, at $55\frac{1}{2}$ d. each?

s.	d.		731 piaftres, at $55\frac{1}{2}$ d.
4 or	48	$= \frac{1}{2}$	146 4
	6	$= \frac{1}{8}$	18 5 6
	$1\frac{1}{2}$	$= \frac{1}{4}$	4 11 $4\frac{1}{2}$

L. 169 0 $10\frac{1}{2}$ *Anf.*

Sterling money is reduced to money of Leghorn, by reverfing the former operation; and exchange money is reduced to lire money by multiplying by 6, and lire money to exchange money by dividing by 6.

100 piaftres of Leghorn are

In Naples = 134 ducats. | In Geneva = $185\frac{1}{3}$ crowns.

Soldi of Leghorn.

In Sicily, 1 crown = $133\frac{1}{3}$

In Sardinia, 1 dollar = $95\frac{1}{2}$

The above are the chief places in Europe with which Britain exchanges directly; the exchanges with other places are generally made by bills on Hamburgh, Holland, or Venice. I fhall here however fubjoin the par of exchange betwixt Britain and moft of the other places in Europe, with which we have any commercial intercourfe.

	Par in Sterling.	L.	s.	d.
Rome,	1 crown	=	6	$1\frac{2}{3}$
Naples,	1 ducat	=	3	$4\frac{1}{2}$
Florence,	1 crown	=	5	$4\frac{5}{8}$
Milan,	1 ducat	=	4	7
Bologna,	1 dollar	=	4	3
Sicily,	1 crown	=	5	0
Vienna,	1 rixdollar	=	4	8
Aufburgh,	1 florin	=	3	$1\frac{1}{2}$
Francfort,	1 florin	=	3	0
Bremen,	1 rixdollar	=	3	6
Brefflau,	1 rixdollar	=	3	3
Berlin,	1 rixdollar	=	4	0
Stetin,	1 mark	=	1	6
Emden,	1 rixdollar	=	3	6
Bolfenna,	1 rixdollar	=	3	8
Dantzic,	$13\frac{1}{2}$ florins	=	1	0 0
Stockholm,	$34\frac{4}{7}$ dollars	=	1	0 0
Ruffia,	1 ruble	=	4	5
Turkey,	1 asper	=	4	6
	D d			

The

The following places, *viz.* Switzerland, Nuremburgh, Leipfic, Dresden, Osnaburgh, Brunswic, Cologne, Leige, Straßburgh, Cracow, Denmark, Norway, Riga, Revel, Narva, exchange with Britain, when direct exchange is made, upon the rixdollar, the par being 4s. 6d. Sterling.

IX. *Exchange with America and the West Indies.*

In North America and the West Indies, accounts, as in Britain, are kept in pounds, shillings and pence. In North America they have few coins circulating among them, and on that account have been obliged to substitute a paper-currency for a medium of their commerce; which having no intrinsic value, is subjected to many disadvantages, and generally suffers a great discount. In the West Indies coins are more frequent, owing to their commercial intercourse with the Spanish settlements.

Exchange betwixt Britain and America, or the West Indies, may be computed as in the following examples.

1. The neat proceeds of a cargo from Britain to Boston amount to 845l. 17s. 6d. currency: How much is that in Sterling money, exchange at 80 *per cent.*?

If 180 : 100

18 : 10 L. s. d.
9 : 5 :: 845 17 6

9)4229 7 6

L. 469 18 7½ Sterling. *Anf.*

2. Boston remits to Britain a bill of 469l. 18s. 7½d. Sterling: How much currency was paid for the bill at Boston, exchange at 80 *per cent.*?

If 100 : 180 L. s. d.

5 : 9 :: 469 18 7½

5)4229 7 6

845 17 6 currency. *Anf.*

3. Philadelphia is indebted to Britain in 985l. 12s. 3d. currency: How much Sterling will that amount to, exchange at 75 *per cent.*?

If

$$\begin{array}{rcl} \text{If } 175 : 100 & & \\ 35 : 20 & L. \ s. \ d. & \\ 7 : 4 :: 985 \ 12 \ 3 & & \\ & & 4 \end{array}$$

$$\begin{array}{r} 7 \overline{)3942 \ 9} \end{array}$$

563 4 1 $\frac{5}{7}$ Sterling. *Ans.*

4. Philadelphia grants a bill on Britain for 563l. 4s. 1 $\frac{5}{7}$ d. Sterling: What is the value of the bill in currency, exchange at 75 per cent.?

$$\begin{array}{rcl} \text{If } 100 : 175 & L. \ s. \ d. & \\ 4 : 7 :: 563 \ 4 \ 1\frac{5}{7} & & \\ & & 7 \end{array}$$

$$\begin{array}{r} 4 \overline{)3942 \ 9} \end{array}$$

Ans. 985 12 3 cur.

By Practice thus:

	$L. \ s. \ d.$
100 =	563 4 1 $\frac{5}{7}$
50 = $\frac{1}{2}$	281 12 0 $\frac{6}{7}$
25 = $\frac{1}{4}$	140 16 0 $\frac{3}{7}$
175	985 12 3 cur.

5. A parcel of goods from Britain, amounting, by invoice, to 323l. 5s. Sterling, are sold by a factor in Virginia for L. 469 : 2 : 6 currency: What Sterling ought the factor to remit, deducting 5 per cent. for commission? and how much does the employer gain per cent. when exchange is at 30 per cent.?

Exch. Com.

$$\begin{array}{rcl} \text{If } 130 + 5 = 135 : 100 & L. & \\ 27 : 20 :: 469.125 & & \\ & & 20 \\ 3 \overline{)9382.500} & & \\ 9 \overline{)3127.5} & & \end{array}$$

347.5 Ster. to be remitted.
323.25 value consigned.
24.25 gain.

$$\begin{array}{rcl} \text{If } 323.25 : 24.25 :: 100 & & \\ & & 100 \end{array}$$

$$\begin{array}{rcl} 323.25 \ 2425.00 (7.5 \text{ or } 7\frac{1}{2} \text{ per cent.} & & \\ 226275 & & \end{array}$$

$$\begin{array}{r} 162250 \\ 161625 \\ \hline 625 \end{array}$$

D d 2

6. How

6. How much currency in Virginia will purchase a bill of 345l. 13s. 4d. Sterling, when exchange is $33\frac{1}{3}$ per cent.?

If 100 : $133\frac{1}{3}$

3

300 : 400

L. s. d.

3 : 4 :: 345 13 4

3)1382 13 4

Ans. currency 460 17 $9\frac{1}{3}$

By Practice.

L. s. d.

100 = 345 13 4

$33\frac{1}{3} = \frac{1}{3}$ 115 4 $5\frac{1}{3}$

133 $\frac{1}{3}$ 460 17 $9\frac{1}{3}$ currency.

7. If an ounce of gold, worth 4l. Sterling be rated in Carolina at 9l. 15s. 7d. currency, how much currency will 100l. Sterling purchase?

L. currency.

If 4 : 9 15 7 :: 100

1

12

L.

25 = 12 + 12 + 1

117 7

117 7

9 15 7

L. 244 9 7 currency. Ans.

8. How much Sterling money will 1780l. Jamaica currency amount to, exchange at 40 per cent.?

If 140 : 100

14 : 10 L.

7 : 5 :: 1780

5

7)8900

1271 8 $6\frac{1}{4}$

Sterling. Ans.

Bills of exchange from America, the rate being high, is an expensive way of remitting money to Britain; and therefore merchants in Britain generally chuse to have the debts due to them remitted home in sugar, rum, or other produce.

X. Exchange with Ireland.

At Dublin, and all over Ireland, books and accounts are kept in pounds, shillings and pence, as in Britain; and they exchange on the 100l. Sterling.

The

The par of one shilling Sterling is one shilling and one penny Irish; and so the par of 100l. Sterling is 108l. 6s. 8d. Irish. The course of exchange runs from 6 to 15 per cent.

E X A M P L E S.

1. London remits to Dublin 586l. 10s. Sterling: How much Irish money will that amount to, exchange at $9\frac{1}{8}$ per cent.?

L.		By Practice.	
If 100	: 109 $\frac{1}{8}$:: 586.5		586.5
8	877	p.cent.	
		10 = $\frac{1}{10}$	58.65
800	: 877	2 = $\frac{2}{5}$	11.73 sub.
	41055		
	41055	8 =	46.92
	46920	1 = $\frac{1}{8}$	5.865
		$\frac{1}{4}$ = $\frac{1}{4}$	2.9325
		$\frac{1}{8}$ = $\frac{1}{8}$	.733125
	800)514360.5		
	642.950625	9 $\frac{1}{8}$	56.450625 add.
			642.950625

Anf. 642l. 19s. Irish.

2. How much Sterling will 625l. Irish amount to, exchange at $10\frac{3}{8}$ per cent.?

If 110 $\frac{3}{8}$	: 100 :: 625		
8	800		
		L.	s. d.
883	800	883)500000	(566 5 0 $\frac{1}{4}$ Ster. Anf.

3. Britain remitted to Ireland 10000l. Sterling, when exchange was at 12 per cent. but, exchange falling to 8 per cent. Britain draws back the whole remittance: How much Sterling was drawn back, and how much did Britain gain?

As 108 : 112 :: 10000

		L.	s.	d.
108)	1120000	(10370	7	4 $\frac{1}{4}$ Sterling drawn back.
		10000		

370 7 4 $\frac{1}{4}$ gained.

Which is more than $3\frac{1}{2}$ per cent.

4. When exchange was up at 12 per cent. Dublin drew on London for 10000l. Sterling; and when exchange fell to 8 per cent. Dublin remitted 10000l. Sterling to London: How much Irish money did Dublin gain?

D d 3

Irish

	<i>Irish.</i>	
	112	
	108	
<i>Ster.</i>	—	<i>Ster.</i>
If 100 :	4 ::	10000
	10000	

100)40000(400l. Irish gained.

XI. Exchange betwixt London and other places in Britain.

The several towns in Britain exchange with London for a small premium in favour of London; such as, 1, $1\frac{1}{4}$, &c. *per cent.* The premium is more or less according to the demand for bills.

E X A M P L E.

Edinburgh draws on London for 860 l. exchange at $1\frac{3}{8}$ *per cent.*: How much money must be paid at Edinburgh for the bill?

	<i>L.</i>
	860
<i>per cent.</i>	—
1 = $\frac{1}{100}$	8 12
$\frac{2}{8} = \frac{1}{4}$	2 3
$\frac{1}{8} = \frac{1}{2}$	1 1 6
	—
	11 16 6 premium.
	—
	871 16 6 paid for the bill.

To avoid paying the premium, it is an usual practice to take the bill payable at London a certain number of days after date; and in this way of doing, 73 days is equivalent to 1 *per cent.*

XII. Arbitration of Exchanges.

The course of exchange betwixt nation and nation naturally rises or falls according as the circumstances and balance of trade happen to vary. Now to draw upon, and remit to foreign places, in this fluctuating state of exchange, in the way that will turn out most profitable, is the design of arbitration. Which is either simple or compound.

I. Simple arbitration.

In simple arbitration the rates or prices of exchange from one place to other two are given; whereby is found the correspondent price between the said two places, called the *arbitrated price*, or *par of arbitration*: and hence is derived a method of drawing and remitting to the best advantage.

E X A M P L E S.

1. If exchange from London to Amsterdam be 33s. 9d. *per* l. Sterling; and if exchange from London to Paris be 32 d. *per* crown; what must be the rate of exchange from Amsterdam to Paris, in order to be on a par with the other two?

<i>Ster.</i>	<i>Flem.</i>	<i>Ster.</i>
<i>s.</i>	<i>s. d.</i>	<i>d.</i>
If 20 :	33 9 ::	32
12	12	
240	405	
	32	
	810	
	1215	

240)12960(54 d. Flemish *per* crown. *Ans.*

2. If exchange from Paris to London be 32 d. Sterling *per* crown; and if exchange from Paris to Amsterdam be 54 d. Flemish *per* crown; what must be the rate of exchange between London and Amsterdam, in order to be on a par with the other two?

<i>Ster.</i>	<i>Flem.</i>	<i>Ster.</i>
<i>d.</i>	<i>d.</i>	<i>d.</i>
If 32 :	54 ::	240
	240	
	216	
	108	
	12)	<i>s. d.</i>

32)12960(405 (33 9 Flemish *per* l. Sterling. *Ans.*

3. If exchange from Amsterdam to Paris be 54 d. Flemish *per* crown; and if exchange from Amsterdam to London be 33 s. 9d. Flemish *per* l. Sterling; what must be the rate of exchange between Paris and London, in order to be on a par with the other two?

<i>Flem.</i>	<i>Ster.</i>	<i>Flem.</i>
<i>s. d.</i>	<i>d.</i>	<i>d.</i>
If 33 9 :	240 ::	54
12	54	
405	96	
	120	

405)12960(32 d. Sterling *per* crown. *Ans.*

D d 4

From

From these three operations it appears, that if any sum of money be remitted, at the rates of exchange mentioned, from any one of the three places to the second, and from the second to the third, and again from the third to the first, the sum so remitted will come home entire, without increase or diminution.

From the par of arbitration thus found, and the course of exchange given, is deduced a method of drawing and remitting to advantage, as in the following examples.

4. If exchange from London to Paris be 32 d. Sterling *per* crown, and to Amsterdam 405 d. Flemish *per* l. Sterling; and if, by advice from Holland or France, the course of exchange between Paris and Amsterdam is fallen to 52 d. Flemish *per* crown; what may be gained *per cent.* by drawing on Paris, and remitting to Amsterdam?

The par of arbitration between Paris and Amsterdam in this case, by Ex. 1. is 54 d. Flemish *per* crown. Work as under.

$$\begin{array}{rcl}
 d. St. Cr. & L. St. & Cr. \\
 \text{If } 32 : 1 :: 100 : 750 & \text{debit at Paris.} & \\
 \\
 Cr. d. Fl. & Cr. & d. Fl. \\
 \text{If } 1 : 52 :: 750 : 39000 & \text{credit at Amsterdam.} & \\
 \\
 d. Fl. L. St. & d. Fl. & L. s. d. Ster. \\
 \text{If } 405 : 1 :: 39000 : 96 & 5 & 11\frac{5}{8} \text{ to be remitted.} \\
 & 100 & \\
 \hline
 & & 3 \ 14 \ 0\frac{8}{9} \text{ gained per cent.}
 \end{array}$$

But if the course of exchange between Paris and Amsterdam, instead of falling below, rise above the par of arbitration, suppose to 56 d. Flemish *per* crown; in this case, if you propose to gain by the negotiation, you must draw on Amsterdam, and remit to Paris. The computation follows:

$$\begin{array}{rcl}
 L. St. d. Fl. & L. St. & d. Fl. \\
 \text{If } 1 : 405 :: 100 : 40500 & \text{debit at Amsterdam.} & \\
 \\
 d. Fl. Cr. & d. Fl. & Cr. \\
 \text{If } 56 : 1 :: 40500 : 723\frac{3}{4} & \text{credit at Paris.} & \\
 \\
 Cr. d. St. & Cr. & L. s. d. Ster. \\
 \text{If } 1 : 32 :: 723\frac{3}{4} : 96 & 8 & 6\frac{6}{7} \text{ to be remitted.} \\
 & 100 & \\
 \hline
 & & 3 \ 11 \ 5\frac{1}{7} \text{ gained per cent.}
 \end{array}$$

In negotiations of this sort, a fund for remittance is afforded out of the sum you receive for the draught; and your credit at the one foreign place pays your debit at the other.

5. London

5. London is indebted to Peterburgh 5000 rubles, and Peterburgh can draw for them directly on England at 50 d. Sterling *per* ruble, or on Holland at 90 d. Flemish *per* ruble : Which of these two ways will be most advantageous to London, supposing the course of exchange between London and Holland to be 36 s. 4 d. Flemish *per* l. Sterling ?

Find the par of arbitration, by saying,

d.St. d.Fl. d.St. d.Fl.

If 50 : 90 :: 240 : 432 or 36 s. Flemish.

If the course of exchange between London and Holland had been the same with the par of arbitration, *viz.* 36 s. payment in either way would be the same to London ; but since the course is 36 s. 4 d. it is plainly an advantage to make payment by the way of Holland ; for it is better to receive 36 s. 4 d. than 36 s. for a pound Sterling. The computation follows :

Rub. d.St. Rub. L. s. d.Ster.

If 1 : 50 :: 5000 : 1041 13 4 London to Peterburgh.

Rub. d.Fl. Rub. d.Fl.

If 1 : 90 :: 5000 : 450000 Holland to Peterburgh.

s. d.F. L.St. d.Fl. L. s. d.Ster.

If 36 4 : 1 :: 450000 : 1032 2 2 $\frac{1}{4}$ London to Holland.

9 11 1 $\frac{3}{4}$ gain.

If the course of exchange to Holland had fallen below 36 s. the direct exchange would have been preferable.

The following, or like questions, are solved, without finding the par of arbitration.

6. London was ordered to remit 500 ducats to Venice, at 50 d. Sterling *per* ducat, and to draw for the value upon Spain, at 40 d. Sterling *per* piastré ; but when the order came to hand, bills on Venice were at 52 $\frac{1}{2}$ d. : At what rate of exchange must London draw upon Spain, to compensate the advance upon the remittance ?

d. d. d.
If 50 : 52 $\frac{1}{2}$:: 40

Or, If 100 : 105 :: 40
40

100)4200(42 d. *Ans.*

Proof.

Duc. d.St. Duc. d. St.
If 1 : 50 :: 500 : 25000

d. Piastré. d. Piastré.
If 40 : 1 :: 25000 : 625

Then

Then, by the course,

<i>Duc.</i>	<i>d.</i>	<i>Duc.</i>	<i>d.</i>		<i>d. Piaft.</i>	<i>d.</i>	<i>Piaft.</i>
If 1	: 52½	::	500	:	26250		If 42 : 1 :: 26250 : 625 as before.

If the course of exchange with Spain be lower than 42 d. the order cannot be executed without loss ; and if it be higher, there will be gain.

7. London was ordered to remit to Paris 800 crowns, at 31⅜ d. Sterling *per* crown, and to draw for the value upon Amsterdam, at 36 s. 9 d. Flemish *per* pound Sterling ; but when the order came up, bills on Paris were at 31⅝ d. Sterling *per* crown : What must be the rate of exchange with Amsterdam to compensate the advance on the remittance ?

<i>d.</i>	<i>s. d.</i>	<i>d.</i>	<i>s. d.</i>
If 31⅝	: 36 9	::	31⅜ : 36 5½ nearly. <i>Anf.</i>

The proportion is inverse ; for the rate of exchange with Holland must be lessened to answer our purpose ; which may be proved as in the former example. The proportions for the remittance and draught follow :

<i>Cr.</i>	<i>d. St.</i>	<i>Cr.</i>	<i>d. Ster.</i>
If 1	: 31⅝	::	800 : 25300
<i>d. St.</i>	<i>s. d. Fl.</i>	<i>d. St.</i>	<i>d. Fl.</i>
If 240	: 36 5½	::	25300 : 46119½ nearly. <i>Anf.</i>

8. A merchant in London had 6000 guilders in the bank at Amsterdam, and was offered 22 d. Sterling for each guilder ; but as this offer did not please, he indorses a bill for the whole to his factor at Paris ; who soon brought the money to France, by exchanging at 55 d. Flemish *per* crown : he allows the factor ½ *per cent.* commission for his trouble, and then draws upon him for the whole, exchange at 32 d. Sterling *per* crown : How much was this better than the offer of 22 d. *per* guilder ?

	6000 guilders.
	40 d. Flemish in 1 guilder.
<i>d. Fl.</i>	<i>Cr.</i>
If 55	: 1 :: 240000 : 4363.63, crowns.
	21.81, com. = ⅓
	4341.81, neat.

Guild.

	<i>Guild.</i>	<i>Cr. d.</i>	<i>Cr.</i>
	6000, at 22 d.	If 1 : 32 ::	4341.81,
<i>d.</i>			32
$20 = \frac{1}{12}$	500		
$2 = \frac{1}{10}$	50		8683,63,
			130254,54,
	5501. Ster.		<i>L. s. d.</i>
		138938,18, =	578 18 2 by exch.
			550 0 0 by offer.
			Ster. 28 18 2 gained.

II. Compound arbitration.

In compound arbitration the rate or price of exchange between three, four, or more places, is given, in order to find how much a remittance passing through them all will amount to at the last place; or to find the arbitrated price, or par of arbitration, between the first place and the last. And this may be done by the following

R U L E S.

I. Distinguish the given rates or prices into antecedents and consequents; place the antecedents in one column, and the consequents in another on the right, fronting one another by way of equation.

II. The first antecedent, and the last consequent to which an antecedent is required, must always be of the same kind.

III. The second antecedent must be of the same kind with the first consequent, and the third antecedent of the same kind with the second consequent, &c.

IV. If to any of the numbers a fraction be annexed, both the antecedent and its consequent must be multiplied into the denominator.

V. To facilitate the operation, terms that happen to be equal or the same in both columns, may be dropped or rejected, and other terms may be abridged.

VI. Multiply the antecedents continually for a divisor, and the consequents continually for a dividend, and the quot will be the answer, or antecedent required.

E X A M P L E S.

1. If London remit 1000l. Sterling to Spain, by way of Holland, at 35s. Flemish *per* l. Sterling; thence to France, at 58 d. Flemish *per* crown; thence to Venice, at 100 crowns *per* 60 ducats; and thence to Spain, at 360 mervadies *per* ducat; how many piaftres, of 272 mervadies, will the 1000l. Sterling amount to in Spain?

Antecedents.

<i>Antecedents.</i>		<i>Consequents.</i>	<i>Abridged.</i>
1 l. Sterling	=	35 s. or 420 d. Flem.	1 = 210
58 d. Flemish	=	1 crown France	29 = 1
100 crowns France	=	60 ducats Venice	1 = 30
1 ducat Venice	=	360 mervadies Spain	1 = 45
272 mervadies	=	1 piaftre	17 = 1
How many piaftres	=	1000 l. Sterling ?	= 10

In order to abridge the terms, divide 58 and 420 by 2, and you have the new antecedent 29, and the new consequent 210; reject two ciphers in 100 and 1000; divide 272 and 360 by 8, and you have 34 and 45; divide 34 and 60 by 2, and you have 17 and 30; and the whole will stand abridged as above.

Then, $29 \times 17 = 493$ divisor; and, $210 \times 30 \times 45 \times 10 = 2835000$ dividend; and, $493 \overline{)2835000} (5750\frac{1}{2}$ piaftres. *Anf.*

Or, the consequents may be connected with the sign of multiplication, and placed over a line by way of numerator, and the antecedents, connected in the same manner, may be placed under the line, by way of denominator, and then abridged, as follows:

$$\frac{420 \times 60 \times 360 \times 1000}{58 \times 100 \times 272} = \frac{210 \times 60 \times 360 \times 10}{29 \times 1 \times 272} = \frac{210 \times 60 \times 45 \times 10}{29 \times 34}$$

$$= \frac{210 \times 30 \times 45 \times 10}{29 \times 17} = \frac{2835000}{493},$$

And, $493 \overline{)2835000} (5750\frac{1}{2}$ piaftres. *Anf.*

The placing the terms by way of antecedent and consequent, and working as the rules direct, saves so many statings of the rule of three, and greatly shortens the operation. The proportions at large for the above question would stand as under:

<i>L. St.</i>	<i>d. Fl.</i>	<i>L. St.</i>	<i>d. Fl.</i>
If 1	: 420 ::	1000	: 420000
<i>d. Fl.</i>	<i>Cr.</i>	<i>d. Fl.</i>	<i>Cr.</i>
If 58	: 1 ::	420000	: $7241\frac{11}{19}$
<i>Cr.</i>	<i>Duc.</i>	<i>Cr.</i>	<i>Duc.</i>
If 100	: 60 ::	$7241\frac{11}{19}$	: $4344\frac{24}{19}$
<i>Duc.</i>	<i>Mer.</i>	<i>Duc.</i>	<i>Mer.</i>
If 1	: 360 ::	$4344\frac{24}{19}$	: $1564137\frac{27}{19}$
<i>Mer.</i>	<i>Piaft.</i>	<i>Mer.</i>	<i>Piaft.</i>
If 272	: 1 ::	$1564137\frac{27}{19}$	: $5750\frac{250}{19}$

If we suppose the course of direct exchange to Spain to be $42\frac{1}{2}$ d. Sterling *per* piaſtre, the 1000 l. remitted would only amount to $5647\frac{1}{2}$ piaſtres; and, conſequently, 103 piaſtres are gained by the negotiation; that is, about 2 *per cent*.

2. A banker in Amſterdam remits to London 400 l. Flemiſh; firſt to France, at 56 d. Flemiſh *per* crown; from France to Venice, at 100 crowns *per* 60 ducats; from Venice to Hamburgh, at 100 d. Flemiſh *per* ducat; from Hamburgh to Liſbon, at 50 d. Flemiſh *per* cruſade of 400 rees; and, laſtly, from Liſbon to London, at 64 d. Sterling *per* millree: How much Sterling money will the remittance amount to? and how much will be gained or ſaved, ſuppoſing the direct exchange from Holland to London at 36 s. 10 d. Flemiſh *per* l. Sterling?

<i>Antecedents.</i>	<i>Conſequents.</i>
56 d. Flemiſh	= 1 crown.
100 crowns	= 60 ducats.
1 ducat	= 100 d. Flemiſh.
50 d. Flemiſh	= 400 rees.
1000 rees	= 64 d. Sterling.
How many d. Sterling	= 400 l. or 96000 d. Flemiſh?

This, in the fractional form, will ſtand as follows :

$$\frac{60 \times 100 \times 400 \times 64 \times 96000}{56 \times 100 \times 50 \times 1000} = \frac{368640}{7}, \text{ and}$$

7)368640(52662 $\frac{6}{7}$ d. Sterling. = 219 l. 8 s. 6 $\frac{6}{7}$ d. Sterling. *Anſ.*

To find how much the exchange from Amſterdam directly to London, at 36 s. 10 d. Flemiſh *per* l. Sterling, will amount to, ſay,

s. d.	d. Fl. L. St.	d. Fl.	L.	s. d. Ster.
36 10	If 442 : 1 :: 96000 :	217	3	10 $\frac{1}{2}$
12		219	8	6 $\frac{3}{4}$
442			2 4	8 $\frac{1}{2}$
	Gained or ſaved,			

In the above example, the par of arbitration, or the arbitrated price, between London and Amſterdam, *viz.* the number of Flemiſh pence given for 1 l. Sterling, may be found thus :

Make 64 d. Sterling, the price of the millree, the firſt antecedent; then all the former conſequents will become antecedents, and all the antecedents will become conſequents. Place 240, the pence in 1 l. Sterling, as the laſt conſequent, and then proceed as taught above, *viz.*

Antecedents.

<i>Antecedents.</i>	<i>Consequents.</i>
64 d. Sterling	= 1000 rees.
400 rees	= 50 d. Flemish.
100 d. Flemish	= 1 ducat.
60 ducats	= 100 crowns.
1 crown	= 56 d. Flemish.
How many d. Flemish = 240 d. Sterling?	

$$\frac{1000 \times 50 \times 100 \times 56 \times 240}{64 \times 400 \times 100 \times 60} = \frac{875}{2}, \text{ and}$$

$$2)875(437\frac{1}{2} \text{ d.} = 36 \text{ s. } 5\frac{1}{2} \text{ d. Flemish per l. Sterling. Ans.}$$

Or the arbitrated price may be found from the answer to the question, by saying,

$$\begin{array}{r} \text{d. Ster.} \quad \text{d. Flem.} \quad \text{d. Ster.} \\ \text{If } \frac{368640}{7} : 96000 :: 240 \end{array}$$

$$\begin{array}{r} 7 \\ \hline 672000 \\ 240 \end{array}$$

$$\hline 2688$$

$$\hline 1344$$

$$\begin{array}{r} \hline \text{d.} \quad \text{s.} \quad \text{d. Flem.} \\ 368640)161280000(437\frac{1}{2} = 36 \ 5\frac{1}{2} \text{ as before.} \end{array}$$

The work may be proved by the arbitrated price thus: As 1 l. Sterling to 36 s. 5½ d. Flemish, so 219 l. 8 s. 6⅞ d. Sterling to 400 l. Flemish.

The arbitrated price compared with the direct course, shows whether the direct or circular remittance will be most advantageous, and how much. Thus the banker at Amsterdam will think it better exchange to receive 1 l. Sterling for 36 s. 5½ d. Flemish, than for 36 s. 10 d. Flemish.

3. A merchant at London has credit for 680 piaftres at Leghorn, for which he can draw directly at 50 d. Sterling *per* piaftre; but chusing to try the circular way, they are by his order remitted, first to Venice, at 94 piaftres *per* 100 ducats Banco; thence to Cadiz, at 320 mervadies *per* ducat; thence to Lisbon, at 630 rees *per* piaftre of 272 mervadies; thence to Amsterdam, at 50 d. Flemish *per* crusade of 400 rees; thence to Paris, at 56 d. Flemish *per* crown; thence to London, at 31⅓ d. Sterling *per* crown: What is the arbitrated price between London and Leghorn *per* piaftre? and how much is the circular remittance better than the direct draught, without reckoning charges?

Antecedents.

<i>Antecedents.</i>	<i>Consequents.</i>
94 piaftres	= 100 ducats.
1 ducat	= 320 mervadies.
272 mervadies	= 630 rees.
400 rees	= 50 d. Flemish.
56 d. Flemish	= 1 crown.
3 crowns	= 94 d. Sterling = $3 \times 31\frac{2}{3}$
How many d. Sterling	= 1 piaftre ?

$$\frac{100 \times 320 \times 630 \times 50 \times 94}{94 \times 272 \times 400 \times 56 \times 3} = \frac{1875}{34}, \text{ and}$$

$$34)1875(55\frac{5}{14} \text{ d. Sterling per piaftre. } \textit{Ans.}$$

	<i>L.</i>	<i>s.</i>	<i>d.</i>
680 piaftres, at $55\frac{5}{14}$ d.	= 156	5	0
at 50 d.	= 141	13	4
Gained	14	11	8

If the arbitrated price of any intermediate antecedent be required, put the question at the void place, fill up the place of the first question with its answer ; or, to avoid the fraction, make 680 piaftres the last consequent, and their value in pence Sterling the last antecedent, and then work as before.

Ex. How many rees are equal in value to 50 d. Flemish ?

<i>Antecedents.</i>	<i>Consequents.</i>
94 piaftres	= 100 ducats.
1 ducat	= 320 mervadies.
272 mervadies	= 630 rees.
How many rees	= 50 d. Flemish ?
56 d. Flemish	= 1 crown.
3 crowns	= 94 d. Sterling.
37500 d. Sterling	= 680 piaftres.

$$\frac{100 \times 320 \times 630 \times 50 \times 94 \times 680}{94 \times 272 \times 56 \times 3 \times 37500} = \frac{5 \times 2 \times 10 \times 4}{1} = 400 \text{ rees. } \textit{Ans.}$$

If the arbitrated price of a consequent be required, at the void place put the question, and work as before ; only in this case the product of the antecedents is the dividend, and the product of the consequents the divisor.

Ex. How many rees are equal in value to a piaftre of 272 mervadies ?

Antecedents.

<i>Antecedents.</i>	<i>Consequents.</i>
94 piaftres	= 100 ducats.
1 ducat	= 320 mervadies.
272 mervadies	= how many rees ?
400 rees	= 50 d. Flemish.
56 d. Flemish	= 1 crown.
3 crowns	= 94 d. Sterling.
37500 d. Sterling	= 680 piaftres.

$$\frac{94 \times 272 \times 400 \times 56 \times 3 \times 37500}{100 \times 320 \times 50 \times 94 \times 680} = \frac{14 \times 3 \times 15}{1} = 630 \text{ rees. } \textit{Ans.}$$

Note. The value of an antecedent is always of the same denomination with the preceding consequent ; and the value of a consequent is of the same denomination with the subsequent antecedent.

In this circular exchange, the agents or factors at the different places through which the money passes, retain so much in name of commission ; and the regular accurate method of computation is, to deduce the commission from the several consequents.

Thus, if we resume the former question, and suppose the commission to be $\frac{1}{2}$ per cent. the antecedents and consequents, with commission deduced, will stand as under :

<i>Antecedents.</i>	<i>Consequents.</i>
94 piaftres	= 99.5 ducats, commission deduced.
1 ducat	= 318.4 mervadies, commission deduced.
272 mervadies	= 626.85 rees, commission deduced.
400 rees	= 49.75 d. Flemish, commission deduced.
56 d. Flemish	= .995 crowns, commission deduced.
3 crowns	= 94 d. Sterling.
How many d. Sterling	= 1 piaftre ?

By working as formerly, the arbitrated price, or value of the piaftre, will be found to be 53.78 d. Sterling nearly, which is something less than the answer found, exclusive of charges. But still there will be profit by the circular remittance : for

	<i>L.</i>	<i>s.</i>	<i>d.</i>
680 piaftres, at 53.78 d. Sterling,	= 152	7	6
———— at 50 d. Sterling,	= 141	13	4
Gain	10	14	2

The value of the above piaftre, with commission allowed, may be found easily, and pretty nearly, by deducting five commissions, equal to $2\frac{1}{2}$ per cent. from the arbitrated price, exclusive of charges, thus :

$$2\frac{1}{2} \text{ per cent. } = \frac{55\frac{1}{4}}{40} = 55.147 \quad | \quad 1.378$$

53.769 value nearly.

Thus a person who knows at what rate he can draw or remit directly, and at the same time has advice of the course of exchange in foreign places, may chalk out a path for circulating his money, so as to make a benefit of his skill and credit: and herein lies the great art of such negotiations.

Not only may different sorts of money be equated in the manner above described, but also weights and measures.

E X A M P L E I.

If 102 lb. of Hamburgh be equal to 100 lb. of Amsterdam, and 100 of Amsterdam to 98 at Francfort, and 98 at Francfort to 105 at Leipzig, and 105 of Leipzig to 145 at Leghorn, and 145 at Leghorn to 106 at Cadiz, and 100 at Cadiz to 103 $\frac{1}{2}$ at London; how many lbs. at London are equal to 3060 at Hamburgh?

<i>Antecedents.</i>	<i>Consequents.</i>
102 at Hamburgh	= 100 at Amsterdam,
100 at Amsterdam	= 98 at Francfort,
98 at Francfort	= 105 at Leipzig,
105 at Leipzig	= 145 at Leghorn,
145 at Leghorn	= 106 at Cadiz,
300 at Cadiz	= 310 at London,
How many at London	= 3060 at Hamburgh?

$$\frac{100 \times 98 \times 105 \times 145 \times 106 \times 310 \times 3060}{102 \times 100 \times 98 \times 105 \times 145 \times 300} = \frac{106 \times 310 \times 3060}{102 \times 300} = \frac{55862}{17}, \text{ and}$$

17)55862(3286 lb. *Ans.*

E X A M P L E II.

If 1 $\frac{1}{2}$ ells, or aunes, of Hamburgh, make 1 ell in Holland, and 7 in Holland make 4 in France, and 7 in France make 5 yards in England; how many yards in England are equal to 588 ells or aunes at Hamburgh? and what their price, at the rate of 4l. Sterling for 5 yards English?

<i>Antecedents.</i>	<i>Consequents.</i>
1 $\frac{1}{2}$ × 5 = 6 of Hamburgh	= 5 of Holland = 1 × 5
7 of Holland	= 4 of France,
7 of France	= 5 of England,
How many yards of England	= 588 of Hamburgh?

$$\frac{5 \times 4 \times 5 \times 588}{6 \times 7 \times 7} = \frac{5 \times 4 \times 5 \times 2}{1} = 200 \text{ yards. } \textit{Ans.}$$

Then for the price :

<i>Antecedents.</i>	<i>Consequents.</i>
6 of Hamburgh	= 5 of Holland,
7 of Holland	= 4 of France,
7 of France	= 5 of England,
5 of England	= 4l. Sterling,
How much Sterling	= 588 of Hamburgh?

$$\frac{5 \times 4 \times 5 \times 4 \times 588}{6 \times 7 \times 7 \times 5} = \frac{4 \times 5 \times 4 \times 2}{1} = 160 \text{ l. Sterl. } \textit{Ans.}$$

E X A M P L E III.

<i>Antecedents.</i>	<i>Consequents.</i>
If 82 bushels at London	= 27 muddles at Amsterdam,
27 muddles at Amsterdam	= 17 setiers at Paris,
38 setiers at Paris	= 65 veertels at Antwerp,
65 veertels at Antwerp	= 104 hanegas at Cadiz,
52 hanegas at Cadiz	= 216 alquiers at Lisbon,
How many alquiers at Lisbon	= 410 bushels at London?

$$\frac{27 \times 19 \times 65 \times 104 \times 216 \times 410}{82 \times 27 \times 38 \times 65 \times 52} = 108 \times 10 = 1080 \text{ alquiers at Lisbon. } \textit{Ans.}$$

Though foreign weights and measures may be equated with those of Britain by arbitration, as in the above examples; yet this is done more easily and readily by the rule of three, from the following tables, which exhibit the proportion or conformity that the weights, long measures, and corn-measures, of the principal places in Europe, have with those of London and Amsterdam.

TABLE

TABLE I.
OF WEIGHTS.

100 lb. of England are equal to lb. oz.	100 lb. of Amster- dam are equal to lb. oz.	At
100 0	109 8	London
91 8	100 0	Amsterdam
96 8	105 8	Antwerp
88 0	96 4	Rouen
106 0	116 0	Lyons
90 9	99 0	Rochelle
107 11	118 0	Toulouse
113 0	123 8	Marseilles
81 7	89 0	Geneva
93 5	102 0	Hamburgh
89 7	98 0	Francfort
96 1	105 0	Leipsic
137 4	150 0	Genoa
132 11	145 0	Leghorn
153 11	168 0	Milan
152 0	166 0	Venice
154 10	169 0	Naples
97 0	106 0	Seville
97 0	106 0	Cadiz
104 13	114 8	Portugal
96 5	105 4	Liege
112 0 $\frac{2}{3}$	123 0 $\frac{1}{2}$	Russia
107 0 $\frac{1}{4}$	117 0	Sweden
89 0 $\frac{1}{2}$	97 13	Denmark

As the weights of Amsterdam, Paris, Bourdeaux, Strasburgh, Befançon, and several other places, have but a very minute difference, they are all comprehended under that of Amsterdam, as that of Nuremberg is under Francfort, and others in like manner.

TABLE II.
OF LONG MEASURE.

100 ells or aunes, or 125 yards of England are equal to	100 aunes of Am- sterdam are equal to	At
<i>Aunes.</i>	<i>Aunes.</i>	
100	60	London
166 $\frac{2}{3}$	100	Amsterdam
162 $\frac{4}{9}$	98 $\frac{3}{4}$	Brabant
97 $\frac{1}{2}$	58 $\frac{1}{2}$	France
200 $\frac{1}{4}$	120	Hamburgh
208 $\frac{1}{3}$	125	Breslau in Silesia
187 $\frac{1}{2}$	112 $\frac{1}{2}$	Dantzick
183 $\frac{1}{4}$	110	Bergue and Drontheim
194 $\frac{1}{2}$	116 $\frac{2}{3}$	Sweden
143 $\frac{1}{7}$	86	St Gall for linen
186 $\frac{2}{3}$	112	St Gall for cloth
100	60	Geneva
164 $\frac{1}{2}$	98 $\frac{3}{4}$	Brussels, Antwerp
<i>Canes.</i>	<i>Canes.</i>	
58 $\frac{1}{4}$	35	Marseilles
62 $\frac{3}{4}$	37 $\frac{1}{2}$	Toulouse
50 $\frac{1}{3}$	30 $\frac{1}{2}$	Genoa
55 $\frac{1}{3}$	33	Rome
<i>Varas.</i>	<i>Varas.</i>	
133 $\frac{1}{3}$	80	Spain
101 $\frac{1}{3}$	61	Portugal
<i>Cavidos.</i>	<i>Cavidos.</i>	
166 $\frac{2}{3}$	100	Portugal
<i>Brasses.</i>	<i>Brasses.</i>	
170 $\frac{1}{3}$	102	Venice
174 $\frac{1}{2}$	105 $\frac{1}{4}$	Bergamo
194 $\frac{1}{7}$	117	Florence, Leg- horn, Lucca
214 $\frac{1}{8}$	128 $\frac{1}{4}$	Milan
<i>Arsheens.</i>	<i>Arsheens.</i>	
178 $\frac{4}{7}$	29 $\frac{1}{2}$	Petersburgh

The aunes or ells of Haerlem, Leyden, the Hague, Rotterdam, and other places of Holland, also that of Nuremberg, are all equal to that of Amsterdam, and comprehended under it; the aune of Osnaburgh is equal to that of France; and the aunes of Bern, Basil, Francfort, and Leipsic, are equal to that of Hamburgh.

TABLE III. OF CORN-MEASURES.

$10\frac{1}{4}$ quarters, or 82 Winchester bushels of England, or 1 last of Amsterdam, make, at

Aiguillon, 41 sacks	Hamburgh, $\frac{1}{3}$ of a last
Albi, 25 setiers	Heusden, $17\frac{1}{4}$ muddes
Alicant, 12 cahizes	Hoorn, 44 sacks
Alkinaar, 36 sacks	Ireland, 38 bushels
Amersfort, 16 muddes	La Brille, 40 sacks
Amsterdam, 27 muddles, or 1 last	La Reole, 30 sacks
Antwerp, $32\frac{1}{2}$ veertels	Lavour, 21 setiers
Arles, 49 setiers	Leyden, 44 sacks
Bayonne, 36 sacks	Libourne, 35 sacks
Beaucaire, 28 setiers	Liege, 96 setiers
Beaumont, 38 sacks	Lille, 38 razieres
Bergen-op-zoom, 63 sifters	Lisbon, 216 alquiers
Bois-le-Duc, $20\frac{1}{2}$ mouwers	Leghorn, 40 sacks
Bommel, 18 muddes	Louvain, 27 muddes
Bordeaux, 38 boisseux	Lubeck, 95 schepels
Breda, 33 veertels	Lyons, $14\frac{1}{4}$ anees
Bruges, $17\frac{1}{2}$ hoedts	Middlebourgh, $41\frac{1}{2}$ sacks
Brussels, 25 sacks	Montfort, 21 muddes
Bueren, 21 muddes	Muyden, 44 sacks
Cadillac, $33\frac{1}{2}$ sacks	Naerden, 44 sacks
Cadiz, 52 hanegas	Nerac, $33\frac{1}{2}$ sacks
Cahors, 100 cartes	Nieuport, $17\frac{1}{2}$ razieres
Campen, $24\frac{1}{2}$ muddes	Oudewater, 21 muddes
Carcassone, 35 setiers	Paris, 19 setiers
Clairac, $34\frac{1}{2}$ sacks	Porto Port, 180 alquiers
Cleves, $16\frac{1}{4}$ mouwers	Purmerent, 27 muddes
Condom, 41 sacks	Rabaftens, 17 setiers
Coningsberg, 1 last	Rhenen, 20 muddes
Copenhagen, 42 tuns	Ruremonde, 68 schepels
Dantzick, 1 last	Riga, 46 loopens
Delft, 29 sacks	Rotterdam, 29 sacks
Deventer, 36 muddes	St Giles, 40 charges
Doesbourg, 22 mouwers	St Omer, $22\frac{1}{2}$ razieres
Dort, 24 sacks	St Valery, 19 setiers
Dunkirk, 18 razieres	Saumur, 19 setiers
Edam, 27 muddes	Steenbergen, 35 veertels
Elbing, 1 last	Stockholm, 23 tuns
Emden, $15\frac{1}{4}$ tuns	Terveer, 39 sacks
Erfelsteyn, 21 muddes	Thiel, 21 muddes
Francfort, 27 malders	Toulouse, 26 setiers
Ghent, 56 halsters	Tongres, 15 muddes
Genoa, 25 mines	Toningen, 24 tuns
Gimond, 20 sacks	Venloo, $21\frac{1}{4}$ mouwers
Graveline, 22 razieres	Vianen, 20 muddes
Haerlem, 38 sacks	Utrecht, 25 muddes
	Zurickzee, 40 sacks

The use of the Tables.

E X A M P L E I.

What are 800 lb. of Lyons equal to in England?

Say, from Table I. As 106 lb. of Lyons : 100 lb. of England :: 800 lb. of Lyons : $754\frac{1}{2}$ of England.

E X A M P L E II.

How many aunes of Amsterdam are equal to 500 aunes of Dantzick?

Say, from Table II. As $112\frac{1}{2}$ aunes of Dantzick : 100 aunes of Amsterdam :: 500 aunes of Dantzick : $444\frac{1}{4}$ aunes of Amsterdam.

E X A M P L E III.

How many setiers will 400 bushels of England make at Paris?

Say, from Table III. As 82 bushels of England : 19 setiers of Paris :: 400 bushels of England : $92\frac{1}{2}$ setiers of Paris.

C H A P. VIII.

A L L I G A T I O N.

ALLIGATION serves to solve questions that relate to the mixing of simples; and is either *medial* or *alternate*.

I. *Alligation Medial.*

Alligation medial, from the rates and quantities of the simples given, discovers the rate of the mixture.

R U L E.

As the total quantity of the simples,
To their price or value;
So any quantity of the mixture,
To the rate.

E X A M P L E S.

1. A grocer mixeth 30 lb. of currants, at 4d. *per* lb. with 10 lb. of other currants, at 6d. *per* lb.: What is the value of 1 lb. of the mixture? *Ans.* $4\frac{1}{2}$ d.

<i>lb.</i>	<i>d.</i>	<i>d.</i>	<i>lb.</i>	<i>d.</i>	<i>lb.</i>	<i>d.</i>
30,	at 4	amounts to	120		If 40 :	180 :: 1 : 4½
10,	at 6	_____	60			
—			—			
40			180			

2. A farmer mingleth several sorts of wheat as follows, *viz.* 20 bushels, at 5s. *per* bushel; 36 bushels, at 3s.; and 40 bushels, at 2s.: What is a bushel of the mixture worth? *Ans.* 3s.

<i>Bush.</i>	<i>s.</i>	<i>s.</i>	<i>Bush.</i>	<i>s.</i>	<i>Bush.</i>	<i>s.</i>
20,	at 5,	come to	100		If 96 :	288 :: 1 : 3
36,	at 3	_____	108			
40,	at 2	_____	80			
—			—			
96			288			

3. A tobacconist mixeth 30 lb. of tobacco, at 9d. *per* lb. with 60 lb. at 14d. and 24½ lb. at 2s. *per* lb.: What is a pound of the mixture worth?

<i>lb.</i>	<i>Rates.</i>	<i>L.</i>	<i>lb.</i>	<i>L.</i>	<i>lb.</i>	<i>L.</i>
30	× .0375	= 1.125	If 114.75 :	7.1 :: 1 :	.0618	
60	× .0583	= 3.5				
24.75	× .1	= 2.475				
—		—				
114.75		7.1				

114.75	7.1000	(.0618 = 1 2¼
	68850	
	—	
	21500	
	11475	
	—	
	100250	
	91800	
	—	
	8450	

Note 1. When the quantity of each simple is the same, the rate of the mixture is readily found by adding the rates of the simples, and dividing their sum by the number of simples, thus:

Suppose a grocer mixes several sorts of sugar, and of each an equal quantity, *viz.* at 50s. at 54s. and at 60s. *per* C. the rate of the mixture will be 54s. 8d. *per* C.; for,

$$50 + 54 + 60 = 164, \text{ and } 3)164(54 \text{ } 8$$

Note 2. If it be required to increase or diminish the quantity of the mixture, say, As the sum of the given quantities of the simples, to the several quantities given; so the quantity of the mixture proposed, to the quantities of the simples sought. Thus:

				<i>Bush.</i>	<i>s.</i>
96	: 20	::	120	: 25,	at 5
96	: 36	::	120	: 45,	at 3
96	: 40	::	120	: 50,	at 2

Proof 120

Suppose a grocer mixes 10lb. of raisins, with 30lb. of almonds, and 40lb. of currants, and it be demanded, how many ounces of each sort are found in every pound, or in every 16 ounces of the mixture, say,

oz.

80 : 10 :: 16 : 2 raisins.
80 : 30 :: 16 : 6 almonds.
80 : 40 :: 16 : 8 currants.

Proof 16

Suppose a grocer has a mixture of 400lb. weight, that cost him 7l. 10s. consisting of raisins, at 4d. *per* lb. and almonds at 6d. how many pounds of almonds were in the mixture?

	<i>lb.</i>	<i>Rates.</i>	
	400	6 d.	
<i>L. s.</i>	4	4 d.	
7 10 = 1800			
1600	1600 d.	2 d.	
2)200(100 lb. of almonds, at 6 d. is,			<i>L. s.</i>
And 300 lb. of raisins, at 4 d. is			2 10
			5 0
Total 400		Proof	7 10

MORE EXAMPLES.

4. A merchant mingled three different sorts of sugar, viz. 3 C. at 2l. 16s. *per* C. and 3 C. at 3l. 14s. 8d. with 6 C. at 1l. 17s. 4d.: What is a C. of the mixture worth? *Ans.* L. 2 : 11 : 4.

5. A vintner mingles several sorts of wine, viz. 36 gallons at 8s. *per* gallon, 2 gallons at 7s. *per* gallon, 13 gallons at 1s. *per* gallon, with 12 gallons of water: What is a gallon of the mixture worth? *Ans.* 5s.

6. If, with 40 bushels of corn at 4s. *per* bushel, there are mixed 10 bushels at 6s. *per* bushel, 30 bushels at 5s. *per* bushel, and 20 bushels at 3s. *per* bushel, what will 10 bushels of the mixture be worth? *Ans.* 2l. 3s.

7. A goldsmith mixes 30 ounces of gold 22 caracts fine, 10 ounces 20 caracts fine, and 20 ounces 18 caracts fine: How many caracts fine is the mixture? *Ans.* $20\frac{1}{3}$; that is, of the 24 parts or caracts into which the ounce is supposed to be divided, $20\frac{1}{3}$ are pure gold, and the remaining $3\frac{2}{3}$ are alloy.

Medicines are sometimes mixed or compounded by this rule; for understanding the manner whereof it will be necessary to observe,

That medicines, drugs, or simples, with respect to their qualities, are divided into five sorts, viz. hot, cold, dry, moist, temperate; and all, except the last, admit of four degrees, represented by indices, as in the following table:

Ind.	1 2 3 4 5 6 7 8 9	
Deg.	4 3 2 1 0 1 2 3 4	The index 3 denotes cold or moist in the 2d degree, and 9 denotes hot or dry in the 4th degree.
	of cold and moisture.	

EXAMPLE.

If I have four simples, A, B, C, D, and mix them as follows: viz. 4 oz. of A hot in the 4th degree, 1 oz. of B hot in the 2d, 1 oz. of C temperate, and 3 oz. of D cold in the 3d degree; what will be the quality of the mixture or compound? *Ans.* Hot in the 1st degree.

$$\begin{array}{rcl}
 A & 4 \times 9 & = 36 \\
 B & 1 \times 7 & = 7 \\
 C & 1 \times 5 & = 5 \\
 D & 3 \times 2 & = 6 \\
 \hline
 & & 54
 \end{array}$$

9 9)54(6 hot in the 1st degree.

II. *Alligation Alternate.*

Alligation alternate, being the converse of alligation medial, from the rates of the simples, and rate of the mixture given, finds the quantities of the simples.

R U L E S.

I. Place the rate of the mixture on the left side of a brace, as the root; and on the right side of the brace set the rates of the several simples, under one another, as the branches.

II. Link or alligate the branches, so as one greater and another less than the root may be linked or yoked together.

III. Set the difference betwixt the root and the several branches, right against their respective yoke-fellows. These alternate differences are the quantities required.

Note 1. If any branch happen to have two or more yoke-fellows, the difference betwixt the root and these yoke-fellows must be placed right against the said branch, one after another, and added into one sum.

Note 2. In some questions, the branches may be alligated more ways than one; and a question will always admit of so many answers as there are different ways of linking the branches.

Alligation alternate admits of three varieties, *viz.* 1. The question may be unlimited, with respect both to the quantity of the simples, and that of the mixture. 2. The question may be limited to a certain quantity of one or more of the simples. 3. The question may be limited to a certain quantity of the mixture.

Variety I.

When the question is unlimited, with respect both to the quantity of the simples, and that of the mixture. This is called *Alligation Simple*.

E X A M P L E S.

1. A grocer would mix sugars, at 5d. 7d. and 10d. *per lb.* so as to sell the mixture or compound at 8d. *per lb.* : What quantity of each must he take?

			<i>lb.</i>
8	{	5	2
		7	2
		10	3, 1
			4

Here the rate of the mixture 8 is placed on the left side of the brace, as the root; and on the right side of the same brace are set the rates of the several simples, *viz.* 5, 7, 10, under one another, as the branches; according to Rule I.

The

The branch 10 being greater than the root, is alligated or linked with 7 and 5, both these being less than the root, as directed in Rule II.

The difference between the root 8 and the branch 5, *viz.* 3, is set right against this branch's yoke-fellow 10. The difference between 8 and 7 is likewise set right against the yoke-fellow 10. And the difference between 8 and 10, *viz.* 2, is set right against the two yoke-fellows 7 and 5; as prescribed by Rule III.

As the branch 10 has two differences on the right, *viz.* 3 and 1, they are added; and the answer to the question is, that 2 lb. at 5 d. 2 lb. at 7 d. and 4 lb. at 10 d. will make the mixture required.

The truth and reason of the rules will appear by considering, that whatever is lost upon any one branch is gained upon its yoke-fellow. Thus, in the above example, by selling 4 lb. of 10 d. sugar at 8 d. *per* lb. there is 8 d. lost: but the like sum is gained upon its two yoke-fellows; for by selling 2 lb. of 5 d. sugar at 8 d. *per* lb. there is 6 d. gained, and by selling 2 lb. of 7 d. sugar at 8 d. there is 2 d. gained; and 6 d. and 2 d. make 8 d.

Hence it follows, that the rate of the mixture must always be mean or middle with respect to the rates of the simples; that is, it must be less than the greatest, and greater than the least; otherwise a solution would be impossible. And the price of the total quantity mixed, computed at the rate of the mixture, will always be equal to the sum of the prices of the several quantities cast up at the respective rates of the simples. This may be used by way of proof; and the above example proved in this manner follows:

$$\begin{array}{r|l}
 \left. \begin{array}{l} 5 \\ 7 \\ 10 \end{array} \right\} \begin{array}{l} 2 \\ 2 \\ 3, 1 \end{array} & \begin{array}{l} 2 \times 5 = 10 \\ 2 \times 7 = 14 \\ 4 \times 10 = 40 \\ \hline 8 \times 8 = 64 \end{array} \\
 & 64 \text{ proof.}
 \end{array}$$

Or say,

$$\text{If } 8 : 64 :: 1 : 8$$

That is, the sum of the prices or price of the mixture, divided by the sum of the quantities, quotes the rate of the mixture.

From the above proof it is obvious, that the rates of the simples, their total quantity, and the price of the mixture, being given; the rate of the mixture, and consequently the quantities of the several simples, may be found: for the price of the mixture divided by the total quantity of the simples, quotes the rate of the mixture; the alternate differences of which, and the several rates, are the quantities sought.

The answer in the above example comes out to be 2, 2, 4; and as the branches can be linked but one way, we can, by the operation,

tion, have only this single answer : but then any other three numbers that have the same proportion to one another, found by division or multiplication, will also answer the question. By this means the answer found may be increased or diminished, till it suit the stock of simples on hand for making the mixture; as in the following specimen :

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	} &c.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	
2.	4.	6.	8.	10.	12.	14.	16.	18.	20.	22.	24.	26.	

But we may also find innumerable other answers, by increasing or diminishing the alternate differences of any pair of simples, without varying the rest, by one or other of the two rules following :

I. When the rates of the variable pair of simples are both greater, or both less, than the rate of the mixture, increase the alternate difference of the one, by adding to it the difference betwixt the rate of the mixture and the rate of the other variable simple; and diminish the alternate difference of the other, by subtracting from it the difference betwixt the rate of the mixture and the rate of the former variable simple.

The alternate difference to be increased is optional, or which of them you please; and instead of the alternate differences, you may use any other numbers that are in the same proportion; and the higher these numbers are, the more answers they will afford. We shall exemplify the rule by the proportional numbers, 13, 13, 26, found above, making 13, 13, the variable pair, their rates, 5 and 7, being both less than 8, the rate of the mixture, and shall continue 26 unvaried.

By increasing 13 in the upper line, we have,

13.	14.	15.	16.	17.
13.	10.	7.	4.	1.
26.	26.	26.	26.	26.

By increasing 13 in the second line, we have,

13.	12.	11.	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.
13.	16.	19.	22.	25.	28.	31.	34.	37.	40.	43.	46.	49.
26.	26.	26.	26.	26.	26.	26.	26.	26.	26.	26.	26.	26.

II. When the rate of one of the variable simples is greater than the rate of the mixture, and the rate of the other less, their alternate differences, or, rather, the numbers proportional to them, must, as directed in the former rule, be both increased, or both diminished, and either or each of these ways may be used.

We shall exemplify the rule, as formerly, by the proportional numbers

numbers 13, 13, 26, making the first 13 and 26 the variable pair, whose rates are 5 and 10, the one below and the other above 8, the rate of the mixture, and shall continue the other 13 unvaried.

By increasing both, we have,

13.	15.	17.	19.	21.	23.	25.	27.	29.	} to infinity.
13.	13.	13.	13.	13.	13.	13.	13.	13.	
26.	29.	32.	35.	38.	41.	44.	47.	50.	

By diminishing both, we have,

13.	11.	9.	7.	5.	3.	1.
13.	13.	13.	13.	13.	13.	13.
26.	23.	20.	17.	14.	11.	8.

Again, we may make the second 13 and 26 the variable pair, their rates being 7 and 10, the one below and the other above 8, the rate of the mixture, and continue the first 13 unvaried, as under :

By increasing both we have,

13.	13.	13.	13.	13.	13.	13.	13.	13.	} to infinity.
13.	15.	17.	19.	21.	23.	25.	27.	29.	
26.	27.	28.	29.	30.	31.	32.	33.	34.	

By diminishing both, we have,

13.	13.	13.	13.	13.	13.	13.
13.	11.	9.	7.	5.	3.	1.
26.	25.	24.	23.	22.	21.	20.

The reason of the rules is obvious : for by increasing or diminishing the alternate differences, or other numbers proportional to them, in the manner prescribed, the value of every pound of increase or diminution is the rate of the mixture.

2. A vintner would mix wines at 14 s. 15 s. 19 s. and 22 s. *per* gallon, so as the mixture may be worth 18 s. *per* gallon : What quantity of each must he take ?

First, thus. Gal.

18	{	14	4	
		15	1	
		19	3	
		22	4	

Sec. thus. Gal.

18	{	14	1	
		15	4	
		19	4	
		22	3	

Thirdly, thus. Gal.

18	{	14	1.4		5
		15	1		1
		19	4.3		7
		22	4		4

Fourthly,

Fourthly, thus. Gal.

$$18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{l} 4 \\ 1,4 \\ 3 \\ 4,3 \end{array} \left| \begin{array}{l} 4 \\ 5 \\ 3 \\ 7 \end{array} \right.$$

Fifthly, thus. Gal.

$$18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{l} 1,4 \\ 4 \\ 4 \\ 4,3 \end{array} \left| \begin{array}{l} 5 \\ 4 \\ 4 \\ 7 \end{array} \right.$$

Sixthly, thus. Gal.

$$18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{l} 1 \\ 1,4 \\ 4,3 \\ 3 \end{array} \left| \begin{array}{l} 1 \\ 5 \\ 7 \\ 3 \end{array} \right.$$

Seventhly, thus. Gal.

$$18 \left\{ \begin{array}{l} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{l} 1,4 \\ 1,4 \\ 4,3 \\ 4,3 \end{array} \left| \begin{array}{l} 5 \\ 5 \\ 7 \\ 7 \end{array} \right.$$

Here the simples are linked seven different ways; and accordingly we have seven different answers to the question: and, by multiplication or division, each of these will produce other answers to infinity. Again, by taking each pair of simples successively, and increasing or diminishing their alternate differences, or other numbers proportional to them, in the manner taught above, we shall have other answers innumerable.

3. A grocer mixes teas, at 7 s. at 8 s. at 10 s. and at 13 s. *per lb.*: How much of each sort must he take, that a pound of the mixture may be worth 9 s. 6 d.?

$$\begin{array}{ll} \text{First, lb.} & \text{Secondly, lb.} \\ 9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} 3.5 \\ .5 \\ 1.5 \\ 2.5 \end{array} & 9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5 \\ 3.5 \\ 2.5 \\ 1.5 \end{array} \end{array}$$

$$\text{Thirdly, lb.} \\ 9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5, 3.5 \\ .5 \\ 2.5, 1.5 \\ 2.5 \end{array} \left| \begin{array}{l} 4 \\ .5 \\ 4 \\ 2.5 \end{array} \right.$$

$$\text{Fourthly, lb.} \\ 9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} 3.5 \\ .5, 3.5 \\ 1.5 \\ 2.5, 1.5 \end{array} \left| \begin{array}{l} 3.5 \\ 4 \\ 1.5 \\ 4 \end{array} \right.$$

$$\text{Fifthly, lb.} \\ 9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5, 3.5 \\ 3.5 \\ 2.5 \\ 2.5, 1.5 \end{array} \left| \begin{array}{l} 4 \\ 3.5 \\ 2.5 \\ 4 \end{array} \right.$$

$$\text{Sixthly, lb.} \\ 9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5 \\ .5, 3.5 \\ 2.5, 1.5 \\ 1.5 \end{array} \left| \begin{array}{l} .5 \\ 4 \\ 4 \\ 1.5 \end{array} \right.$$

$$\text{Seventhly, lb.} \\ 9.5 \left\{ \begin{array}{l} 7 \\ 8 \\ 10 \\ 13 \end{array} \right\} \begin{array}{l} .5, 3.5 \\ .5, 3.5 \\ 2.5, 1.5 \\ 2.5, 1.5 \end{array} \left| \begin{array}{l} 4 \\ 4 \\ 4 \\ 4 \end{array} \right.$$

Other answers innumerable may be found by the methods explained above.

4. An apothecary would mix four simples, *viz.* A hot in the fourth degree, B hot in the second, C temperate, and D cold in the third, so as the mixture may be hot in the first degree: What quantity

quantity of each simple must he take? See the table on composition of medicines in allegation medial.

$$\begin{array}{r|l}
 \left. \begin{array}{l} 9 \\ 7 \\ 5 \\ 2 \end{array} \right\} & \begin{array}{l} 1 \text{ A} \\ 4 \text{ B} \\ 3 \text{ C} \\ 1 \text{ D} \end{array} \\
 \hline & \begin{array}{l} 1 \times 9 = 9 \\ 4 \times 7 = 28 \\ 3 \times 5 = 15 \\ 1 \times 2 = 2 \end{array} \\
 \hline & 6 \times 9 \times 54 \text{ Proof } 54
 \end{array}$$

The branches may be alligated other six ways, which will afford the like number of other answers.

Variety II.

When the question is limited to a certain quantity of one or more of the simples. This is called *Alligation Partial*.

If the quantity of one of the simples only be limited, alligate the branches, and take their differences, as if there had been no such limitation; and then work by the following proportion:

As the difference right against the rate of the simple whose quantity is given,
 To the other differences respectively;
 So the quantity given,
 To the several quantities sought.

EXAMPLES.

1. A distiller would, with 40 gallons of brandy at 12 s. *per* gallon, mix rum at 7 s. *per* gallon, and gin at 4 s. *per* gallon: How much of the rum and gin must he take, to sell the mixture at 8 s. *per* gallon?

$$\begin{array}{r|l}
 \left. \begin{array}{l} 12 \\ 7 \\ 4 \end{array} \right\} & \begin{array}{l} 1, 4 \\ 4 \\ 4 \end{array} \\
 \hline & \begin{array}{l} 5 \\ 4 \\ 4 \end{array} \\
 \hline & \begin{array}{l} 40 \text{ of brandy.} \\ 32 \text{ of rum.} \\ 32 \text{ of gin.} \end{array}
 \end{array} \quad \left. \vphantom{\begin{array}{l} 12 \\ 7 \\ 4 \end{array}} \right\} \text{Ans.}$$

The operation gives for answer 5 gallons of brandy, 4 of rum, and 4 of gin. But the question limits the quantity of brandy to 40 gallons; therefore say,

$$\text{If } 5 : 4 :: 40 : 32$$

The quantity of gin, by the operation being also 4, the proportion needs not be repeated. The proof is the same as formerly.

As the branches can be linked only one way, we can have but one answer by the operation; and because the quantity of brandy is limited to 40 gallons, there can be no recourse to proportional numbers; but by proceeding with the remaining pair of simples, in the manner formerly taught, we may have a great many other answers, as follows, *viz.*

By

By increasing the first 32, we have,

40. 40. 40. 40. 40. 40. 40. 40. }
 32. 36. 40. 44. 48. 52. 56. 60. } &c.
 32. 31. 30. 29. 28. 27. 26. 25. }

By increasing the last 32, we have,

40. 40. 40. 40. 40. 40. 40. 40.
 32. 28. 24. 20. 16. 12. 8. 4.
 32. 33. 34. 35. 36. 37. 38. 39.

2. An innkeeper would, with 72 bushels of his best corn, at 60 d. per bushel, mix other corns at 48 d. at 36 d. at 24 d. per bushel : What quantity of the last three must he take, that the mixture may be worth 42 d. per bushel ?

First, *Ans.* *Secondly,* *Ans.* *Thirdly,* *Ans.*

$$\begin{array}{l} 42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 36 \\ 6 \\ 6 \\ 18 \end{array} \left| \begin{array}{l} 72 \\ 12 \\ 12 \\ 36 \end{array} \right. \\ 42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6 \\ 18 \\ 18 \\ 6 \end{array} \left| \begin{array}{l} 72 \\ 216 \\ 216 \\ 72 \end{array} \right. \\ 42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6, 18 \\ 6 \\ 18, 6 \\ 18 \end{array} \left| \begin{array}{l} 24 \\ 6 \\ 24 \\ 18 \end{array} \right| \begin{array}{l} 72 \\ 18 \\ 72 \\ 54 \end{array} \end{array}$$

Fourthly, *Ans.* *Fifthly,* *Ans.*

$$\begin{array}{l} 42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 18 \\ 6, 18 \\ 6 \\ 18, 6 \end{array} \left| \begin{array}{l} 18 \\ 24 \\ 6 \\ 24 \end{array} \right| \begin{array}{l} 72 \\ 96 \\ 24 \\ 96 \end{array} \\ 42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6, 18 \\ 18 \\ 18 \\ 18, 6 \end{array} \left| \begin{array}{l} 24 \\ 18 \\ 18 \\ 24 \end{array} \right| \begin{array}{l} 72 \\ 54 \\ 54 \\ 72 \end{array} \end{array}$$

Sixthly, *Ans.* *Seventhly,* *Ans.*

$$\begin{array}{l} 42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6 \\ 6, 18 \\ 18, 6 \\ 6 \end{array} \left| \begin{array}{l} 6 \\ 24 \\ 24 \\ 6 \end{array} \right| \begin{array}{l} 72 \\ 288 \\ 288 \\ 72 \end{array} \\ 42 \left\{ \begin{array}{l} 60 \\ 48 \\ 36 \\ 24 \end{array} \right\} \begin{array}{l} 6, 18 \\ 6, 18 \\ 18, 6 \\ 18, 6 \end{array} \left| \begin{array}{l} 24 \\ 24 \\ 24 \\ 24 \end{array} \right| \begin{array}{l} 72 \\ 72 \\ 72 \\ 72 \end{array} \end{array}$$

In each of the seven answers here given, 72 is limited ; but then any two of the three remaining simples may be esteemed a pair, affording in all three pairs, from which innumerable other answers may be found.

3. A goldsmith would, with 85 oz. of gold, worth 5 l. per oz. mix other sorts of gold, at 4 l. 10 s. per oz. and at 4 l. per oz. : What quantity of the last two, and what quantity of alloy must he take, that the mass may be worth 4 l. 5 s. per oz. ?

L. *oz.*

$$\begin{array}{l} 4.25 \left\{ \begin{array}{l} 5 \\ 4.5 \\ 4 \\ 0 \end{array} \right\} \begin{array}{l} 4.25 \\ .25 \\ .25 \\ .75 \end{array} \end{array} \quad \text{As } 4.25 : 85 :: \left\{ \begin{array}{l} 4.25 : 85 \\ .25 : 5 \\ .25 : 5 \\ .75 : 15 \end{array} \right\} \text{Ans.}$$

 By

By other ways of linking, and from the pairs that arise from the three variable simples, other answers innumerable may be found.

4. A vintner would, with 10 gallons of wine, at 12s. *per* gallon, and 20 gallons, at 15s. *per* gallon, mix other wines, at 18s. and at 20s. *per* gallon: What quantity of the last two must he take, that the mixture may be worth 16s. *per* gallon?

This question is limited to a certain quantity of two of the simples, and, previous to the operation, the two given quantities must be supposed to be mixed by themselves; and the rate of the mixture found by alligation medial, as under.

Gallons.	s.
10, at 12s. come to	120
20, at 15s. amount to	300
—	—
30	30)420(14s. <i>per</i> gallon.

The question now may be considered as limited only in the quantity of one of the simples, and may be proposed and solved as follows:

A vintner would, with 30 gallons of wine, at 14s. *per* gallon, mix other wines, at 18s. and at 20s. *per* gallon: What quantity of the last two must he take, that the mixture may be worth 16s. *per* gallon?

$$16 \left\{ \begin{array}{l} 14 \\ 18 \\ 20 \end{array} \right. \begin{array}{l} 2, 4 \\ 2 \\ 2 \end{array} \left| \begin{array}{l} 6 \\ 2 \\ 2 \end{array} \right| \begin{array}{l} 30 \\ 10 \\ 10 \end{array} \quad \text{Ans.}$$

That is, 30 gallons of wine, at 14s. *per* gallon; or, resolving this compound into its simples, the answer is, that 10 gallons of wine at 12s. *per* gallon, and 20 gallons at 15s. together with 10 gallons of each of the other two sorts of wine, will make the mixture proposed.

$$\begin{array}{rcl} 10 \times 12 & = & 120 \\ 20 \times 15 & = & 300 \\ 10 \times 18 & = & 180 \\ 10 \times 20 & = & 200 \\ \hline 50 & & 50)800(16 \text{ proof.} \end{array}$$

The quantity 30, or rather 10 and 20, are limited; but from the remaining pair a great many other answers may be found.

The procedure is the same when the question is limited in the quantity of three or more of the simples.

Variety

Variety III.

When the question is limited to a certain quantity of the mixture, this is called *Alligation Total*.

After linking the branches, and taking the differences, work by the proportion following :

As the sum of the differences,
To each particular difference ;
So the given total of the mixture,
To the respective quantities required.

EXAMPLES.

1. A vintner hath wine at 3 s. per gallon, and would mix it with water, so as to make a composition of 144 gallons, worth 2 s. 6 d. per gallon : How much wine and how much water must he take ?

30	$\begin{array}{r} 36 \overline{) 30} \\ 6 \\ \hline 36 \end{array}$	Gal.	$\begin{array}{l} 120 \text{ of wine.} \\ 24 \text{ of water.} \\ \hline 144 \text{ total.} \end{array}$	} Ans.	$\begin{array}{r} 120 \times 36 = 4320 \\ 24 \times 6 = 144 \\ \hline \text{Proof } 144) 4320(30 \end{array}$
----	---	------	--	--------	---

As 36 : 30 :: 144 : 120

As 36 : 6 :: 144 : 24

There being here only two simples, and the total of the mixture limited, the question admits but of one answer.

2. A distiller would mix brandy at 8 s. wine at 7 s. and cyder at 1 s. per gallon, so that the mixture may contain 26 gallons, and be worth 5 s. per gallon : What quantity of each must he take ?

5	$\begin{array}{r} 8 \\ 7 \\ 1 \end{array} \begin{array}{l} 4 \\ 4 \\ 3, 2 \end{array}$	} Ans.	$\begin{array}{r} 4 \\ 4 \\ 5 \\ \hline 13 \end{array} \begin{array}{l} 8 \\ 8 \\ 10 \\ \hline 26 \end{array}$	If 13 : 4 :: 26 : 8	If 13 : 5 :: 26 : 10
---	--	--------	--	---------------------	----------------------

From the alternate differences, an immense variety of other unlimited answers may be obtained by the methods explained above ; and any one of these unlimited answers, that suits the purpose best, may be reduced to the limits of the question by the proportion assigned above. And even, from the limited answer, other answers may be found, by observing the directions following, *viz.*

When three simples, as here, are mixed ; though the total be limited, yet from the answer found by the operation, other answers may be deduced in the manner following :

F f

First,

First, Increase or diminish the quantity of that simple whose rate alone is greater or less than the rate of the mixture, by the difference of the differences between the rate of the mixture and the rates of the other two simples. Thus, in the above example, the rate of the cyder alone is less than the rate of the mixture, and the differences betwixt the rate of the mixture and the rates of the other two simples, are respectively 3 and 2, whose difference is 1; and accordingly the quantity of the cyder found by the operation, *viz.* 10, may be increased or diminished by 1, that is, it may be made 11 or 9.

Secondly, Of the two remaining simples, increase or diminish the quantity of that one whose rate is farthest from the rate of the mixture, by the sum of the differences betwixt the rate of the mixture, and the rates of the other two simples. Thus, in the above example of the two remaining simples, the rate of the brandy is farthest from the rate of the mixture, and the rate of the mixture differs from the rates of the other two simples, *viz.* the wine and cyder respectively, by 2 and 4, whose sum is 6; and accordingly the quantity of the brandy, *viz.* 8, may be increased or diminished by 6, that is, it may be made 14 or 2.

Thirdly, Diminish or increase the quantity of the remaining simple, by the sum of the differences betwixt the rate of the mixture and the rates of the other two simples. But here observe, that the quantity of this simple is to be diminished when those of the former two are increased; and the contrary. Thus the quantity of the wine, *viz.* 8, is to be diminished or increased by 7, the sum of 3 and 4, the respective differences betwixt the rate of the mixture and the rates of the brandy and cyder; that is, it may be made 1 or 15.

By this means other two answers to the above question are obtained, which, with the answer found by the operation, are here subjoined.

Brandy	14.	8.	2.
Wine	1.	8.	15.
Cyder	11.	10.	9.
	26	26	26

3. A goldsmith hath several sorts of gold, viz. N^o 1. of 24 carats fine, N^o 2. of 24 carats fine, N^o 3. of 18 carats fine, and N^o 4. of 16 carats fine: How much of each sort must he take, to make a mass of 60 ounces, of 21 carats fine?

		<i>Ans.</i>		
21	{	24	5	25
		22	3	15
		18	1	5
		16	3	15
		12		60

As 12 : 5 :: 60 : 25

As 12 : 3 :: 60 : 15

As 12 : 1 :: 60 : 5

The

The branches in the above example may be linked other six ways, which will accordingly afford so many other answers.

When four simples, as here, are mixed, and it is required to deduce other answers from that found by the operation, in this case one of the simples must be considered as invariable, and the three variable simples must be such, that the rate of one of them may, with respect to the rate of the mixture, be opposite to the rates of the other two; that is, if the rate of one of the variable simples be greater than the rate of the mixture, then the rates of the other two must both be less; and the contrary.

Thus, in the above example, making N^o 1. invariable, the rate of N^o 2. is greater than the rate of the mixture, and the rates of the other two are both less; and accordingly by the three directions subjoined to Ex. 2. we may, from the answer in Ex. 3. deduce other answers, as follows:

N^o 1. invariable.

N ^o 1.	25.	25.	25.	25.
N ^o 2.	15.	13.	11.	9.
N ^o 3.	5.	11.	17.	23.
N ^o 4.	15.	11.	7.	3.
	—	—	—	—
	60	60	60	60

Next, making N^o 2. invariable, we shall have,

N ^o 1.	25.	23.	21.
N ^o 2.	15.	15.	15.
N ^o 3.	5.	13.	21.
N ^o 4.	15.	9.	3.
	—	—	—
	60	60	60

Again, making N^o 3. invariable, we shall have,

N ^o 1.	31.	25.	19.	13.	7.	1.
N ^o 2.	7.	15.	23.	31.	39.	47.
N ^o 3.	5.	5.	5.	5.	5.	5.
N ^o 4.	17.	15.	13.	11.	9.	7.
	—	—	—	—	—	—
	60	60	60	60	60	60

Lastly, making N^o 4. invariable, we shall have,

N ^o 1.	17.	21.	25.	29.	33.
N ^o 2.	27.	21.	15.	9.	3.
N ^o 3.	1.	3.	5.	7.	9.
N ^o 4.	15.	15.	15.	15.	15.
	—	—	—	—	—
	60	60	60	60	60

By taking the six answers that will arise from the other six ways of linking, and proceeding in like manner with each of them, a great many other answers may be found.

4. A merchant having wines at 12d. 14d. 15d. 18d. and 22d. per pint, would make a mixture of 1240 pints, to sell at 17d. per pint: What quantity of each must he take?

		<i>Ans.</i>	
17 {	12	1,5	6
	14	5	5
	15	5	5
	18	5	5
	22	5,3,2	10
		—	—
		31	1240

As 31 : 6 :: 1240 : 240
As 31 : 5 :: 1240 : 200
As 31 : 10 :: 1240 : 400

The branches in this example may be alligated a great many other ways; each of which will afford a different answer.

When five simples, as here, are mixed; in order to deduce other answers from that found by the operation, two of the simples, taken by turns, must always be made invariable; and the three remaining simples must be such, that the rate of one of them being greater or less than the rate of the mixture, the rates of the other two must respectively be both less, or both greater, than the rate of the mixture; and then the procedure is the same as formerly. In like manner, when six, seven, eight, &c. simples, are mixed, all the simples must by turns be made invariable, except three; and these three must be qualified as above.

5. Suppose 9 lb. of pure gold immersed in a vessel full of water to expel 3 lb. of water, 9 lb. of pure silver to expel 6 lb. of water, and 9 lb. of a mass made up of gold and silver to expel 4 lb. of water; required the quantity of gold and silver in the said mass?

In order to solve this question, the three several quantities of water expelled, which denote the specific bulks or gravities of the simples, must be considered as the rates; that of the mass being esteemed the root, and the other two the branches, as follows:

$$\begin{array}{r} \text{Ans.} \\ 4 \left\{ \begin{array}{l} 3)^2 \\ 6) 1 \end{array} \right. \left| \begin{array}{l} 6 \text{ lb. of pure gold} \\ 3 \text{ lb. of pure silver} \end{array} \right\} \text{ in the mass.} \\ \hline 3 \quad 9 \end{array}$$

Hence is solved the famous question with respect to the crown of Hiero King of Syracuse. He had ordered a crown to be made of pure gold; but suspecting that the founder had mixed silver with the gold, he desired Archimedes to examine the affair, and tell him, how much gold and how much silver was in the crown. Archimedes, by immersing the crown, and also the like quantities of pure gold and silver, in water, severally, and observing the respective quantities of water expelled, found an answer to the question.

C H A P. IX.

R U L E of F A L S E.

THE Rule of False is so called, because, by the help of false supposed numbers, the true ones are discovered; and is divided into Single and Double, or into Single Position and Double Position.

Before the knowledge of algebra came to be common, this rule was in high esteem, and found to be of great service; and though not

not so universally practised now as formerly, yet still it continues to be of considerable use.

I. *Single Position.*

Single position is, when, by one supposition, or one position, the number sought is discovered: in the finding of which we work first with the position, as if it was the true number; and, having brought out the result, we then proceed by the following proportion:

As the result by operation, or false result,
To the false position;
So the true result,
To the true position, or number sought.

EXAMPLES.

1. A, B, and C, buy a quantity of wine for 340l.; of which A pays three times more than B, and B four times more than C: What paid each?

L.	
Suppose A paid 36	As 51 : 36 :: 340 : 240
Then B paid 12	
And C paid 3	
Result 51	
	L.
	So A paid 240
	B paid 80
	C paid 20
	340 proof.

2. A person having about him a certain number of crowns, said, If a third, a fourth, and a sixth of them were added together, the sum would be 45: How many crowns had he?

Suppose 48, whereof a third is 16	As 36 : 48 :: 45 : 60
a fourth is 12	
a sixth is 8	A third of 60 is 20
Result 36	a fourth is 15
	a sixth is 10
	Proof 45

3. Suppose the sum of 20l. was to be paid by A, B, and C, in the following proportion, viz. A $\frac{1}{2}$, B $\frac{1}{3}$, C $\frac{1}{4}$: What must each pay?

	L. s. d.
$\frac{1}{2}$ of 20l. is	10 0 0
$\frac{1}{3}$ is	6 13 4
$\frac{1}{4}$ is	5 0 0
Result	21 13 4
	F f 3

L.

$$\begin{array}{rcl} & L. \quad s. \quad d. & \\ \text{As } 21 \quad 13 \quad 4 : & \left\{ \begin{array}{l} 10 \quad 0 \quad 0 \\ 6 \quad 13 \quad 4 \\ 5 \quad 0 \quad 0 \end{array} \right\} & :: 20 : \left\{ \begin{array}{l} 9 \quad 4 \quad 7\frac{1}{2} \\ 6 \quad 3 \quad 0\frac{1}{2} \\ 4 \quad 12 \quad 3\frac{1}{2} \end{array} \right\} \end{array}$$

Proof 20 0 0

4. A schoolmaster being asked how many scholars he had, answered, If I had as many, half as many, and one fourth as many, I should have 99: How many had he? *Ans.* 36.

5. A, B, and C, discoursing of their age, B says to A, I am as old, and half as old again, as you; says C to B, I am twice as old as you; and A affirms, that the sum of their three ages amounted to 165: What was each man's age? *Ans.* A 30, B 45, and C 90 years of age.

6. A gentleman bought a chaise, horse, and harness, for 55l.; the horse came to twice the price of the chaise, and the harness cost one third the price of the horse: What did he pay for each? *Ans.* For the chaise 15l. for the horse 30l. and for the harness 10l.

II. Double Position.

Double position is when two suppositions, or two positions, must be used, in order to discover the number sought; in finding whereof, work with each position, as if it was the true number, till by this means you bring out the results, and consequently the errors; and then proceed by the following

R U L E.

Multiply the two positions into their altern errors; then, errors alike, that is, both excesses, or both defects, divide the difference of the products by the difference of the errors. But if the errors are unlike, that is, the one an excess and the other a defect, divide the sum of the products by the sum of the errors.

Or, instead of the above rule, you may use the following proportion:

As the difference of the errors, if alike, or their sum, if unlike,
To the difference of the positions;
So either error,
To a fourth number.

Which fourth number added to, or subtracted from, the position producing the error, gives the number sought.

E X A M P L E S.

1. A person buys 30 pints of liquor for 50 shillings; part beer, at

at 6 d. *per* pint, and part wine, at 3 s. *per* pint : How much was there of each ?

<i>Pts.</i>	<i>s.</i>	<i>Pts.</i>	<i>s.</i>
<i>Position 1.</i> Beer 6, at 6 d. is,	3	<i>Position 2.</i> Beer 10, at 6 d. is	5
Then wine 24, at 3 s. is	72	Then wine 20, at 3 s. is	60
	—		—
Result	75	Result	65
	50		50
	—		—
Error of excess + 25		Error of excess + 15	
<i>Pos. Er.</i>		<i>Pts.</i>	<i>s.</i>
6 × 15 = 90		Beer 16, at 6 d. is,	8
10 × 25 = 250		Wine 14, at 3 s. is,	42
	<i>Pts.</i>		—
10) 160 (16 beer.		Proof	50

Or, work by the proportion, as follows :

$$\text{As } 10 : 4 :: 25 : 10$$

$$\text{Or, As } 10 : 4 :: 15 : 6$$

$$\text{And } 10 + 6 = 16 \text{ pints of beer.}$$

$$\text{And } 6 + 10 = 16 \text{ pints of beer, as before.}$$

It is easy to perceive, that the fourth number must here be added to the position, because, by increasing the quantity of beer, the quantity of wine is lessened ; and this of course lessens the result, which is too great.

The positions might have been so chosen, that the errors thence arising would have been both defects, or the one an excess, and the other a defect ; but the doing of this is left as a proper exercise to the learner. These varieties shall be illustrated in other examples.

It is worth while here also to observe, that double position takes place, or becomes necessary, when there is no partition of prices or quantities given, whereby to found or apply a proportion ; for the question says, A person bought 30 pints of liquor, part beer, part wine, without partitioning the quantities ; whereas, had the question said, That for every 8 pints of beer, he had 7 pints of wine, the answer might have been found by single position.

The reason of the assigned proportion will appear by considering, that the result varies with the position ; and that the error of a result is ever proportional to the error of its position ; so that the error of any one result is to the error of its own position, as the error of any other result is to the error of its position ; and the difference of any two results has the same proportion to the difference

of the positions from which they flow, as the difference of any other two results has to the difference of their positions. But the difference of the errors is equal to the difference of the results; therefore, As the difference of the errors, if alike, to the difference of the positions, so either error to a fourth number, *viz.* the error of the position.

When the errors are unlike, one of the false results is greater and the other less than the true result given in the question; and consequently the difference of the errors is found the same way as the difference of an affirmative and negative quantity, namely, by taking their sum; or, in the same manner as is found the difference of latitude of two places, whereof one lies to the north, and the other to the south of the equator; that is, by adding the latitudes of the two places into one sum.

The truth of the rule will appear, by observing, that the difference of the two products arising from the multiplication of the positions into their altern errors, may be conceived as made up of two small parts or products, *viz.* first, the product of half the sum of the positions into the difference of the errors; secondly, the product of half the sum of the errors into the difference of the positions. Now, the former of these small products, divided by the difference of the errors, quotes half the sum of the positions, the divisor being one of the factors; the latter of these small products, divided also by the difference of the errors, quotes half the sum of the errors of the positions, by the proportion; and consequently the sum of these two small products, (equal to the difference of the alternate products), divided by the difference of the errors, quotes the true position, or answer sought, when the errors are alike.

When the errors are unlike, their difference, as also the difference of the alternate products, is found by taking their sum, as has been already shown.

2. A, B, and C, built a house, which cost 76 l.; whereof A paid a sum unknown, B paid 10 l. more than A, and C paid as much as A and B: What did each partner pay?

Position 1.		Position 2.	
A	6	A	9
B	16	B	19
C	22	C	28
	—		—
Result	44	Result	56
	76		76
	—		—
Error of defect—	32	Error of defect—	20

Pos. Er.

$$6 \times 20 = 120$$

$$9 \times 32 = 288$$

L.

$$12)168(14 \text{ A}$$

L.

$$\text{A } 14$$

$$\text{B } 24$$

$$\text{C } 38$$

76 proof.

By the proportion.

As 12 : 3 :: 32 : 8. And 8 + 6 = 14 } A's share, as
Or, As 12 : 3 :: 20 : 5. And 5 + 9 = 14 } before.

It is obvious, that the fourth numbers here must be added to the positions, because the results were both too small.

3. A labourer having threshed out 40 quarters of grain, part wheat, part barley, received, in name of wages, 28s. or 336d. being paid at the rate of 12d. for every quarter of wheat, and 6d. for every quarter of barley : How many of the 40 quarters were wheat, and how many were barley ?

Pos. 1. Wheat 10, at 12d. is, 120
Barley 30, at 6d. is, 180
Result 300
336
Error of defect -36

Pos. 2. Wheat 26, at 12d. is, 312
Barley 14, at 6d. is, 84
Result 396
336
Error of excess +60

Pos. Er.

$$10 \times 60 = 600$$

$$26 \times 36 = 936$$

$$96) 1536(16 \text{ quarters of wheat.}$$

$$576$$

$$576$$

Pos. 3. Wheat 16, at 12d. is, 192
Barley 24, at 6d. is, 144

Proof 336

By the proportion.

As 96 : 16 :: 36 : 6
Or, As 96 : 16 :: 60 : 10

And 6 + 10 = 16 } quarters of wheat,
And 26 - 10 = 16 } as before.

The

The fourth number of the first proportion is added to the position, because the result was too little. But the fourth number of the second proportion is subtracted from the position, the result being too great.

4. What number is that whereof $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{8}$, make 19?

1 pos. 30.

$$\begin{array}{r} \frac{1}{2} = 10 \\ \frac{1}{4} = 7.5 \\ \frac{1}{5} = 6 \\ \frac{1}{8} = 5 \end{array}$$

Result 28.5

19

Error + 9.5

2 pos. 10.

$$\begin{array}{r} \frac{1}{2} = 3.33 \\ \frac{1}{4} = 2.5 \\ \frac{1}{5} = 2. \\ \frac{1}{8} = 1.66 \end{array}$$

Result 9.50

19.

Error - 9.5

Pos. Er.

$$30 \times 9.5 = 285$$

$$10 \times 9.5 = 95$$

19)380(20 Ans.

$$\frac{1}{2} \text{ of } 20 = 6.6$$

$$\frac{1}{4} = 5$$

$$\frac{1}{5} = 4$$

$$\frac{1}{8} = 3.3$$

Proof 19

Hence observe, that any question, such as this, that properly belongs to single position, may also be solved by double position.

Again, observe, that when the errors are equal and unlike, half the sum of the positions is the answer, or number sought.

5. A, B, and C, discoursing of their age, A affirmed, that he was 18 years old; B said, his age was equal to that of A, and half the age of C; and C affirmed, he was as old as both A and B: What was the age of each person? *Ans.* A 18, B 54, and C 72.

6. A father dying, left to his three sons, A, B, C, his estate in money, dividing it as follows, viz. To A he gave the half, wanting 44l.; to B he gave a third, and 14l. over; and to C he gave the remainder, which was 82l. less than the share of B: What was the sum left, and what was each son's share? *Ans.* The sum left was 588l. whereof A had 250l. B 210l. and C 128l.

7. A and B had each a certain number of crowns, and said A to B, If you give me one of your crowns, I shall then have five times as many as you have behind; but said B to A, If you give me one of your crowns, then shall each of us have an equal number: How many crowns had each person? *Ans.* A had 4 and B 2.

8. A gentleman had two horses, Chestnut and Swift, and a saddle

saddle worth 50*l*. which set on the back of Chesnut, makes his value double that of Swift; but the saddle set on the back of Swift, makes his value triple that of Chesnut: What was the value of each horse? *Ans.* Chesnut 30*l*. and Swift 40*l*.

9. The money staked by A, B, and C, playing at Hazard, was 324 crowns; but they happening to disagree, each seized as many of the crowns as he could: A got a number unknown; B as many as A, and 15 over; C got a fifth part of both their sums added together: How many crowns got each? *Ans.* A 127½, B 142½, and C 54.

10. A gentleman caught a fish, whose head was 6 inches long, the tail as long as the head and half the body, the body was just the length of the head and tail: What was the length of the fish, what the length of the body, and what the length of the tail? *Ans.* Length of the fish 48 inches, length of the body 24 inches, and length of the tail 18 inches.

11. When first the marriage-knot was tied
 Betwixt my wife and me,
 My age did hers as far exceed,
 As three times three does three;
 But after ten and half ten years,
 We man and wife had been,
 Her age came up as near to mine,
 As eight is to sixteen.
 Now, Tyro skill'd in numbers, say,
 What were our ages on the wedding-day?

Answer.
 Sir, Forty-five years you had been,
 Your bride no more than just fifteen.

C H A P. X.

E X T R A C T I O N of R O O T S.

IF unity be multiplied continually by any given number, the products thence arising are called *powers of that number*; and the given number is called *the root, or first power*.

Thus, if 2 be the given number, then $1 \times 2 = 2$ is the root or first power; and $2 \times 2 = 4$ is the square or second power; and $4 \times 2 = 8$ is the cube or third power; and $8 \times 2 = 16$ is the biquadrate or fourth power; and $16 \times 2 = 32$ is the fursolid or fifth power; and $32 \times 2 = 64$ is the sixth power, or cube squared, &c.

The natural numbers, 1, 2, 3, &c. are sometimes placed over these powers, denoting the number of multiplications used in producing them, or showing what powers they are; and are called *indices or exponents*, as in the following scheme.

Indices,

Indices, 0, 1, 2, 3, 4, 5, 6, 7, &c.
 Powers, 1, 2, 4, 8, 16, 32, 64, 128, &c.

The raising any root or number given to any power required, is called *involution*; and is performed by multiplying the given root into unity continually, as taught above. But the finding the root of a given power is called *evolution*, or *extraction of roots*.

If the root of any power not exceeding the seventh power, be a single digit, it may be obtained by inspection, from the following table of powers.

T A B L E.

1st power of root.	2d power of square.	3d power of cube.	4th power of biquadrate.	5th power of solid.	6th power of cube square.	7th power.
1	1	1	1	1	1	1
2	4	8	16	32	64	128
3	9	27	81	243	729	2187
4	16	64	256	1024	4096	16384
5	25	125	625	3125	15625	78125
6	36	216	1296	7776	46656	279936
7	49	343	2401	16807	117649	823543
8	64	512	4096	32768	262144	2097152
9	81	729	6561	59049	531441	4782969

I. *Extraction of the Square Root.*

R U L E S.

1. Divide the given number into periods of two figures, beginning at the right hand in integers, and pointing toward the left. But in decimals, begin at the place of hundreds, and point toward the right. Every period will give one figure in the root.

2. Find by the table of powers, or by trial, the nearest lesser root of the left-hand period, place the figure so found in the quot, subtract its square from the said period, and to the remainder bring down the next period for a dividend or resolvend.

3. Double the quot for the first part of the divisor; inquire how often this first part is contained in the whole resolvend, excluding the units place; and place the figure denoting the answer both in the quot and on the right of the first part; and you have the divisor complete.

4. Multiply

4. Multiply the divisor thus completed by the figure put in the quot, subtract the product from the resolvend, and to the remainder bring down the following period for a new resolvend, and then proceed as before.

Note 1. If the first part of the divisor, with unity supposed to be annexed to it, happen to be greater than the resolvend, in this case place 0 in the quot, and also on the right of the partial divisor; to the resolvend bring down another period; and proceed to divide as before.

Note 2. If the product of the quotient-figure into the divisor happen to be greater than the resolvend, you must go back, and give a lesser figure to the quot.

Note 3. If, after every period of the given number is brought down, there happen at last to be a remainder, you may continue the operation, by annexing periods or pairs of ciphers till there be no remainder, or till the decimal part of the quot repeat or circulate, or till you think proper to limit it.

EXAMPLE I.

Required the square root of 133225.

Square number	133225	(365 root.	
	9		365
1 div. 66)	432	resolvend.	1825
	396	product.	2190
2 div. 725)	3625	resolvend.	1095
	3625	product.	133225 proof.

EXAMPLE II.

Required the square root of 54990 $\frac{1}{4}$.

Square number	54990.25	(234.5 root.
	4	
1 div. 43)	149	resolvend.
	129	product.
2 div. 464)	2090	resol.
	1856	prod.
3 div. 4685)	23425	resol.
	23425	prod.

E X.

EXAMPLE III.

Required the square root of 72, to eight decimal places.

72.00000000 (8.48528137 root.

64

164) 800

656

1688) 14400

13504

16965) 89600

84825

169702) 477500

339404

169704) 138096

..... 135763

2333

1697

636

509

127

118

(9)

After getting half of the decimal places, I work by contracted division for the other half; and obtain them with the same accuracy as if the work had been at large.

EXAMPLE IV.

Required the square root of .2916

.2916 (.54 root.

25

104) 416

416

EXAMPLE V.

Required the square root of .001225

.001225 (.035 root.

9

65) ... 325

325

EX

EXAMPLE VI.

Required the square root of 5.875

$$\begin{array}{r}
 5.875 \text{ (2.423 root.)} \\
 4 \\
 \hline
 44) 187 \\
 176 \\
 \hline
 482) 1150 \\
 964 \\
 \hline
 4843) 18600 \\
 14529 \\
 \hline
 (4071)
 \end{array}$$

$$\begin{array}{r}
 2.423 \\
 2.423 \\
 \hline
 7269 \\
 4846 \\
 \hline
 9692 \\
 4846 \\
 \hline
 5.870929 \\
 4071 \text{ rem.} \\
 \hline
 5.875000 \text{ proof.}
 \end{array}$$

The operation may be continued further by annexing to the remainder more periods of ciphers.

EXAMPLE VII.

Required the square root of the repetend . $\dot{x}$. $\dot{x}$ 1(.3333 &c. root.

9

$$\begin{array}{r}
 63) 211 \\
 189 \\
 \hline
 \end{array}$$

Here, instead of annexing pairs of ciphers to the remainders, I annex periods of the repetend.

$$\begin{array}{r}
 663) 2211 \\
 1989 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 6663) 22211 \\
 19989 \\
 \hline
 \end{array}$$

(222211) and so on to infinity.

EXAMPLE VIII.

Required the square root of the repetend . $\dot{4}$. $\dot{4}$ 4(.666 &c. root.

36

$$\begin{array}{r}
 126) 844 \\
 756 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1326) 8844 \\
 7956 \\
 \hline
 \end{array}$$

(88844)

Note. No repeating digit, except . $\dot{x}$ and . $\dot{4}$ have for their roots a pure single repetend.

E X-

EXAMPLE IX.

Required the square root of 1320.7

$$\begin{array}{r}
 \sqrt{1320.7} \text{ (36.33 } \& \text{c. root.} \\
 9 \\
 \hline
 66) 420 \\
 \underline{396} \\
 24 \\
 723) 2411 \\
 \underline{2169} \\
 242 \\
 7263) 24211 \\
 \underline{21789} \\
 242211 \\
 \hline
 (242211)
 \end{array}$$

EXAMPLE X.

Required the square root of the circulate 138.518, ; or, which is the same thing, of 13,8.51,

$$\begin{array}{r}
 \sqrt{13.8.51} \text{ (11.769389 root.} \\
 1 \\
 \hline
 21) 38 \\
 \underline{21} \\
 17 \\
 227) 1751 \\
 \underline{1589} \\
 162 \\
 2346) 16285 \\
 \underline{14076} \\
 2209 \\
 23529) 220918 \\
 \underline{211761} \\
 9157 \\
 23538) 9157 \\
 \underline{7061} \\
 2096 \\
 1883 \\
 213 \\
 211 \\
 \hline
 (2)
 \end{array}$$

Here I annex to the remainders periods of the circulating figures.

I continue the work till I have three decimal places in the root ; and then, by contracted division, I obtain three more.

If

If the square root of a vulgar fraction be required, find the root of the given numerator for a new numerator, and find the root of the given denominator for a new denominator. Thus, the square root of $\frac{4}{9}$ is $\frac{2}{3}$, and the root of $\frac{16}{25}$ is $\frac{4}{5}$; and thus the root of $\frac{25}{4}$ ($=6\frac{1}{4}$) is $\frac{5}{2} = 2\frac{1}{2}$.

But if the root of either the numerator or denominator cannot be extracted without a remainder, reduce the vulgar fraction to a decimal, and then extract the root, as in Example 4. and 5.

The extraction of roots may be conducted in the way of vulgar fractions; but then the operation becomes involved and troublesome. The following example, done both ways, will shew the superior excellence of decimals.

Required the square root of $20\frac{1}{4}$.

By vulgar fractions.

$$\begin{array}{r} 20\frac{1}{4}(4\frac{1}{2} \text{ root.} \\ 16 \\ \hline 8\frac{1}{2})4\frac{1}{4} \\ 4\frac{1}{4} \\ \hline \end{array}$$

Decimally.

$$\begin{array}{r} 20.25(4.5 \\ 16 \\ \hline 85)425 \\ 425 \\ \hline \end{array}$$

The method of extracting the square root is founded on Euclid, book 2. prop. 4. where it is demonstrated, That if any line, and consequently any number, be divided into two parts, the square of the whole line or number will be equal to the squares of the two parts, together with twice a rectangle or product under the parts. Thus, if the number 10 be divided into two parts, suppose 6 and 4, the square of 10 will be equal to $6 \times 6 = 36 + 4 \times 4 = 16$, + twice $6 \times 4 = 48$; for $36 + 16 + 48 = 100 = 10 \times 10$.

In order to shew the reason of the rules, we must have recourse to algebra, in which a letter, as a , is put to represent any number at pleasure; and aa denotes the square of a , and aaa its cube; ay denotes a product, and $2ay$ denotes a double product; and the factor into which any letter is multiplied, is called its *coefficient*.

Now, let the first two periods of the given square number be considered as a square number by themselves; and as every period gives a figure to the root, their root will consist of two figures, or two parts; call the first a and the other y ; then $a + y$ is their root complete; and by Euclid ii. 4. or by algebraic multiplication, $aa + 2ay + yy$ will be equal to the first two periods of the square number given; and when aa , the square of the first part of the root, is subtracted from the first period, there will remain $2ay + yy$; and to find y , the other part of the root, I divide by its factor or coefficient, *viz.* by $2a + y$; that is, I find a divisor by doubling the quot, and annexing to it the next quotient-figure.

G g

N. B.

N. B. The figure of the quot denoted by *a*, in regard another figure is to follow, is considered as in the place of tens, having a cipher on the right, and consequently so has the first part of the divisor; and therefore the annexing the quotient-figure to the said partial divisor, is the same thing in this case as adding it.

If there be more periods of the given square number, the two figures now in the quot are to be esteemed the first part of the root, and called *a*; and by repeating the operation, as above directed, the other part *y* is found. And if there be still more periods, call the three figures now in the quot *a*, then repeat the operation, and find *y* as before. And thus proceed till every period is brought down.

MORE EXAMPLES.

1. There are two numbers, whereof the lesser is 3456, their difference is 293392: What is the greater, and what the square root of their sum? *Ans.* The greater is 296848, and the square root of their sum 548.

2. There is a round floor, whose content is, 45.938271604 square feet, or 45 feet, 11 inches, 3 lines and 4 fourths: Required the side of a square floor equal to it in area? *Ans.* 6.7 feet, or 6 feet, 9 inches, 4 lines. See duodecimals in Decimal practice, Ex. 3.

3. There are three fields, whereof the first contains 100 acres, 3 roods and 36 poles; the second contains 118 acres, 2 roods and 24 poles; and the third contains 122 acres, 2 roods and 20 poles: Required the side of a square field that shall be equal to all the three in area? *Ans.* The side of the square field is 234 poles nearly, or 58 chains and 2 poles.

4. A round malt-kiln, whose diameter is 12 feet, is to be enlarged, so as to hold three times as much malt as formerly: Required the new diameter? *Ans.* 20.78 feet. The areas of circles are to one another as the squares of their diameters, by Euclid xii. 2.; multiply therefore the square of the given diameter by 3, and extract the square root of the product.

5. A tower is surrounded by a ditch 40 feet wide, and a scaling ladder 50 feet long will reach from the outside of the ditch to the top of the tower: Required the height of the tower? *Ans.* 30 feet. The square of the hypotenuse, or longest side of a right-angled triangle, is equal to the sum of the squares of the other two sides, by Euclid i. 47.; and therefore, from the square of 50 subtract the square of 40, and the square root of the remainder is the height of the tower.

6. Three towns, A, B, C, are so situate, that B lies 80 miles south of A, and C 60 miles west of A: What is the distance between B and C? *Ans.* 100 miles.

II. *Extraction of the Cube Root.*

R U L E S.

1. Divide the given number into periods of three figures, beginning at the right hand in integers, and pointing toward the left. But in decimals, begin at the place of thousands, and point toward the right. The number of periods shews the number of figures in the root.

2. Find by the table of powers, or by trial, the nearest lesser root of the left-hand period; place the figure so found in the quot; subtract its cube from the said period; and to the remainder bring down the next period for a dividend or resolvend.

The divisor consists of three parts, which may be found as follows:

3. The first part of the divisor is found thus: Multiply the square of the quot by 3, and to the product annex two ciphers; then inquire how often this first part of the divisor is contained in the resolvend, and place the figure denoting the answer in the quot.

4. Multiply the former quot by 3, and the product by the figure now put in the quot; to this last product annex a cipher; and you have the second part of the divisor. Again, square the figure now put in the quot for the third part of the divisor; place these three parts under one another, as in addition; and their sum will be the divisor complete.

5. Multiply the divisor, thus completed, by the figure last put in the quot, subtract the product from the resolvend, and to the remainder bring down the following period for a new resolvend, and then proceed as before.

Note 1. If the first part of the divisor happen to be equal to, or greater than the resolvend, in this case, place 0 in the quot, annex two ciphers to the said first part of the divisor, to the resolvend bring down another period, and proceed to divide as before.

Note 2. If the product of the quotient-figure into the divisor happen to be greater than the resolvend, you must go back, and give a lesser figure to the quot.

Note 3. If, after every period of the given number is brought down, there happen at last to be a remainder, you may continue the operation by annexing periods of three ciphers till there be no remainder, or till you have as many decimal places in the root as you judge necessary.

E X A M P L E I.

Required the cube root of 12812904?

G g 2

Cube

Cube number 12812904 (234 root.
8

1st part 1200 } 4812 resolvend.
2d part 180 }
3d part 9 }

1 divisor 1389 $\times 3 = 4167$ product.

1st part 158700 } 645904 resolvend.
2d part 2760 }
3d part 16 }

2 divisor 161476 $\times 4 = 645904$ product.

P R O O F.

234	Square 54756
234	234
936	219024
702	164268
468	109512
Square 54756	Cube 12812904

The reason of the rules will appear, if we take the first two periods of the given cube number, and consider them as a cube number by themselves, whose root will consist of two figures or parts, which call a and y ; and then, by algebraic multiplication, the first two periods of the cube number will be equal to $aaa + 3aay + 3aay + yyy$. Now, when aaa , the cube of the first part of the root, is subtracted from the left-hand period, there remains $3aay + 3aay + yyy$; and to find y , the other part of the root, I divide by its coefficient, viz. by $3aa + 3ay + yy$; and as the divisor consists of three parts, I begin with the first part, viz. $3aa$, and try how often it is contained in the resolvend; and then, by the help of the figure that goes to the quot, I make up the other two parts, as follows:

1st part 1200 $= 3aa = 3 \times 2 \times 2$
2d part 180 $= 3ay = 3 \times 2 \times 3$
3d part 9 $= yy = 3 \times 3$

1 divisor 1389

The reason of annexing one cipher for every a , viz. two ciphers in

in the first part of the divisor, and one in the second, is because the first figure of the root 2, another figure being to follow on the right, is really and truly 20.

If, as in the above example, there be more periods of the given cube number, the two figures now in the quote are to be esteemed the first part of the root, and called *a*; and by repeating the operation for a new divisor, the other part *y*, may be found. Thus,

$$\begin{array}{l} \text{1st part } 158700 = 3aa = 3 \times 23 \times 23 \\ \text{2d part } 2760 = 3ay = 3 \times 23 \times 4 \\ \text{3d part } 16 = yy = 4 \times 4 \\ \hline \text{2 divisor } 161476 \end{array}$$

In the first and second part of the divisor, the ciphers are annexed for the reason assigned above,

If there be still more periods in the cube number, call the three figures now in the quot *a*, repeat the operation, and find *y* as before; and thus proceed till every period is brought down.

The reason of dividing the given number into periods of two figures in the square, of three in the cube, of five in the sursolid, &c. and of there being as many figures in the root as there are periods, arises from the nature of involution; and will appear by resolving any number into its constituent parts, and raising them separately to the second, third, fourth, fifth power, &c. Let the number be $7842 = 7000 + 800 + 40 + 2$. Now, 7 has three places after it; but in the square it will have six, and in the cube nine. Again, 8 has two places after it; but in the square it will have four, and in the cube six, &c.

$$\begin{array}{r} \text{Thus, } 7\ 000 \\ \quad 7\ 000 \\ \hline \text{Square } 49,00,00,00 \\ \quad 7\ 000 \\ \hline \text{Cube } 343,000,000,000 \end{array}$$

$$\begin{array}{r} \text{And thus, } 8\ 00 \\ \quad 8\ 00 \\ \hline \text{Square } 64,00,00 \\ \quad 8\ 00 \\ \hline \text{Cube } 512,000,000 \end{array}$$

Hence it is plain, that the square of 7842 will contain four periods of two figures, and its cube will contain four periods of three figures; and so the number of periods will be equal to the number of figures in the root. And the square and cube of 842 will in like manner contain three periods each.

EXAMPLE II.

Required the cube root of $28\frac{1}{4}$.

G g 3

28.750000

$\dot{2}8.\dot{7}50000(3.06 \text{ root,}$
 $\underline{27}$

$\left. \begin{array}{r} 270000 \\ 5400 \\ 36 \end{array} \right\}$

$\underline{)1750000 \text{ resolv.}}$

Div. $275436 \times 6 = 1652616 \text{ prod.}$

$\underline{97384 \text{ rem.}}$

P R O O F.

$\begin{array}{r} 3.06 \\ 3.06 \\ \hline \end{array}$

Sq. $\begin{array}{r} 9.3636 \\ 3.06 \\ \hline \end{array}$

$\begin{array}{r} 1836 \\ 918 \\ \hline \end{array}$

$\begin{array}{r} 561816 \\ 280908 \\ \hline \end{array}$

Sq. 9.3636

$\begin{array}{r} 28.652616 \\ 97384 \text{ rem.} \\ \hline \end{array}$

28.750000 cube.

EXAMPLE III.

Required the cube root of $.000485613$.

$\dot{.}000485613(\dot{.}078 \text{ root,}$
 $\underline{343}$

$\left. \begin{array}{r} 14700 \\ 1680 \\ 64 \end{array} \right\}$

$\underline{) \dots 142613 \text{ resolv.}}$

Div. $16444 \times 8 = 131552 \text{ prod.}$

$\underline{11061 \text{ rem.}}$

EXAMPLE IV.

Required the cube root of 7584 .

7584(19.64 root.
I

300 }
270 }
81 }
6584 resolv.

1 div. 651 $\times 9 = 5859$ prod.

108300 }
3420 }
36 }
725000 resolv.

2 div. 111756 $\times 6 = 670536$ prod.

11524800 }
23520 }
16 }
54464000 resolv.

3 div. 11548336 $\times 4 = 46193344$ prod.

Rem. 8270656

If the cube root of a vulgar fraction be required, find the cube root of the given numerator for a new numerator, and the cube root of the given denominator for a new denominator. Thus, the cube root of $\frac{8}{27}$ is $\frac{2}{3}$, and the cube root of $\frac{27}{64}$ is $\frac{3}{4}$; and thus, the cube root of $\frac{125}{512}$ ($= 15\frac{5}{8}$) is $\frac{5}{4} = 2\frac{1}{2}$.

But if the root of either the numerator or denominator cannot be extracted without a remainder, reduce the vulgar fraction to a decimal, and then extract the root, as in Ex. 3.

MORE EXAMPLES.

1. There are two numbers, whereof the greater is 2579890752, their difference is 1152: What is the lesser, and what the cube root of their sum? *Ans.* The lesser is 2579889600, and the cube root of their sum is 1728.

2. The content of an oblong cellar is 1953.125 cubic feet: Required the side of a cubical cellar that shall contain just as much. *Ans.* 12.5 feet.

3. There are three boxes, the content of one is 10000 solid inches, of another 16656, and of the third 20000: Required the side of a cubical box that shall contain as much as all the three. *Ans.* 36 inches, or 3 feet.

4. Required the side of a cubical vessel that will contain 80 wine gallons. *Ans.* 26.43 inches.

231 cubic inches in a wine gallon.
80

18480 whose cube root is 26.43 inches.

5. A ship's length is 50 feet, breadth 20, and depth of the hold 10 : Required the dimensions of a like vessel triple her burthen.

Length 50, its cube $125000 \times 3 = 375000$, its cube root 71.54

Breadth 20, its cube $8000 \times 3 = 24000$, its cube root 29.24

Depth 10, its cube $1000 \times 3 = 3000$, its cube root 14.44

6. A brass bullet of 5 inches diameter weighs 20lb. : What is the diameter of a like bullet that weighs 160lb. ? *Ans.* 10 inches.

<i>lb.</i>		<i>lb.</i>
As 20 : 5 × 5 × 5 ::	160	
4 : 5 × 5 ::	160	
1 : 25 ::	40 : 1000,	whose cube root is 10 inches.

III. Extraction of the biquadrate Root.

R U L E.

Extract the square root of the given number ; and again extract the square root of the root so found, and the last of these roots is the root sought.

E X A M P L E.

Required the biquadrate root of 5308416.

5308416	(2304(48 root.
4	16
43)130	88)704
129	704
4604) 18416	
18416	

If, in the first extraction, there happen to be a remainder, continue the operation, by annexing pairs of ciphers, till you have twice as many decimal places in the square or first root, as you propose to have in the last root.

IV. Ex-

IV. *Extraction of the root of the fifth power, or fursolid.*

R U L E S.

1. Divide the given number into periods of five figures, find the nearest lesser root of the left-hand period, put the figure so found in the quot, subtract its fifth power, and to the remainder bring down the next period for a resolvend.

2. Put $a+y$ for the root, and then the fursolid or fifth power will be $aaaaa + 5aaaay + 10aaaay + 10aayyy + 5ayyyy + yyyyy$. Now, $aaaaa$ being already subtracted, there remains the other five parts; and to find y I divide by its coefficient, viz. by $5aaaa + 10aaaay + 10aayyy + 5ayyyy + yyyyy$; that is, I try how often $5aaaa$ is contained in the resolvend; and, by the help of the quotient-figure, I make up the other four parts of the divisor.

E X A M P L E.

Required the fursolid root of 33554432.

$$\begin{array}{r}
 33554432 \text{ (32 root.} \\
 \underline{243} \\
 9254432 \text{ resolv.} \\
 4050000 = 5aaaa \\
 540000 = 10aaaay \\
 36000 = 10aayyy \\
 1200 = 5ayyyy \\
 16 = yyyyy \\
 \hline
 \end{array}$$

Divisor $4627216 \times 2 = 9254432$ prod.

(o)

V. *Extraction of the Root of the sixth power, or cube squared.*

R U L E.

Extract the square root of the given number, and then extract the cube root of that root, the last is the root sought. Or, first extract the cube root, and then extract the square root of that root.

E X A M P L E.

Required the root of 191102976, being the sixth power.

$$\begin{array}{r}
 191102976 \\
 \hline
 \end{array}$$

VII. *Extraction of the Root of the eighth power.*

R U L E.

Extract the square root of the given number continually till you have three roots; the last of these is the root sought.

Thus, let 1785793904896 be the eighth power; by extracting the square root you get the biquadrate or fourth power, *viz.* 1336336; and by extracting the square root of the biquadrate, you get the square or second power, *viz.* 1156, whose square root is 34, the root sought.

VIII. *Extraction of the Root of the ninth power.*

R U L E.

Extract the cube root of the given number, and you have the cube or third power, whose cube root is the root sought.

Thus, let 5159780352 be the ninth power; by extracting the cube root you get the cube or third power, *viz.* 1728, whose cube root 12 is the root sought.

Universally, whatever the given power be, put $a + y$ for the root, and by involution raise $a + y$ to the power of the given number; then, with this as your guide or canon, extract the root in the manner prescribed and exemplified in the extraction of the root of the fifth and seventh powers.

But if the index of the given power be a multiple of 2, the work may be rendered easier; for by extracting the square root of the given number, you obtain a power whose index is one half of the index of the given power. Thus, by extracting the square root of the tenth power, you have the fifth power; and the square root of the twelfth power is the sixth power, &c.

Again, if the index of the given power be a multiple of 3, by extracting the cube root you obtain a power whose index is one third of the index of the power given. Thus the cube root of the ninth power is the cube or third power; and the cube root of the twelfth power is the biquadrate or fourth power, &c.

Involution is directly contrary to extraction or evolution; and therefore, if a square number be squared, it will give the biquadrate or fourth power; and if a biquadrate be squared, it will give the eighth power. Again, if a cube number be cubed, it will give the ninth power; and if the biquadrate be cubed, it will give the twelfth power.

C H A P. XI.

P R O G R E S S I O N.

THE comparing of numbers with one another is called *Comparative Arithmetic*; and in comparing any two numbers together,

gether, the first number, or the number compared, is called the *antecedent*, and the other number with which it is compared, is called the *consequent*.

In comparing two numbers, we may, by subtracting the lesser from the greater, find their difference; and the difference thus found is called the *arithmetical ratio* of the two numbers; and if two or more pairs of numbers have all the same ratio or difference, such numbers are said to be in *arithmetical proportion*; and the ratio in reference to the several proportional numbers, is usually called the *common difference*; the first and last term are called *extremes*, and the intermediate ones are denominated *means*.

If a rank of numbers, without interruption, have all the same difference, the proportion is said to be *continued* or *continual*; and such a rank is called an *arithmetical progression*, or an *arithmetical series*; such as, 2, 4, 6, 8, 10, 12.

But if, in the rank of proportional numbers, there be a breach or chasm, the proportion is said to be *discontinued*, *disjunct* or *interrupted*; such as, 3, 5, 7, — 19, 21, 23.

Again, in comparing two numbers, we may find how often the one contains the other, by dividing the antecedent by the consequent; and the quotient thence arising is called the *geometrical ratio* of the two numbers; and if two or more pairs of numbers have all the same ratio, such numbers are said to be in *geometrical proportion*. The first and last of the proportional numbers are called *extremes*, and the other terms are denominated *means*.

If a rank of numbers, without breach or chasm, have all the same ratio, the proportion is said to be *continued* or *continual*; and such a rank is called a *geometrical progression*, or a *geometrical series*; such as, 2 : 4 : 8 : 16 : 32.

But if, in the rank of proportional numbers, there be a chasm, the proportion is said to be *discontinued*, *disjunct*, or *interrupted*; such as, 12 : 6 :: 30 : 15.

I. *Arithmetical Progression.*

An arithmetical progression, or series, is formed either from 0, or from some number assumed as the first term, by continual addition or subtraction; and is either increasing or decreasing.

Thus 0, 3, 6, 9, 12, 15, 18, is an arithmetical progression or series, formed from 0, assumed as the first term, and increasing by the continual addition of 3; and such a series may be continued upward to infinity.

And 14, 12, 10, 8, 6, 4, 2, is an arithmetical progression, or series, formed from 14, assumed as the first term, and decreasing by the continual subtraction of 2; but such a series can be continued downward only till the last term becomes equal to or less than the common difference.

In an arithmetical progression, or series, five things occur to be considered.

I. The

- | | |
|------------------------------|-------------|
| I. The least term. | } Extremes. |
| II. The greatest term. | |
| III. The number of terms. | |
| IV. The common difference. | |
| V. The sum of all the terms. | |

The properties of numbers in arithmetical progression are various and numerous; but the chief, or most useful, which alone we propose to take notice of in this place, are explained in the following theorems:

T H E O R E M I.

Any term of an arithmetical series is equal to the sum of the least term, and the product of the common difference into the number of terms before or after it.

Thus, in the increasing series, 2, 5, 8, 11, 14, 17, the term $17 = 2 + 3 \times 5 = 2 + 15$.

Again, in the decreasing series 18, 16, 14, 12, 10, 8, 6, 4, 2, the term $18 = 2 + 8 \times 2 = 2 + 16$.

The reason is plain; because every term subsequent or prior to the least is made up by the continual addition or subtraction of the common difference.

Hence the product of the common difference into the number of terms minus unity, added to the least term gives the greatest, or subtracted from the greatest term leaves the least.

T H E O R E M II.

In an arithmetical series, consisting of three, five, or any odd number of terms, the double of the middle term is equal to the sum of the two extremes, or to the sum of any two means equally distant from the said middle term.

Thus, in the series, 1, 2, 3, 4, (5), 6, 7, 8, 9, the middle term 5 doubled is $10 = 1 + 9$, the two extremes, or $= 2 + 8$, or $3 + 7$, or $4 + 6$.

The reason is obvious: for of two terms equally distant from the middle term, the one exceeds the middle term just as much as the other is below it.

C O R O L L A R I E S.

1. Hence an arithmetical mean betwixt two given extremes is found by taking half the sum of the extremes. Thus, the mean betwixt 6 and 10 is 8; for $6 + 10 = 16$, and $2)16(8$. So 6, 8, 10, are in arithmetical proportion.

2. Hence also, to an extreme and mean given, a third proportional, or the other extreme, is found by subtracting the given extreme from the mean doubled. Thus, to 3, 6, given, the third proportional is 9; for $2 \times 6 = 12$, and $12 - 3 = 9$: so 3, 6, 9, are

are in arithmetical proportion. In like manner, to 10, 7, given, the third proportional is 4; for $2 \times 7 = 14$, and $14 - 10 = 4$: so 10, 7, 4, are arithmetical proportionals.

THEOREM III.

In an arithmetical series, consisting of 4, 6, or any even number of terms, the sum of the extremes is equal to the sum of the two middle terms, or to the sum of any two means equally distant from the extremes.

Thus, in the series, 3, 5, 7, 9, 11, 13, the sum of $3 + 13 = 7 + 9$, or $= 5 + 11$.

The reason is plain: for if betwixt the two middle terms in the series, a middle term be inserted, the sum of the extremes, as also the sum of every pair of equidistant means, will, by Theorem II. be equal to the double of the inserted middle term; and things equal to one and the same thing are equal to one another.

COROLLARIES.

1. Hence, to an extreme, and two means given, the other extreme, or fourth proportional, is found, by subtracting the given extreme from the sum of the two given means. Thus, to 4, 6, 8, the fourth proportional is 10; for $6 + 8 = 14$, and $14 - 4 = 10$; so 4, 6, 8, 10, are in arithmetical proportion. And this takes place though the proportion be disjunct. Thus, to 18, 14, —6, the fourth proportional is 2; for $14 + 6 = 20$, and $20 - 18 = 2$; so 18, 14, —6, 2, are in arithmetical proportion disjunct.

2. Hence likewise, when two extremes and a mean are given, the other mean is found by subtracting the given mean from the sum of the two extremes. Thus, to 13, 9, 7, the other mean is 11; for $13 + 7 = 20$, and $20 - 9 = 11$; so 13, 11, 9, 7, are arithmetical proportionals. This also takes place, though the proportion be disjunct. Thus, to 9, 15, —36, the other mean is 30; for $9 + 36 = 45$, and $45 - 15 = 30$; so 9, 15, —30, 36, are in arithmetical proportion disjunct.

THEOREM IV.

In an arithmetical series, the difference of the extremes, divided by the common difference, gives a quot, which, increased by unity, is the number of terms.

Thus, in the series, 2, 4, 6, 8, 10, 12, 14, the difference of the extremes 2 and 14 is 12, and 12, divided by the common difference 2, quotes 6, and $6 + 1 = 7$, the number of terms.

The reason is obvious: for the difference of the extremes is made up by a repetition of the common difference as often as there are terms save one; that is, the difference of the extremes is a product of the common difference into the number of terms minus unity.

THEO-

THEOREM V.

In an arithmetical series, the difference of the extremes, divided by the number of terms minus unity, quotes the common difference.

Thus, in the series, 3, 5, 7, 9, 11, 13, the difference of the extremes 3 and 13 is 10, and 10 divided by the number of terms minus unity, *viz.* 5, quotes 2, the common difference.

The reason is the same here as in the former theorem.

THEOREM VI.

In an arithmetical series, the sum of all the terms may be made out several ways, as follows :

1. The sum of the extremes, multiplied by the number of terms, gives a product, which divided by 2, quotes the sum of all the terms.

Thus, in the series, 3, 6, 9, 12, 15, 18, 21, the sum of 3 and 21 is 24 ; and $24 \times 7 = 168$; and $2)168(84$, the sum of all the terms.

The reason will appear by inverting the series, and adding equidistant pair of means, as follows :

$$\begin{array}{r}
 3, \quad 6, \quad 9, \quad 12, \quad 15, \quad 18, \quad 21 \\
 21, \quad 18, \quad 15, \quad 12, \quad 9, \quad 6, \quad 3 \\
 \hline
 24, \quad 24, \quad 24, \quad 24, \quad 24, \quad 24, \quad 24
 \end{array}$$

Now, as the sum of every pair is 24, and the number of terms 7, it is plain, the product of 7 into 24 will be double the sum of the series.

2. The sum of the extremes multiplied by half the number of terms, gives a product equal to the sum of all the terms.

Thus, in the series, 12, 10, 8, 6, 4, 2, the sum of 12 and 2 is 14, and $14 \times 3 = 42$, the sum of all the terms.

The reason is the same as before.

3. The half sum of the extremes multiplied by the number of terms, gives a product equal to the sum of all the terms.

Thus, in the series, 1, 4, 7, 10, 13, 16, 19, the half sum of 1 and 19 is 10, and $10 \times 7 = 70$, the sum of all the terms.

The reason is the same as above.

4. When a series consists of an odd number of terms, the middle term multiplied by the number of terms, gives a product equal to the sum of all the terms.

Thus, in the series, 3, 6, (9), 12, 15, the middle term $9 \times 5 = 45$, the sum of all the terms.

The reason is plain ; because the middle term, by Theorem II. is equal to half the sum of the extremes.

5. In a series of the natural numbers, 1, 2, 3, 4, 5, &c. the greatest

greatest given term multiplied into the next greater, gives a product, which divided by 2, quotes the sum of the whole series.

Thus, in the series, 1, 2, 3, 4, 5, 6, 7, 8, 9, the greatest given term, 9, multiplied into the next greater, 10, gives 90; and $2)90(45$, the sum of the whole series.

The reason is, because the greatest term is, in this case, the number of terms, and the next greater term is the sum of the extremes; therefore, &c.

6. In a series of the natural odd numbers, 1, 3, 5, 7, &c. the square of the number of terms is equal to the sum of the whole series.

Thus, in the series, 1, 3, 5, 7, 9, 11, 13, the number of terms is 7, and $7 \times 7 = 49$, the sum of the whole series.

The reason is, because, in this case, the half sum of the extremes is always equal to the number of terms.

7. In a series of the natural even numbers, 2, 4, 6, 8, &c. the number of terms multiplied into the number of terms plus unity, gives a product equal to the sum of the whole series.

Thus, in the series, 2, 4, 6, 8, 10, 12, the number of terms is 6, and $6 \times 7 = 42$, the sum of the whole series.

The reason is, because, in this case, the half sum of the extremes always exceeds the number of terms by unity.

T H E O R E M VII.

In an arithmetical series, the sum of all the terms divided by the number of terms, quotes half the sum of the extremes; and half the product of the common difference multiplied into the number of terms minus unity, is equal to half the difference of the extremes.

The reason of the first part of the proposition is evident; for by Theorem VI. 2. the sum of the extremes multiplied into half the number of terms, gives a product equal to the sum of all the terms; and consequently the sum of all the terms divided by half the number of terms, quotes the sum of the extremes; and doubling the divisor, that is, dividing by the number of terms, the quotient will be half the sum of the extremes.

The second part of the proposition is likewise plain: for, by Theorem V. the difference of the extremes, divided by the number of terms minus unity, quotes the common difference; but the quotient multiplied into the divisor, produces the dividend; which, in this case, divided by 2, quotes half the difference of the extremes.

Hence the sum of a series, the number of terms, and the common difference being given, the extremes may be found: for the theorem gives their half sum, and half difference; and the half difference of two quantities added to half their sum, gives the greater quantity; and the half difference subtracted from half their sum, leaves the lesser quantity.

Of

Of the five things that occur to be considered in an arithmetical series, if any three be given, the other two may be found; the exemplification whereof at large would open a wide field; but we propose only to give a brief specimen, in a few problems, whose solutions are founded upon, and flow directly from the above theorems.

P R O B. I.

Given the greatest term, the number of terms, and common difference, to find the least term; that is, Given II. III. IV. to find I.

R U L E.

Multiply the common difference into the number of terms minus unity, subtract the product from the greatest term, and the remainder is the least term, by Theorem I.

E X A M P L E I.

A man travels 6 days, and increases each day's journey by 4 miles; his last day's journey was 40 miles: What was the first? *Anf.* 20 miles.

$$4 \times 5 = 20, \text{ and } 40 - 20 = 20 \text{ miles. } \textit{Ans.}$$

Series 20, 24, 28, 32, 36, 40, complete.

E X A M P L E II.

A man takes out of his pocket, at eight several times, so many several numbers of shillings, every one exceeding the former by 6; the last was 54: What was the first? *Anf.* 12 shillings.

$$6 \times 7 = 42, \text{ and } 54 - 42 = 12 \text{ s. } \textit{Ans.}$$

P R O B. II.

Given the least term, the number of terms, and common difference, to find the greatest term; that is, Given I. III. IV. to find II.

R U L E.

Multiply the common difference into the number of terms minus unity, add the product to the least term, and the sum is the greatest term, by Theorem I.

E X A M P L E I.

There is an arithmetical series consisting of 18 terms or places; the first term is 4, the common difference 12: What is the greatest term? *Anf.* 208.

$$12 \times 17 = 204, \text{ and } 204 + 4 = 208 \text{ } \textit{Ans.}$$

H h

E X-

E X A M P L E II.

A man had 12 children; the youngest was three years old, and the common difference of their ages was 4 years: What was the age of the eldest? *Ans.* 47 years.

$$4 \times 11 = 44, \text{ and } 44 + 3 = 47 \text{ years. } \textit{Ans.}$$

P R O B. III.

Given the extremes, and common difference, to find the number of terms; that is, Given I. II. IV. to find III.

R U L E.

Divide the difference of the extremes by the common difference, and the quot plus unity is the number of terms, by Theorem IV.

E X A M P L E I.

A man setting out on a journey, travels 12 miles the first day, and increases every day's journey by 4 miles, till at last he travelled 64 miles in one day: How many days did he travel? *Ans.* 14 days.

$$64 - 12 = 52, \text{ and } 4)52(13, \text{ and } 13 + 1 = 14 \text{ days. } \textit{Ans.}$$

E X A M P L E II.

The youngest child of a large family was 4 years old, the eldest 40, and the common difference of the childrens ages was 3 years: How many children were there? *Ans.* 13.

$$40 - 4 = 36, \text{ and } 3)36(12, \text{ and } 12 + 1 = 13 \text{ } \textit{Ans.}$$

P R O B. IV.

Given the extremes, and number of terms, to find the common difference; that is, Given I. II. III. to find IV.

R U L E.

Divide the difference of the extremes by the number of terms minus unity, and the quot is the common difference, by Theorem V.

E X A M P L E I.

A man had 12 sons, whose ages were in arithmetical progression; the youngest was 2 years old, and the eldest 35: What was the common difference of their ages? *Ans.* 3 years.

$$35 - 2 = 33, \text{ and } 11)33(3 \text{ years. } \textit{Ans.}$$

E X A M P L E II.

A gentleman distributes a sum of money in arithmetical progression

sion among 20 beggars; to the first he gave 3 shillings, and to the last 41 shillings: What was the common difference? *Ans.* 2 shillings.

$$41 - 3 = 38, \text{ and } 19)38(2s. \text{ } Ans.$$

P R O B. V.

Given the extremes, and number of terms, to find the sum of the series; that is, Given I. II. III. to find V.

The several solutions of this problem assigned in Theorem VI. supply the place of rules; and to these we shall constantly refer.

E X A M P L E I.

13 persons give their charity to a poor man in arithmetical progression; the first gave 2 d. the last 26 d.: How much did the poor man get? *Ans.* 182 d. or 15 s. 2 d. Theorem VI. 1.

$$2 + 26 = 28, \text{ and } 28 \times 13 = 364, \text{ and } 2)364(182d. \text{ } Ans.$$

E X A M P L E II.

Placing 100 eggs in a straight line, at a yard's distance one from another, and the first a yard from a basket: How far must one travel to bring the eggs, one by one, into the basket? *Ans.* 10100 yards, or 5 English miles and 1300 yards. Theorem VI. 2.

$$2 + 200 = 202, \text{ and } 202 \times 50 = 10100 \text{ yards. } Ans.$$

E X A M P L E III.

A man bought 12 growing trees, the first for 2 s. the last for 40 s.; and their prices being in arithmetical progression, what paid he for the whole? *Ans.* 252 s. or 12 l. 12 s. Theorem VI. 3.

$$2 + 40 = 42, \text{ and } 2)42(21, \text{ and } 21 \times 12 = 252s. \text{ } Ans.$$

E X A M P L E IV.

A person completes a journey in 13 days, the number of miles travelled each day being in arithmetical progression, the seventh day he travelled 22 miles: What was the length of his journey? *Ans.* 286 miles. Theorem VI. 4.

$$22 \times 13 = 286 \text{ miles. } Ans.$$

E X A M P L E V.

How many strokes does a clock strike in one revolution of the index, viz. in 12 hours? *Ans.* 78 strokes. Theorem VI. 5.

$$12 \times 13 = 156, \text{ and } 2)156(78 \text{ strokes. } Ans.$$

EXAMPLE VI.

A butcher bought a dozen of fat lambs, and was to pay 1 shilling for the first, 3 for the second, and 5 for the third, &c. still advancing 2 shillings more for every following one: What did the 12 lambs cost him? *Ans.* 144 shillings, or 7l. 4s. Theorem VI. 6.

$$12 \times 12 = 144 \text{ shillings. } \textit{Ans.}$$

EXAMPLE VII.

A man buys 10 horses, and is to pay 2l. for the first, 4 for the second, 6 for the third, &c. still paying 2l. more for every following one: What will the 10 horses cost him? *Ans.* 110l. Theorem VI. 7.

$$10 \times 11 = 110 \text{ l. } \textit{Ans.}$$

P R O B. VI.

Given the sum of the series, the number of terms, and common difference, to find the extremes; that is, Given V. III. IV. to find I. and II.

R U L E.

Divide the sum of the series by the number of terms, and the quot is half the sum of the extremes; then multiply the common difference into the number of terms minus unity, and the product divided by 2 quotes half the difference of the extremes. The half difference added to the half sum gives the greater extreme, and subtracted leaves the lesser, by Theorem VII.

EXAMPLE.

A man received 300l. at 12 payments, each payment exceeding the former by 4l.: What was the first, and what the last payment?

$$12)300(25 \text{ half sum of the extremes.}$$

$$4 \times 11 = 44; \text{ and } 2)44(22 \text{ half difference of extremes.}$$

$$\text{And } 25 + 22 = 47 \text{ greatest extreme, or last payment.}$$

$$\text{And } 25 - 22 = 3 \text{ least extreme, or first payment.}$$

MIXT QUESTIONS.

Quest. 1. A man travels 15 days, increasing every day's journey by 3 miles, his last day's journey was 44 miles: What was his first day's journey? and how many miles did he travel?

$$3 \times 14 = 42, \text{ and } 44 - 42 = 2, \text{ the miles he travelled the first day, by Prob. I.}$$

$$\text{And } 2 + 44 = 46, \text{ and } 2)46(23, \text{ and } 23 \times 15 = 345, \text{ the miles travelled in all, by Prob. V. See Theorem VI. 3.}$$

Quest. 2. A man clears a debt by payments in arithmetical proportion, the common difference being 5l.; the first payment was 12l. the last 57l.: How many payments were there? and what the amount of the debt?

$57 - 12 = 45$, and $5)45(9$, and $9 + 1 = 10$, the number of payments, by Prob. III.

And $12 + 57 = 69$, and $69 \times 5 = 345$, the amount of the debt, by Prob. V. See Theorem VI. 2.

II. Geometrical Progression.

A geometrical progression, or series, is formed from some number assumed as the first term, by continual multiplication or division; and is either increasing or decreasing.

Thus, $1 : 2 : 4 : 8 : 16 : 32 : 64$, is a geometrical progression or series, formed from 1 assumed as the first term, and increasing, being continually multiplied by 2; and may be continued upward to infinity.

And $243 : 81 : 27 : 9 : 3 : 1$, is a geometrical progression or series, formed from 243 assumed as the first term, and decreasing, being continually divided by 3, and may likewise be continued downward to infinity.

The multiplier or divisor whereby the series is continued upward or downward, is called the *common ratio*.

Note, the natural numbers, 1, 2, 3, 4, &c. are sometimes set over a geometrical series, to shew the distance of any term from unity, or from the first term; and in this case the natural numbers are called *indices*, or *exponents*, thus:

1.	2.	3.	4.	5.	6.	exponents.
3	: 6	: 12	: 24	: 48	: 96	series.

But if the series proceed from unity, it is usual and convenient to place 0 over 1, thus:

0.	1.	2.	3.	4.	5.	6.	exponents.
1	: 2	: 4	: 8	: 16	: 32	: 64	series.

In a geometrical progression, or series, five things occur to be considered; any three of which being given, the other two may be found. The five things are,

- | | |
|------------------------------|-------------|
| I. The least term. | } Extremes. |
| II. The greatest term. | |
| III. The number of terms. | |
| IV. The common ratio. | |
| V. The sum of all the terms. | |

The more useful properties of numbers in geometrical progression are explained in the following theorems.

T H E O R E M I.

Any term of a geometrical series is equal to the product of the least term, multiplied continually into the common ratio, repeated as often as there are terms before or after it.

Thus, in the increasing series, $3 : 6 : 12 : 24 : 48 : 96$, whose common ratio is 2, the term 96 is equal to $3 \times 2 \times 2 \times 2 \times 2 \times 2$.

Again, in the decreasing series, $162 : 54 : 18 : 6 : 2$, whose ratio is 3, the term 162 is equal to $2 \times 3 \times 3 \times 3 \times 3$.

The reason is evident from the nature of the series, and the manner of its formation.

In practice it will be convenient, in the first place, to multiply the common ratio continually into itself; that is, to raise it to a power whose index or exponent is equal to the number of terms minus unity, and then multiply this power into the least term.

Thus, in the first of the above examples, $2 \times 2 \times 2 \times 2 \times 2 = 32$, and $32 \times 3 = 96$. Again, in the second example, $3 \times 3 \times 3 \times 3 = 81$, and $81 \times 2 = 162$.

Hence, in any geometrical series, that power of the common ratio whose index is equal to the number of terms minus unity, multiplied into the least term, produces the greatest; and the greatest term divided by that power quotes the least.

T H E O R E M II.

In a geometrical series, consisting of three, five, or any odd number of terms, the square of the middle term is equal to the product of the extremes, or to the product of any two means equally distant from the said middle term.

Thus, in the series, $1 : 2 : 4 : (8) : 16 : 32 : 64$, the square of the middle term 8, viz. $8 \times 8 = 1 \times 64$, the extremes, or $= 2 \times 32$, or $= 4 \times 16$, the equidistant means.

The reason will appear by considering, that in four proportional numbers the product of the extremes is equal to the product of the means, as was demonstrated in Multiplication of Vulgar Fractions; and when there are only three proportional numbers given, the middle term supplies the place of two, being the consequent to the first, and the antecedent to the third term. Thus, $1 : 8 : 64$, expressed at more length, is $1 : 8 :: 8 : 64$; and therefore $8 \times 8 = 1 \times 64$. Again, $2 : 8 : 32$, expressed more at large, is $2 : 8 :: 8 : 32$, and so $8 \times 8 = 2 \times 32$, &c.

C O R O L L A R I E S.

I. Hence a geometrical mean betwixt two given extremes is found by multiplying the extremes into one another, and extracting the square root of the product. Thus, the mean betwixt 2 and

and 32 is 8; for $2 \times 32 = 64$, and the square root of 64 is 8. So $2 : 8 : 32$.

2. Hence also, to an extreme and mean given, a third proportional, or the other extreme, is found by dividing the square of the mean by the given extreme. Thus, to $4 : 8$ given, the third proportional is 16; for $8 \times 8 = 64$, and $4)64(16$, so $4 : 8 : 16$. In like manner, to $32 : 8$ given, the third proportional is 2; for $8 \times 8 = 64$, and $32)64(2$; so $32 : 8 : 2$.

3. Hence likewise, in a series not proceeding from unity, the square of any term divided by the first term quotes a term of a double exponent minus unity. Thus, in the 1 series following, $12 \times 12 = 144$, and $3)144(48$, the fifth term; and in the 2 series, $48)144(3$, the fifth term,

	1.	2.	3.	4.	5.
1 series	3	6	12	24	48.
2 series	48	24	12	6	3.

THEOREM III.

In a geometrical series, consisting of four, six, or any even number of terms, the product of the extremes is equal to the product of the two middle terms, or to the product of any two means equally distant from the extremes.

Thus, in the series, $2 : 4 : (8 : 16) : 32 : 64$, the product of $2 \times 64 = 8 \times 16$, or $= 4 \times 32$.

The reason is plain: for in four proportional numbers, the product of the extremes is equal to the product of the means; and any pair of equidistant means may be esteemed middle terms.

COROLLARIES.

1. Hence, to an extreme and two means given, the other extreme, or fourth proportional, is found, by dividing the product of the means by the given extreme. Thus, to $4 : 8 : 16$, the fourth proportional is 32; for $8 \times 16 = 128$, and $4)128(32$; so $4 : 8 : 16 : 32$. And this takes place though the proportion be disjunct. Thus, to $48 : 24 :: 6$, the fourth proportional is 3; for $24 \times 6 = 144$, and $48)144(3$; so $48 : 24 :: 6 : 3$.

2. Hence likewise, when two extremes and a mean are given, the other mean is found by dividing the product of the two extremes by the given mean. Thus, to $2 : 4 : 16$, the other mean is 8; for $2 \times 16 = 32$; and $4)32(8$; so $2 : 4 : 8 : 16$. This likewise takes place, though the proportion be disjunct. Thus, to $2 :: 32 : 64$, the other mean is 4; for $2 \times 64 = 128$, and $32)128(4$; so $2 : 4 :: 32 : 64$.

3. From this and the preceding problem, it follows, that in a series proceeding from unity, any term squared produces a term of a double exponent; and any two or more terms multiplied into

H h 4

one

one another, produce a term whose exponent is the sum of all their exponents. Thus, in the following series, $8 \times 8 = 64$, whose exponent is $6 = 2 \times 3$; and $4 \times 8 = 32$, whose exponent is $5 = 2 + 3$; and $2 \times 4 \times 8 = 64$, whose exponent is $6 = 1 + 2 + 3$.

0. 1. 2. 3. 4. 5. 6.
1 : 2 : 4 : 8 : 16 : 32 : 64

T H E O R E M IV.

In a geometrical series, the quot of the greatest extreme, divided by the least, is equal to that power of the common ratio whose exponent is the number of terms minus unity.

Thus, in the series, $3 : 6 : 12 : 24 : 48 : 96$, the quot of $3)96$ ($32 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, the fifth power of 2, the common ratio of the series).

The reason is obvious : for, by Theorem I. the greatest term divided by such a power of the ratio, quotes the first term, and any dividend divided by the quot, gives the divisor.

C O R O L L A R I E S.

1. Hence the extremes and number of terms being given, the common ratio may be found, *viz.* divide the greatest extreme by the least, the quot is a power whose exponent is the number of terms minus unity, and the root extracted is the common ratio.

2. Hence likewise we have a method of finding several mean proportionals betwixt two given numbers ; *viz.* divide the greater by the lesser, esteem the quot a power whose index is greater by unity than the number of means proposed, and the root of this power extracted is the ratio ; by which multiply the lesser of the given numbers continually, and the several products are the means required.

T H E O R E M V.

In a geometrical series, the difference of the extremes, divided by the common ratio minus unity, quotes the sum of all the terms except the greatest.

Thus, in the series, $3 : 9 : 27 : 81 : 243$, the greatest term $243 - 3 = 240$, and the ratio $3 - 1 = 2$ $240(120 = 3 + 9 + 27 + 81$.

The truth of the proposition will be evident from the following considerations, *viz.* In a series whose ratio is 2, any term minus the least is equal to the sum of all the lesser terms. Thus, in the series, $1 : 2 : 4 : 8 : 16$, the second term $2 - 1 = 1$; and $4 - 1 = 1 + 2$; and $8 - 1 = 1 + 2 + 4$; and $16 - 1 = 1 + 2 + 4 + 8$. If the ratio be 3, any term minus the least is double to the sum of all the lesser terms. Thus, in the series, $18 : 6 : 2$, the second

term $\frac{6-2}{2} = 2$, and $\frac{18-2}{2} = 6 + 2$. If the ratio be 4, any term minus the least is triple of all the lesser terms, &c. And therefore,

fore, universally, the difference of the extremes divided by the common ratio minus unity, quotes the sum of all the terms except the greatest.

COROLLARIES.

1. The least term multiplied into that power of the ratio whose index is the number of terms, gives the next higher term of the series continued; from which, therefore, if the least term be subtracted, the remainder divided by the ratio minus unity, will quote the sum of the given series.

2. If a decreasing series be supposed infinite, the least extreme vanishes, or becomes 0; and in this case, if the greatest extreme be multiplied into the common ratio, the product divided by the ratio minus unity, will quote the sum of the series. Thus $1 + \frac{1}{2}$

$$+ \frac{1}{4} + \frac{1}{8} + \frac{1}{16}, \text{ &c.} = \frac{1 \times 2}{2-1} = 2; \text{ and } 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}, \text{ &c.}$$

$$= \frac{1 \times 3}{3-1} = \frac{3}{2} = 1\frac{1}{2}.$$

3. The difference of the extremes of a series divided by the difference of the sum and greater extreme, quotes the ratio minus unity.

We shall now subjoin a few problems, whose solutions flow directly from the above theorems, or their corollaries.

PROB. I.

Given the greatest term, the number of terms, and common ratio, to find the least term; that is, Given II. III. IV. to find I.

RULE.

Raise the common ratio to a power whose index is the number of terms minus unity; by this divide the greatest term, and the quot is the least term, by Theorem I.

EXAMPLE.

A farmer buys six cows, whose prices were in geometrical progression, the common ratio being 2, the price of the last was 96 crowns: What was the price of the first? *Ans.* 3 crowns.

$$1, 2, 3, 4, 5 \\ 2 \times 2 \times 2 \times 2 \times 2 = 32, \text{ and } 32)96(3 \text{ crowns. } \textit{Ans.}$$

PROB. II.

Given the least term, the number of terms, and common ratio, to find the greatest term; that is, Given I. III. IV. to find II.

RULE.

R U L E.

Raise the common ratio to a power whose index is the number of terms minus unity, multiply this by the least term, and the product is the greatest, by Theorem II.

E X A M P L E.

A nobleman had nine sons, to whom he left his estate, divided into portions in geometrical progression, the common ratio being 3; the youngest son got the least portion, being only 50 l. : What did the eldest son get? *Ans.* 328050 l.

$$\begin{array}{cccccccc} 1. & 2. & 3. & 4. & 5. & 6. & 7. & 8. \\ 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 & = & 6561, & \text{and } 6561 \times 50 & = & 328050. \end{array} \text{Ans.}$$

P R O B. III.

The first term and common ratio of a series not proceeding from unity being given, to find any remote term, without producing all the intermediate terms.

R U L E.

Find a few of the terms, by multiplying or dividing the first term continually by the common ratio, and over the terms thus found place their exponents; square the greatest of the found terms; which square, divided by the first term, will quote a term of a double exponent minus unity. Again, the square of the term last found, divided by the first, will quote another term of a double exponent minus unity. Thus proceed, till you either find the term sought, or one near it; and from a near term the one sought may be found by means of the ratio. Theorem II. cor. 3.

E X A M P L E.

A sum of money was divided among ten persons; their shares in geometrical progression, and the common ratio, 3; the first person got 4 l. : What was the tenth person's share? *Ans.* 78732 l.

$$\begin{array}{ccccccc} 1. & 2. & 3. & 5. & 9. & 10. & \\ 4 : 12 : 36 & \text{---} & 324 & \text{---} & 26244 : 78732. \end{array} \text{Ans.}$$

$$36 \times 36 = 1296 \div 4 = 324 \text{ the fifth term.}$$

$$324 \times 324 = 104976 \div 4 = 26244, \text{ the ninth term.}$$

The tenth term is found by multiplying the ninth into the common ratio 3.

P R O B. IV.

The common ratio of a series proceeding from unity being given, to find any remote term, without producing all the intermediate terms.

R U L E.

R U L E.

Find some few terms by means of the ratio, over which place their exponents; then observe what exponents added will make up the exponent of the term sought, or of some term near it; and the terms standing under the said exponents, multiplied into one another, will produce the term answering to the sum of the exponents. Theorem III. cor. 3.

Note. In a series proceeding from unity, as 1 stands over 1, the exponent of every term will be one short of the number of terms from the beginning of the series. Thus, the exponent of the fifth term is 4, of the sixth term 5, &c.

E X A M P L E.

A grafter bought 14 sheep, at a farthing for the first, a half-penny for the second, still doubling the price for every subsequent one; and was to pay only the price of the last sheep for the whole: What sum had he to pay? *Anf.* L. 8 : 10 : 8.

$$\begin{array}{cccccc} 0. & 1. & 2. & 3. & 4. & 5. \\ 1 & : 2 & : 4 & : 8 & : 16 & : 32 \end{array}$$

$$\begin{array}{l} 1 + 3 + 4 + 5 = 13 \quad L. s. d. \\ 2 \times 8 \times 16 \times 32 = 8192 = 8 \text{ } 10 \text{ } 8 \quad \textit{Ans.} \end{array}$$

Or thus :

$$\begin{array}{l} 3 + 5 + 5 = 13 \\ 8 \times 32 \times 32 = 8192 \end{array}$$

Or thus :

$$\begin{array}{l} 1 + 3 + 4 + 5 = 13 \\ 2 \times 8 \times 16 \times 32 = 8192 \end{array}$$

P R O B. V.

Given the extremes, and common ratio, to find the number of terms; that is, Given I. II. IV. to find III.

R U L E.

Divide the greatest extreme by the least, raise the common ratio to a power equal to the quot, and the index of that power plus 1 is the number of terms. Theorem IV.

E X A M P L E.

A gentleman purchased some acres of ground, whose prices were in geometrical progression, the common ratio being 3; the price of the first acre was 3 d. and of the last 59049 d. : How many acres did he purchase? *Anf.* 10 acres.

$$3)59049(19683$$

$$\begin{array}{l} 1. \quad 2. \quad 3. \quad 4. \quad 5. \quad 6. \quad 7. \quad 8. \quad 9. \text{ and } 9 + 1 = 10 \quad \textit{Ans.} \\ \text{And } 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 19683 \end{array}$$

P R O B.

P R O B. VI.

Given the extremes, and common ratio, to find the sum of the series; that is, Given I. II. IV. to find V.

R U L E.

Divide the difference of the extremes by the ratio minus unity, and the quot added to the greatest extreme, gives the sum of the series. Theorem V.

E X A M P L E.

A gentleman who had a daughter married on New-year's day, gave the husband towards her portion 4 shillings, promising to triple that sum the first day of every month, for nine months after the marriage; the sum paid on the first day of the ninth month was 26244 shillings: What was the lady's portion? *Ans.* 39364 shillings, or 1968l. 4s.

$$26244 - 4 = 26240, \text{ and } 3 - 1 = 2) 26240 (13120, \text{ and } 13120 + 26244 = 39364 \text{ s.} = 1968 \text{ l. } 4 \text{ s.}$$

P R O B. VII.

Given the least extreme, the number of terms, and common ratio, to find the sum of the series; that is, Given I. III. IV. to find V.

R U L E.

Find, by Prob. III. or IV. the term next after the greatest extreme; from this term subtract the least term; and the remainder, divided by the ratio minus unity, will quote the sum of the series. Theorem V. cor. 1.

E X A M P L E I.

A corn-merchant buys 12 stacks of wheat, and was to pay 2d. for the first stack, 6d. for the next, tripling the price for every following stack? What sum had he to pay? *Ans.* 531440d. or L. 2214 : 6 : 8.

$$\begin{array}{r} \begin{array}{ccccccc} 1. & 2. & 3. & 4. & 7. & & 13. \\ 2 : 6 : 18 : 54 & \text{---} & 1458 & \text{---} & 1062882 \\ 54 \times 54 = 2916 \div 2 = 1458 \\ 1458 \times 1458 = 2125764 \div 2 = 1062882 \\ 1062882 - 2 = 1062880, \text{ and } 3 - 1 = \\ 2) 1062880 (531440 \text{ d.} = \text{L. } 2214 : 6 : 8. \end{array} \end{array}$$

E X A M P L E II.

A gentleman buys a fine house, in which were 24 thresholds; and,

and, in name of price, was to lay a farthing on the first threshold, a halfpenny on the second, a penny on the third, doubling the sum on every following threshold: What would the house cost him?

Ans. 16777215 farthings, or L. 17476 : 5 : 3 $\frac{1}{2}$.

$$0. \quad 1. \quad 2. \quad 3. \quad 4. \quad 5. \quad 9. \quad 24.$$

$$1 : 2 : 4 : 8 : 16 : 32 \text{ --- } 512 \text{ --- } 16777216$$

$$4 + 5 = 9$$

$$16 \times 32 = 512$$

$$9 + 9 + 2 + 4 = 24$$

$$512 \times 512 \times 4 \times 16 = 16777216 \quad L. \quad s. \quad d.$$

$$16777216 - 1 = 16777215f. = 17476 \quad 5 \quad 3\frac{1}{2}$$

The ratio minus unity being 1, there is no occasion to divide by it.

EXAMPLE III.

What will a horse cost by tripling the 32 nails in his shoes with a farthing? *Ans.* 926510094425920 farthings, or 965114682527l.

EXAMPLE IV.

What will 40 drove of cattle cost, by tripling each drove with a farthing? *Ans.* 6332117426592150l. 8s. 4d.

PROB. VIII.

Given the extremes, and sum of the series, to find the common ratio, and number of terms; that is, Given I. II. V. to find IV. III.

RULE.

Divide the difference of the extremes by the difference of the sum and greater extreme, and the quot. is the ratio minus unity, by Theorem V. cor. 3. The ratio being thus obtained, find the number of terms by Prob. V.

EXAMPLE.

A gentleman sold his estate for 21844l. which was paid by several payments, in geometrical progression; the first was 4l. the last 16384l.: What was the ratio, and how many payments?

Ans. The ratio 4, payments 7.

$$\begin{array}{r} 21844 \quad 16384 \\ 16384 \quad 4 \\ \hline \end{array}$$

$$5460) 16380(3 + 1 = 4 \text{ the common ratio.}$$

$$4) 16384(4096$$

$$1. \quad 2. \quad 3. \quad 4. \quad 5. \quad 6. \text{ and } 6 + 1 = 7 \text{ the number of payments.}$$

$$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096.$$

C H A P. XII.

I N T E R E S T.

INTEREST is a small sum of money paid by the borrower to the lender for the use of a greater sum, at a certain rate *per cent. per annum*; and is either simple or compound.

I. *Simple Interest.*

Simple interest is that which arises purely or only from the principal, or sum lent.

The sum of principal and interest is called the *amount*. The rate, by the laws of Britain, ever since 1714, cannot exceed 5 *per cent.* but may be less.

The year is supposed always to consist of 365 days, and the 29th of February in leap years is not reckoned; for no more interest can be legally charged for leap than for a common year.

The day computed from is not reckoned, but the day computed to is. Few chuse to compute by months, a month being no aliquot part of a year. The computation by years and quarters is usual, but years and days still more so.

The operations may be rendered more simple by assuming the interest of 1*l.* for a year, as the rate. Thus, at 4, $4\frac{1}{4}$, $4\frac{1}{2}$, $4\frac{3}{4}$, and 5 *per cent.* the interest or rate for 1*l.* is .04, .0425, .045, .0475, .05, found by saying, as 100 : 4 :: 1 : .04, &c.

The principal and every year's amount make a series of arithmetical proportionals, the rate being the common difference; as under:

	1	2	3	4	5 years.
Principal 1	1.05	1.10	1.15	1.20	1.25 .05 difference.
Principal 100	105	110	115	120	125 5 difference.

P R O B. I.

Principal, rate, and time, in years, given, to find the interest, and consequently the amount.

R U L E.

Multiply the principal, the rate of 1*l.* and the time continually, and the last product is the interest.

Or,

Multiply the principal, the rate of 100*l.* and the time continually, and the last product divided by 100 quotes the interest.

E X A M P L E I.

What is the interest of 584l. 6s. 8d. for $3\frac{1}{4}$ years, at $4\frac{1}{2}$ per cent.?

Or thus :

$ \begin{array}{r} 584.3 \\ .045 \\ \hline 29216 \\ 233783 \\ \hline 26.2950 \\ 3\frac{1}{4} \\ \hline 4)78.885 \\ 19.72125 \\ \hline \text{Interest } 98.60625 = 98 \text{ } 12 \text{ } 1\frac{1}{2} \\ \text{Principal } 584 \text{ } 6 \text{ } 8 \\ \hline \text{Amount } 682 \text{ } 18 \text{ } 9\frac{1}{2} \end{array} $	$ \begin{array}{r} 584.3 \\ 4\frac{1}{2} \\ \hline 2337.83 \\ 292.16 \\ \hline 2629.50 \\ 3.75 \\ \hline 131475 \\ 184065 \\ 78885 \\ \hline 100)98.60625 \text{ interest.} \end{array} $
---	--

The amount is found by adding the principal to the interest as above ; but the amount may also be found thus : To the product of the rate and time add 1, and multiply by the principal.

Time 3.75	1.16875	
Rate .045	584.3	
$\frac{1875}{1500}$	$\frac{467500}{935000}$	
Product .16875	584375	
	682.55000	
	$.389583 = \frac{1}{2}$	
	$\frac{1}{2}$	
	L. s. d.	
Amount 682.939583	= 682 18 9 $\frac{1}{2}$	

The interest may be found by subtracting the principal from the amount.

If the given time be days, or years and days, reduce the days to the decimal of a year, and then work as before. Or, reduce the years to days, then work, and divide the result by 365.

The reason both of this rule, and of those assigned in the subsequent problems, may be deduced from the compound proportion following.

Prin.	T.	Int.	Prin.	Year.	Int.
100	x	1	:	4 $\frac{1}{2}$	:
584 6 8	x	3 $\frac{1}{4}$	:	98 12 1 $\frac{1}{2}$	:
Or, 1	x	1	:	.045	:
584.3	x	3.75	:	98.60625	:

But

But interest is more usually computed by some practical method: and these methods are numerous, and various in their own nature; and differ also as the rate varies, and as the given time is years or days.

I. *The given Time Years.*

R U L E S.

1. When the rate is 5 *per cent.* and the time 1 year, $\frac{1}{20}$ of the principal is the interest sought, because the rate of 5 is $\frac{1}{20}$ of 100.

This is the most simple case that can happen; and because 5 *per cent.* is 1s. *per* l. you may readily compute the interest by esteeming the pounds so many shillings, and the shillings so many halfpence, adding $\frac{1}{2}$; and if there be any pence, every 6d. is $1\frac{1}{2}$ farthing. Thus the interest of 54l. 16s. for 1 year, at 5 *per cent.* is 54s. 19 $\frac{1}{2}$ halfpence, or 2l. 14s. 9.6d.

2. When the given time is any number of years, for 2 years take $\frac{2}{20} = \frac{1}{10}$; for 3 years take $\frac{3}{20}$ thrice; for 4 take $\frac{4}{20} = \frac{1}{5}$; for 5 take $\frac{5}{20} = \frac{1}{4}$; for 10 take $\frac{10}{20} = \frac{1}{2}$, &c.; or find the interest for 1 year, and multiply the interest so found by the years in the question, as in Ex. 2.

3. When the rate is any other than 5 *per cent.* first work for 5 *per cent.* and then to or from the interest thus found, add or subtract the correspondent part; or multiply $\frac{r}{5}$ of the interest at 5 *per cent.* by the given rate, as in Ex. 3.

4. When the given time is years and quarters, or years and aliquot parts, find the interest at 5 *per cent.* for the years, as taught above; for the quarters or parts take aliquot parts; and then multiply $\frac{1}{4}$ of the interest thus found by the given rate: Or, multiply the rate into the time, esteem the product a new rate, to which compute the interest, as in Ex. 4. 5. and 6.

E X A M P L E II.

What is the interest of 684l. 16s. 8d. for 15 years, at 5 *per cent.*?

Or thus:

	L.	s.	d.	
	684	16	8	
Years.				
10 = $\frac{1}{2}$	342	8	4	
5 = $\frac{1}{4}$	171	4	2	
	Ans. 513	12	6	

20)684 16 8

34 4 10

15 = 5 × 3

171 4 2

3

Ans. 513 12 6

E X.

E X A M P L E III.

What is the interest of 438 l. 14 s. 6 d. for 12 years, at $4\frac{1}{2}$ per cent. ?

Or thus :

	L.	s.	d.
	438	14	6
$r.$			
$10 = \frac{1}{2}$	219	7	3
$2 = \frac{1}{10}$	43	17	5.4
at 5 p.c.	263	4	8.4
Deduce $\frac{1}{10}$	26	6	5.64
Anf.	236	18	2.76

20)	438	14	6
	21	18	8.7
			12
5)	263	4	8.4
	52	12	11.28
			$4\frac{1}{2}$
	210	11	9.12
	26	6	5.64
Anf.	236	18	2.76

E X A M P L E IV.

What is the interest of 317 l. 16 s. for $5\frac{1}{4}$ years, at $3\frac{1}{2}$ per cent. ?

	L.	s.
	317	16
$r.$		
$5 = \frac{1}{4}$	79	9
$\frac{1}{2} = \frac{1}{10}$	7	18 10.8
$\frac{1}{4} = \frac{1}{2}$	3	19 5.4
5)	91	7 4.2
	18	5 5.64
		$3\frac{1}{2}$
	54	16 4.92
	9	2 8.82
Anf.	63	19 1.74

Or thus :

$$5\frac{1}{4} \times 3\frac{1}{2} = 20\frac{3}{8}$$

	L.	s.
	317	16
$p.c.$		
$10 = \frac{1}{10}$	31	15 7.2
$5 = \frac{1}{2}$	15	17 9.6
$5 = \frac{1}{2}$	15	17 9.6
$\frac{1}{8} = \frac{1}{40}$	7	11.34
Anf.	63	19 1.74

E X A M P L E V.

What is the interest of 79 l. 16 s. for $5\frac{1}{2}$ years, at $4\frac{3}{8}$ per cent. ?

I;

L.

	$L.$	$s.$
	79	16
$r.$		
$5 = \frac{1}{4}$	19	19
$\frac{1}{2} = \frac{1}{8}$	1	19 10.8
	5)21	18 10.8
	4	7 9.36
		$4\frac{3}{8}$
	17	11 1.44
$\frac{2}{8} = \frac{1}{4}$	1	1 11.34
$\frac{1}{8} = \frac{1}{8}$		10 11.67
	Ans. 19 4 0.45	

Or thus:

$$5\frac{1}{4} \times 4\frac{3}{8} = 24\frac{1}{8}$$

	$p.c.$	79	16
$20 = \frac{1}{2}$	15	19	2.4
$4 = \frac{1}{5}$	3	3	10.08
$\frac{1}{8} = \frac{1}{84}$			11.97
	Ans. 19 4 0.45		

EXAMPLE VI.

What is the interest of 648 l. 8 s. for $6\frac{3}{4}$ years, at $4\frac{1}{4}$ per cent.?

	$L.$
	648.4
$r.$	
$5 = \frac{1}{4}$	162.1
$1 = \frac{1}{5}$	32.42
$\frac{1}{2} = \frac{1}{2}$	16.21
$\frac{3}{4} = \frac{3}{4}$	8.105
	5)218.835
	43.767
	$4\frac{1}{4}$
	175.068
	10.94175
	Ans. 186.00975 = 186 l. 0 s. 2 d.

Or thus:

$$6\frac{3}{4} \times 4\frac{1}{4} = 28\frac{1}{8}$$

	$p.c.$	648.4
$20 = \frac{1}{2}$	129.68	
$4 = \frac{1}{5}$	25.936	
$4 = \frac{1}{5}$	25.936	
$\frac{1}{8} = \frac{1}{84}$	4.45775	
	Ans. 186.00975	

II. The given time days.

R U L E S.

1. Multiply the principal by the number of days, and the product divided by 7300 quotes the interest at 5 per cent.; and if the rate be any other than 5 per cent., adjust the matter by aliquot parts, as in Ex. 7.

2. Divide any principal by 73, and the quot will be the interest for 100 days at 5 per cent. and for the remaining days take aliquot parts, as in Ex. 8.

3. If the number of the given days be 73, or any multiple of 73, such as, 146, 219, 292, 365, &c. divide the principal by 100, and the quot will be the interest for 73 days at 5 per cent. ; which multiplied by the number denoting the multiple, will produce the interest for the given days, as in Ex. 9.

4. If the principal be 100 l. esteem the days so many pounds principal, divide by 73, and the quot will be the interest of 100 l. for the given number of days, at 5 per cent. as in Ex. 10.

The reason of the above rules will appear from the following compound proportion, in which *p.* is put to denote the principal, and *d.* the number of days.

$$\begin{array}{l} \text{Prin. Days. Int.} \\ 100 \times 365 : 5 :: p. \times d. \end{array}$$

$$\text{That is, } \frac{5 \times p. \times d.}{100 \times 365} = \frac{p. \times d.}{100 \times 73} = \frac{p. \times d.}{7300}, \text{ Hence Rule I.}$$

$$\begin{array}{l} \text{Prin. Days. Int. Days.} \\ \text{Again, } 100 \times 365 : 5 :: p. \times 100 \end{array}$$

$$\text{That is, } \frac{5 \times p. \times 100}{100 \times 365} = \frac{5 \times p.}{365} = \frac{p.}{73}, \text{ Hence Rule II.}$$

The truth of Rule 3. is evident ; for if 73 dividing any principal quotes the interest for 100 days, it follows, that 100 dividing any principal will quote the interest for 73 days.

The reason of Rule 4. is also obvious ; for the interest of 100 l. for 50 days will be equal to the interest of 50 l. for 100 days.

E X A M P L E VII.

What is the interest of 245 l. 13 s. 4 d. from the 21st of March to the 22d of November, at $4\frac{3}{4}$ per cent ?

I i 2

Days.

<i>Days.</i>	<i>L.</i>
Mar. 10	245.6
Apr. 30	246
May 31	—
June 30	14740
July 31	98206
Aug. 31	491333
Sept. 30	—
Oct. 31	73 00)60434.0(8.2786, at 5 per cent.
Nov. 22	584... .4139, at $\frac{1}{4}$ per cent.
246	203 7.8647 = L. 7 17 $3\frac{1}{2}$, at $4\frac{1}{4}$ per cent. <i>Ans.</i>
	146
	—
	574
	511
	—
	630
	584
	—
	460
	438
	—
	(22)

EXAMPLE VIII.

What is the interest of 378 l. 3s. $8\frac{1}{2}$ d. for 275 days, at $4\frac{1}{2}$ per cent.?

<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>L.</i>	<i>s.</i>	<i>d.</i>	<i>Days.</i>
73) 378	3	$8\frac{1}{2}$	(5	3	7.349,	for 100.
365					2	
—						
13			10	7	2.698,	for 200
20			2	11	9.674,	for 50 = $\frac{1}{2}$
—			1	0	8.669,	for 20 = $\frac{1}{5}$
263			5		2.167,	for 5 = $\frac{1}{4}$
219			—			
—			10) 14	4	11.208,	at 5 per cent.
44			1	8	5.920,	at $\frac{1}{2}$ per cent.
12			—			
—			<i>Ans.</i> 12	16	5.288,	at $4\frac{1}{2}$ per cent.
536.5						
511						
—						
255						
219						
—						
360						
292						
—						
680						
657						
—						
(23)						

Or, L. 12 16 $5\frac{1}{4}$

E X A M P L E IX.

What is the interest of 864 l. 19 s. 8 d. for 219 days, at $4\frac{5}{8}$ per cent.?

L.	s.	d.	L.	s.	d.	Days.
100	8	64	19	8	(8 12 11.96,	for 73
	20				3	
12	99		25	18	11.88,	for 219, at 5 per cent.
	12		1	18	11.09,	at $\frac{3}{8}$ per cent. = $\frac{3}{40}$
11	96		24	0	0.79,	at $4\frac{5}{8}$ per cent. Ans.

E X A M P L E X.

What is the interest of 100 l. for 254 days, at $4\frac{7}{8}$ per cent.?

L.	s.	d.	
73	254	(3 9	7.068, at 5 per cent.
	219	1	8.876, at $\frac{3}{8}$ per cent. = $\frac{1}{40}$
35	3	7	10.192, at $4\frac{7}{8}$ per cent. for 254 days. Ans.

20

700

657

43

12

516

511

500

438

620

584

(36)

If the given principal be any multiple of 100, work for 100 as above, and then multiply the result by the number denoting the multiple.

The computing of interest occurs so frequently in practice, that men of business, for the sake of ease and dispatch, generally use tables constructed for that purpose. The following tables give the interest at 5 per cent. ; whence the interest at any other rate may easily be found.

I i 3

The

The interest of one pound at 5 *per cent.*

	TABLE I. <i>per year.</i>	TABLE II. <i>per Day.</i>	TABLE III. <i>per quarter.</i>	TABLE IV. <i>per month.</i>
1	.05	.0001369863	.0125	.00416
2	.1	.0002739726	.025	.0083
3	.15	.0004109589	.0375	.0125
4	.2	.0005479452	.05	.0166
5	.25	.0006849315	.0625	.02083
6	.3	.0008219178	.075	.025
7	.35	.0009589041	.0875	.02916
8	.4	.0010958904	.1	.03
9	.45	.0012328767	.1125	.0375

The above tables are constructed by the rules already assigned for finding the interest of any principal for any given time. The quarter in Table III. is $\frac{1}{4}$ of a year, or 91 days 6 hours; and the month in Table IV. is $\frac{1}{12}$ of a year, or 30 days 10 hours. In using these tables observe the following

R U L E S.

1. Resolve the given number of years or days into its constituent parts.
2. Seek the significant figure of each constituent part on the left; opposite to which, under *per year*, or *per day*, &c. according to the denomination of the given time, you have the decimal to be taken out.
3. Move the decimal point in each decimal so many places to the right as there are ciphers in the constituent part; and, in taking out for decimal figures, move the decimal point so many places to the left as the decimal figure is to the right.
4. Add the decimals thus collected from the tables, and their sum is the interest sought.
5. If the given time be of different denominations, such as years and days, find the interest for the years and days separately, and the sum of the results will be the interest sought. Or, reduce the years to days, and then collect from Table II. Do the like when the time is give in years and quarters, or years and months, &c.
6. If the given rate be not 5 *per cent.* first find the interest at 5 *per cent.* and then multiply one-fifth of the interest so found by the give rate.

The decimals in the tables are complete, and consequently will, when all the figures are used, give a complete answer; but yet, in most cases, four or five figures will be sufficiently accurate.

E X A M P L E I.

What is the interest of 3485 l. for 1 year, at $4\frac{1}{4}$ *per cent.*?

In

In Table I. $\left\{ \begin{array}{l} 3000 \text{ --- } 150 \\ 400 \text{ --- } 20 \\ 80 \text{ --- } 4 \\ 5 \text{ --- } .25 \end{array} \right.$
opposite to —
you find

5) 174.25 at 5 per cent.

34.85 at 1 per cent.

$4\frac{3}{4}$

139.40

17.425

8.7125

$\frac{L. \quad s. \quad d.}{165.5375 = 165 \quad 10 \quad 9 \quad \text{Ans.}}$

E X A M P L E II.

What is the interest of 3485 l. for 1 day, at $4\frac{3}{4}$ per cent.?

In Table II. $\left\{ \begin{array}{l} 3000 \text{ --- } .41095 \\ 400 \text{ --- } .05479 \\ 80 \text{ --- } .01095 \\ 5 \text{ --- } .00068 \end{array} \right.$
opposite to —
you find

5) 47737 at 5 per cent.

.09547 at 1 per cent.

$4\frac{3}{4}$

.38188

.04773

.02386

$\frac{s. \quad d.}{.45347 = 9 \quad 0\frac{3}{4} \quad \text{Ans.}}$

E X A M P L E III.

What is the interest of 284 l. 5 s. for 15 years, at 5 per cent.?

Multiply the principal 284.25
by the number of years 15

Resolve the product 4263.75 into constituent parts, and then proceed as before.

$\left\{ \begin{array}{l} 4000 \text{ --- } 200 \\ 200 \text{ --- } 10 \\ 60 \text{ --- } 3 \\ 3 \text{ --- } .15 \\ .7 \text{ --- } .035 \\ .05 \text{ --- } .0025 \end{array} \right.$

$\frac{L. \quad s. \quad d.}{213.1875 = 213 \quad 3 \quad 9 \quad \text{Ans.}}$

114

EX-

E X A M P L E I V.

What is the interest of 564 l. for 238 days, at $4\frac{1}{2}$ per cent.?

Multiply the principal 564
by the number of days 238

Resolve the product 134232 into constituent parts, and then proceed as as formerly.

100000 — 13.6986
30000 — 4.1095
4000 — .5479
200 — .0273
30 — .0041
2 — .0002

5)18.3876 at 5 per cent.

3.6775 at 1 per cent.
 $4\frac{1}{2}$

14.7100
1.8387

L. s. d.
16.5487 = 16 10 11 $\frac{1}{2}$ Ans.

E X A M P L E V.

What is the interest of 78 l. for $4\frac{1}{4}$ years, at $4\frac{1}{8}$ per cent.?

$78 \times 4.25 = 331.5$

By Table I.

300 — 15
30 — 1.5
1 — .05
.5 — .025

5)16.575 at 5 per cent.

3.315 at 1 per cent.
 $4\frac{1}{8}$

13.260

.414 = $\frac{1}{8}$

L. s. d.
Ans. 13.674 = 13 13 5 $\frac{1}{4}$

Or, 78×17 quarters = 1326

By Table III.

1000 — 12.5
300 — 3.75
20 — .25
6 — .075

5)16.575 at 5 per cent.

3.315 at 1 per cent.
 $4\frac{1}{8}$

13.260

.414 = $\frac{1}{8}$

Ans. 13.674 as before.

E X.

E X A M P L E VI.

What is the interest of 85l. 14s. 6d. for 3 years 11 months, at $4\frac{1}{2}$ per cent.?

$$85.725 \times 47 \text{ months} = 4029.075$$

By Table IV.

$$4000 \text{ --- } 16.6666$$

$$20 \text{ --- } .0823$$

$$9 \text{ --- } .0375$$

$$.07 \text{ --- } .0002$$

$$.005 \text{ --- } .0000$$

$$5)16.7876 \text{ at } 5 \text{ per cent.}$$

$$3.3575 \text{ at } 1 \text{ per cent.}$$

$$4\frac{1}{2}$$

$$13.4300$$

$$1.6787 = \frac{4}{3}$$

$$.4196 = \frac{1}{2}$$

L. s. d.

$$\text{Ans. } 15.5283 = 15 \text{ } 10 \text{ } 6\frac{1}{2}$$

In calculating interest on cash-accounts, or accounts-current, where partial payments are made, and the account cleared within twelve months, multiply the principal and the several balances into the number of days they are at interest, and the sum of these products divided by 7300, will quote the interest at 5 per cent.; and for any other rate multiply one fifth of the interest thus found by the given rate, and the product will be the interest sought; as in the following examples:

E X A M P L E I.

Lent A B, the 10th of June 1785, the sum of 800l. Sterling, and received the same back in partial payments, as follows: What interest is due at 5 per cent.?

1785.		L.	s.	d.	Da.	Products.
June 10.	Principal lent	800			60	48000
Aug. 9.	Received	200	10			
	New principal	599	10		67	40166.5
Oct. 15.	Received	328	15	6		
	New principal	270	14	6	38	10287.55
Nov. 22.	Received	170	14	6		
	New principal	100			31	3100
Dec. 25.	Received in full of principal	100				
						101554.05

7300

E X A M P L E IV.

What is the interest of 564 l. for 238 days, at $4\frac{1}{2}$ per cent.?

Multiply the principal 564
by the number of days 238

Resolve the product 134232 into constituent parts, and then proceed as as formerly.

100000 — 13.6986
30000 — 4.1095
4000 — .5479
200 — .0273
30 — .0041
2 — .0002

5)18.3876 at 5 per cent.

3.6775 at 1 per cent.
 $4\frac{1}{2}$

14.7100
1.8387

16.5487 = 16 L. 10 s. 11 $\frac{1}{2}$ d. *Ans.*

E X A M P L E V.

What is the interest of 78 l. for $4\frac{1}{4}$ years, at $4\frac{1}{8}$ per cent.?

$78 \times 4.25 = 331.5$

By Table I.

300 — 15
30 — 1.5
1 — .05
.5 — .025

5)16.575 at 5 per cent.

3.315 at 1 per cent.
 $4\frac{1}{8}$

13.260

.414 = $\frac{1}{8}$

Ans. 13.674 = 13 L. 13 s. 5 $\frac{3}{4}$ d.

Or, 78×17 quarters = 1326

By Table III.

1000 — 12.5

300 — 3.75

20 — .25

6 — .075

5)16.575 at 5 per cent.

3.315 at 1 per cent.
 $4\frac{1}{8}$

13.260

.414 = $\frac{1}{8}$

Ans. 13.674 as before.

E X.

EXAMPLE VI.

What is the interest of 85l. 14s. 6d. for 3 years 11 months, at $4\frac{5}{4}$ per cent.?

$$85.725 \times 47 \text{ months} = 4029.075$$

By Table IV.

4000	—	16.6666
20	—	.0833
9	—	.0375
.07	—	.0002
.005	—	.0000

5) 16.7876 at 5 per cent.

3.3575 at 1 per cent.

$4\frac{5}{8}$

13.4300

$$1.6787 = \frac{4}{3}$$

$$.4196 = \frac{1049}{2500}$$

L. s. d.

Ans. $15.5283 = 15 \text{ } 10 \text{ } 6\frac{3}{4}$

In calculating interest on cash-accounts, or accounts-current, where partial payments are made, and the account cleared within twelve months, multiply the principal and the several balances into the number of days they are at interest, and the sum of these products divided by 7300, will quote the interest at 5 per cent.; and for any other rate multiply one fifth of the interest thus found by the given rate, and the product will be the interest sought; as in the following examples:

EXAMPLE I.

Lent A B, the 10th of June 1785, the sum of 800l. Sterling, and received the same back in partial payments, as follows: What interest is due at 5 *per cent.*?

1785.		L. s. d. Da.				Products.
June 10.	Principal lent	800			60	48000
Aug. 9.	Received	200	10			
	New principal	599	10		67	40166.5
Oct. 15.	Received	328	15	6		
	New principal	270	14	6	38	10287.55
Nov. 22.	Received	170	14	6		
	New principal	100			31	3100
Dec. 25.	Received in full of principal	100				
						101554.05

$$7300 = 73 \times 100$$

$$73)1015.5405(13.9115 = 13l. 18s. 2\frac{1}{2}d. \text{ Ans.}$$

Banks, or bankers, sometimes borrow at 4 *per cent.* but lend at 5 *per cent.*; and the person to whom they give a cash-credit has no occasion to keep money by him, but gives it into the bank, and receives 4 *per cent.* interest for the balance of cash due to him; but when the balance runs against him, he pays interest at the rate of 5 *per cent.* In this case it will be proper to consider the money lent or paid by the bank as Dr, and the money received by the bank as Cr, and to make two columns for products, *viz.* one for the Dr products, and the other for the Cr products. The interest arising from the Dr products is to be computed at 5 *per cent.* and that from the Cr products at 4 *per cent.*; and the difference of these two is the balance of interest due by the person to the bank, or by the bank to him.

E X A M P L E II.

Given A B a cash-credit at 5 *per cent.* for the balances due by him, allowing him 4 *per cent.* for such balances as may be due to him: What interest will be due, and by whom, in consequence of the following transactions:

				Products.		
1785.		L.	s.	d.	Da.	Dr
Jan. 8.	Dr	845	6	8	102	86224
April 20.	Cr	1265	6	8		
	Cr	420			86	36120
July 15.	Dr	968	10	0		
	Dr	548	10	0	77	42234.5
Sept. 30.	Cr	848	10	0		
	Cr	300			32	9600
Nov. 1.	Dr	500				
	Dr	200			44	8800
Dec. 15.	Cr	200				
					137258.5	45720

A 5 fl 18/16 At 5 *per cent.* = L. 18 15 6 = L. 5 at 4 *per cent.*
 5 00 0
 13 15 6 interest due by A B.

If the rate of interest on both sides be the same, you have only

to divide the difference of the sums of the Dr and Cr products by 7300.

When partial payments are made on bonds or bills at any interval greater than a year, the legal and usual method is, to add the interest at the times of payment to the principal, from that amount deducting the payment. Bankers avoid such dilatory payments, by taking care to have all their cash-accounts settled within the year, this being the shortest or speediest way of converting interest into principal.

E X A M P L E III.

Borrowed on bond, 1781, June 1. the sum of 500 l. at 5 per cent. and made partial payments as follows: Required the state of the affair the 14th of August 1785, when the account comes to be cleared.

		L.	s.	d.
1781, June 1.	Principal borrowed	500		
	Interest for 1 year and 129 days	33	16	8½
	Amount	533	16	8½
1782, Oct. 8.	Paid	120		
	New principal	413	16	8½
	Interest for 1 year and 85 days	25	10	2½
	Amount	439	6	10¾
1784, Jan. 1.	Paid	239	6	10¾
	New principal	200		
	Interest for 1 year and 225 days	16	3	3½
	Amount	216	3	3½
1785, Aug. 14.	Paid in full	216	3	3½

P R O B. II.

Amount, rate, and time, in years given, to find the principal, or present worth.

R U L E.

Add 1 to the product of the time and rate of one pound, by which divide the amount.

E X A M P L E I.

What ready money will pay a bill of 682 l. 18 s. 9½ d. due 3½ years hence, discounting interest at 4½ per cent.?

3.75 time.
 .045 rate of 1l.

1875
 1500

$$\begin{array}{r} 1.16875 \end{array} 682.939583 (584.3 = 584 \begin{array}{l} L. \quad s. \quad d. \\ 6 \quad 8 \end{array} \text{ Ans.} \\ 584375 \dots$$

985645
 935000

506458
 467500

389583
 350625

*38958

Or say, As the amount of 100l. at the rate and time given, is to 100l. principal; so is the given amount to the principal, or present worth, sought.

3.75 time.
 4.5 rate.

1875
 1500

16.875 interest of 100l. for the given time.

100

116.875 amount of 100 l. for the given time.

$$\begin{array}{cccc} L. & L. & L. & L. \\ \text{Then say, } 116.875 : 100 :: 682.939583 : 584.3 \end{array}$$

If the time be given in days, divide the product of the rate and time by 365, and then proceed as before.

The difference between the present worth and the amount, or sum of the bill, is called *rebate*, or *discount*; and is always less than the interest of the amount for the given time, being precisely equal to the interest of the present worth for the said time; so that if the present worth be put to interest, it will, by the end of the given time, with the interest thence arising, be exactly equal to the given amount.

Thus, If I have a bill of 105l. due 1 year hence, the discount at

at 5 *per cent*, will be 5*l.* and the present worth 100*l.*; for if I put the 100*l.* to interest, it will, with the interest, by the end of the year, be equal to 105*l.* the given amount. But the interest of 105*l.* for a year is 5*l.* 5*s.*; and if this be deduced from the sum of the bill, I shall then receive as present worth the sum only of 99*l.* 15*s.* which is 5*s.* too little,

The discount is found by subtracting the present worth from the amount; but it may also be found as follows, *viz.* As the amount of 100*l.* or of 1*l.* for the time given, to the interest; so the given amount or sum of the bill to the discount sought. The discount of the former example found in this manner follows:

$$\begin{array}{rcl}
 116.875 : 16.875 :: 682.939583 & & \\
 23.375 : 3.375 & & .027 \\
 4.675 : .675 & & \hline
 .935 : .135 & & 4780577083 \\
 .187 : .027 & & 13658791666 \\
 .187) & & 18.439368750 \quad (98.60625 = 98 \quad 12 \quad 1\frac{1}{2} \\
 & & 1683 \dots\dots \\
 & & \hline
 & & 1609 \\
 & & 1496 \\
 & & \hline
 & & 1133 \\
 & & 1122 \\
 & & \hline
 & & 1168 \\
 & & 1122 \\
 & & \hline
 & & 467 \\
 & & 374 \\
 & & \hline
 & & 935 \\
 & & 935 \\
 & & \hline
 \end{array}$$

The present worth is found by subtracting the discount from the amount, or sum of the bill.

$$\begin{array}{rcl}
 L. & s. & d. \\
 682 & 18 & 9\frac{1}{2} \text{ amount.} \\
 98 & 12 & 1\frac{1}{2} \text{ discount.} \\
 \hline
 584 & 6 & 8 \text{ present worth.}
 \end{array}$$

EXAMPLE II.

What is the present worth, and what the rebate, of a bill of 85*l.* 10*s.* discounting for 66 days, at 5 *per cent.*?

73)66.0(.9041 interest of 100l. for 66 days.

657

300

292

80

73

7

Then 100.9041 : .9041 :: 85.5

85.5

45205

45205

72328

s. d.

100.9041)77.30055(.766 = 15 3½ rebate.
7063287 85 10 amount.

6667680

6054246

84 14 8½ present worth.

6134340

6054246

(80094)

EXAMPLE III.

A bill of 546l. 16s. dated January 10. payable the 8th of August, was presented March 15. for discount at 4½ per cent. : What will the rebate come to?

73)146(2 interest at 5 per cent.

.1 at ½ per cent. = ½

1.9 interest of 100l. for 146

100 days, at 4½ per cent.

101.9

March 16

April 30

May 31

June 30

July 31

Aug. 8

146 days.

Then

Then $101.9 : 1.9 :: 546.8$ 1.9 49212 5468 *L. s. d.* $101.9)1038.92(10.193 = 10\ 3\ 10\frac{1}{4}$ *Ans.* 1019 1992 1019 9730 9171 5590 5095 (495)

Here it is to be observed, that an alteration, either in the days or the rate, does not produce a proportional alteration in the discount. Thus the discount of any sum for 40 days, will be more than half the discount for 80 days at the same rate; and the discount at 4 *per cent.* will be more than half the discount of the same sum for the same time at 8 *per cent.*

The reason is, because the first term, or divisor, consists of two parts; the one variable, and the other invariable. The variable part is the interest of 100l. for the given time, which increases proportionally with the rate or time; and when this interest comes to be doubled, the dividend is of course doubled also; but the divisor being by this means increased, will not give a double quotient. Thus, $100 + 2 = 102$ $306(3$, but $100 + 4 = 104$ dividing 612 will not quote 6. Hence it is that tables of discount cannot be accurate, unless calculated at every different rate, and for as many days as the case may require; because every day's discount varies, being still less as the days or rate increase.

P R O B. III.

Principal, amount, and time in years, given, to find the rate.

R U L E.

As the product of the principal and time, to the difference of the principal and amount, or total interest; so 100 to the rate *per cent.*

E X A M P L E I.

At what rate of interest will 584l. 6s. 8d. amount to 682l. 18s. 9 $\frac{1}{4}$ d. in 3 $\frac{3}{4}$ years.

584.3

$$\begin{array}{r}
 584.3 \\
 \underline{3\frac{1}{4}} \\
 4)1753.0 \\
 \underline{438.25} \\
 2191.25 : 98.60625 :: 100 \\
 438.25 : 19.72125 \\
 87.65 : 3.94425 \\
 17.53 :) 78.885(4.5 \text{ per cent. } \textit{Ans.} \\
 \underline{7012} \\
 8765 \\
 \underline{8765}
 \end{array}$$

If the time given be days, divide the product of the principal and time by 365, and then proceed as before.

E X A M P L E II.

At what rate of interest will 275l. gain 8l. 5s. in 219 days?

$$\begin{array}{r}
 275 \\
 \underline{219} \\
 2475 \\
 275 \\
 \underline{550} \\
 365)60225(165 : 8.25 :: 100 \\
 \underline{365} \quad 33 :)165(5 \text{ per cent. } \textit{Ans.} \\
 \underline{2372} \quad 165 \\
 2190 \\
 \underline{1825} \\
 1825
 \end{array}$$

P R O B. IV.

Principal, amount, and rate given, to find the time in years.

R U L E.

Divide the difference of the principal and amount by the product of the principal and rate of one pound; that is, as one year's interest of the principal to one year; so the total interest to the time sought.

E X.

E X A M P L E I.

In what time will 584l. 6s. 8d. amount to 682l. 18s. 9½d. at 4½ per cent.?

584.3 .045 29216 233783 26.295)98.60625(3.75 years. <i>Ans.</i> 78885 .. 197212 184065 131475 131475 	682.939583 584.333333 98.606250
--	---

If the time be required in days, multiply the difference of the principal and amount by 365, and then proceed as before.

E X A M P L E II.

In how many days will 275l. principal gain 8l. 5s. at 5 per cent.?

275 .05 13.75)3011.25(219 days. <i>Ans.</i> 2750 .. 2612 1375 12375 12375 	8.25 365 4125 4950 2475 3011.25
---	--

I shall conclude this section with the explication of a rule that stands connected with simple interest, viz.

Equation of Payments.

When two or more debts are payable at different times, the finding a mean time, at which all the debts may be paid at once, without loss to debtor or creditor, is called *equating* the terms or payment; and is commonly performed by the following

K k

R U L E.

R U L E.

Multiply the several debts into their respective times, divide the sum of the products by the total debts, and the quot is accounted the mean time.

E X A M P L E I.

A owes B 600l. to pay at 40 days, 200l. at 60 days, and 200l. at 120 days: When may these debts be paid at once, without injury to either party?

<i>Debts.</i>	<i>Days.</i>	<i>Prod.</i>
600 ×	40 =	24000
200 ×	60 =	12000
200 ×	120 =	24000
1000		)60000(60 days. <i>Ans.</i>

E X A M P L E II.

A owes B 640l. whereof 40l. to be paid presently, 350l. at the end of 3 months, and 250l. at 9 months: Required the equated time for paying the whole.

<i>Debts.</i>	<i>Mon.</i>	<i>Prod.</i>
40 ×	0 =	0000
350 ×	3 =	1050
250 ×	9 =	2250
640		)3300(5 $\frac{3}{4}$ months. <i>Ans.</i>

E X A M P L E III.

A owes B a certain sum; whereof $\frac{1}{3}$ to be paid in ready money, $\frac{1}{3}$ at the end of 6 months, and the other $\frac{1}{3}$ at 8 months: Required the equated time for paying the whole.

<i>Debts.</i>	<i>Mon.</i>	<i>Prod.</i>
$\frac{1}{3} \times$	0 =	0
$\frac{1}{3} \times$	6 =	2
$\frac{1}{3} \times$	8 =	2 $\frac{2}{3}$
1		)4 $\frac{2}{3}$ (4 $\frac{2}{3}$ months. <i>Ans.</i>

The above method is easy, and on that account commonly practised, but is not accurate; for a person, by keeping money unpaid after it becomes due, gains the interest thereof for that time; but by paying money before it is due, he does not lose the interest, as the rule supposes, but only the discount thereof for that time, which is always less than the interest.

They who incline to be more exact, may work by the following

R U L E.

R U L E.

Find, by Prob. 2. the present worth of each debt ; and then, by Prob. 4. find in what time the sum of the present worths will amount to the sum of the debts.

E X A M P L E IV.

A owes B 50l.; whereof 20l. is payable 2 years hence, and 30l. 5 years hence: What is the equated time for paying both debts at once, discounting interest at 5 per cent?

By Prob. 2.

L.

The present worth of 20l. for 2 years is, 18.18

The present worth of 30l. for 5 years is, 24.00

Sum 42.18

Total debts 50.00

Difference 7.82

By Prob. 4.

Principal 42.18

Rate .05

2.1090)7.8200(3.7079 years.

Ans. 3.7079 years, or 3 years and 258 days.

Some still complain, that the rule last assigned is not absolutely perfect; and argue, that the rule for finding the mean or equated time, ought to be such as will make the interest of the debts paid after they are due, exactly equal to the discount of the debts paid before they are due.

II. *Compound Interest.*

Compound interest arises both from the principal and interest; for at the end of one year the amount or sum of principal and interest becomes a new principal for the year ensuing.

The laws in Britain forbid the lending of money at compound interest; but then the lender may exact his interest at the year's end, lend it out again, and so, in effect, have compound interest on his money. Annuities too are commonly reckoned at compound interest; and this makes the knowledge of such computations in some measure necessary. I propose however to be very brief both on compound interest and annuities; and this the rather, because operations of this sort are performed to most advantage by logarithms.

In compound interest, the principal and the several years amounts

mounts make a series of geometrical proportionals, the common ratio being the amount of 1l. for one year, as under :

	1	2	3	4 Years.	Ratio.
Principal	1	1.05	1.1025	1.157625	1.21550625
Principal 100	100	105	110.25	115.7625	121.550625

} 1.05

P R O B. I.

Principal, rate, and time, given, to find the amount and interest.

R U L E.

Multiply the amount of 1l. for a year so often into itself as are the number of years given save one; and the last product multiplied by the principal gives the amount; from which subtract the principal, and the remainder is the interest.

E X A M P L E I.

What is the amount and interest of 500l. for 3 years, at 4 per cent.?

$$\begin{array}{r}
 1.04 \text{ amount of 1l. for a year.} \\
 1.04 \\
 \hline
 1.0816 \\
 1.04 \\
 \hline
 1.124864 \\
 500 \text{ principal.} \\
 \hline
 562.432000 \text{ amount} \\
 500 \\
 \hline
 62.432 \text{ interest}
 \end{array}$$

L.	s.	d.
562	8	$7\frac{1}{2}$
amount =		
62	8	$7\frac{1}{2}$
interest =		

Or, Multiply the given principal by the amount of 1l. for a year, so often as there are years in the question.

$$\begin{array}{r}
 500 \text{ principal given.} \\
 1.04 \text{ amount of 1l. for a year.} \\
 \hline
 520 \text{ amount for 1 year.} \\
 1.04 \\
 \hline
 540.80 \text{ amount for 2 years.} \\
 1.04 \\
 \hline
 562.4320 \text{ amount for 3 years.}
 \end{array}$$

Or,

Or, Multiply the given principal by the rate of $\frac{1}{100}$ and the product is the interest for one year ; which, added to the principal, gives the amount for the first year ; and work in like manner for each of the remaining years.

500 principal.
.04

20.00 interest.

520 amount.
.04

20.80 interest.

540.80 amount.
.04

21.6320 interest.

562.432 amount.

By practice, thus :

25)500 principal.
 20 interest.

25)520 amount for 1 year.
 20.8 interest.

25)540.8 amount for 2 years.
 21.632 interest.

562.432 amount for 3 years.

If the given time consist of years and parts, or years and days, find the interest of the last amount for the given parts or days, and add the interest so found to the last amount.

EXAMPLE II.

What is the amount and interest of 354l. 16s. for $3\frac{1}{2}$ years, at 5 per cent. ?

By practice :

1.05 amount
 1.05 of 1l.

1.1025
1.05

1.157625
 354.8

40)410.7253500
 10.26813375

420.99348375 amount.
 354.8

66.19348375 interest.

20)354.8
 17.74

20)372.54
 18.627

20)391.167
 19.55835

40)410.72535 amount for 3 years.
 10.26813375 interest.

420.99348375 total amount.
 354.8

66.19348375 total interest.

The interest for the half-year, in the above example, is computed in the way of simple interest: but it must be observed, that the interest for the half-year found in this manner will be too much; for in simple interest the several amounts are in arithmetical proportion; but in compound interest, the amounts are in geometrical proportion; and consequently the amount of any principal at compound interest, for any number of years, will be more than at simple interest: for one year they will be equal; but for any time less than a year, the amount, at compound interest, will be less than at simple interest.

To compute the amount for any aliquot part of a year, in this way, observe the following rule, *viz.* Extract that root of the amount of 1*l.* for a year, which is denoted by the denominator of the fraction; and the product of the principal into this root, is the amount for the part of the year required.

Thus, for $\frac{1}{4}$ of a year, extract the biquadrate root; for $\frac{1}{2}$ of a year extract the square root; so, in our example, the square root of 1.05 is 1.024695; and $410.72535 \times 1.024695 = 420.86821251825$; which is somewhat less than the amount found above.

Again, for $\frac{3}{4}$ of a year, extract the biquadrate root, and raise this root to the third power, the numerator being 3. In like manner, if the given time be days, extract the 365th root, and raise this root to the power denoted by the numerator, or number of days given. But such extractions and involutions will prove a troublesome task, without the help of logarithms.

P R O B. II.

Amount, rate, and time, given, to find the principal or present worth.

R U L E.

Divide the given amount by the amount of 1*l.* for the given time and rate, and the quot is the principal or present worth; that is, as the amount of 1*l.* to 1*l.* principal, so the given amount to the principal sought.

E X A M P L E.

What ready money will pay a debt of 562.432*l.* due 3 years hence, discounting at 4 *per cent.* compound interest?

$$\begin{array}{r}
 1.04 \\
 1.04 \\
 \hline
 1.0816 \\
 1.04 \\
 \hline
 1.124864
 \end{array}
 \begin{array}{l}
 562.432000 \\
 5624320
 \end{array}
 \begin{array}{l}
 (500 \text{ l. } \textit{Ans.} \\
 \hline
 \end{array}$$

The

The difference of the given amount and present worth is the discount or rebate, which in the above example is 62.432l.

If the given time be less than a year, the present worth may be found nearly in the way of simple interest; but, to be accurate, find the amount of 1l. for the given time, in the way taught above, by which divide the given amount:

P R O B. III.

Principal, amount, and time, given, to find the rate:

R U L E.

Divide the amount by the principal, and that root of the quot which is denoted by the number of years will be the amount of 1l. for a year, from which subtract 1; and there will remain the rate.

E X A M P L E.

At what rate of compound interest will 500l. amount to 562.432 in three years?

500)562.432(1.124864; and the cube root of 1.124864 is 1.04; and hence .04 is the rate of 1l.; that is 4 *per cent*.

P R O B. IV.

Principal, amount, and rate, given, to find the time.

R U L E.

Divide the amount by the principal, and again divide the quot by the amount of 1l. for a year continually till the quot is 1; and the number of continual divisions is the number of years. Or, Raise the amount of 1l. in a year to a power equal to the first quot; and the index of that power is the time in years.

E X A M P L E.

In what time will 500l. amount to 562.432l. at 4 *per cent*. compound interest?

$$500)562.4320$$

$$1.04)1.124864$$

$$1.04)1.0816$$

$$1.04)1.04$$

$$\text{Or, } 1.04$$

$$1.04$$

$$1.0816 \text{ square.}$$

$$1.04$$

$$1.124864 \text{ cube. } \textit{Ans. 3 years.}$$

For ease and dispatch in calculations of compound interest, tables, framed for years and for days, at all the different rates, are absolutely necessary, and accordingly are universally used. I shall here subjoin three tables of this kind, and then show their construction and use.

T A B L E I.
The amount of 1 l. for years, compound interest.

Y.	2 per c.	2½ per c.	3 per c.	3½ per c.
26	1.6734181	1.9002927	2.1565912	2.4459585
27	1.7068864	1.9478000	2.2212890	2.5315671
28	1.7410242	1.9964950	2.2879276	2.6201719
29	1.7758446	2.0464073	2.3565655	2.7118779
30	1.8113615	2.0975675	2.4272624	2.8067937
31	1.8475888	2.1500067	2.5000803	2.9050314
32	1.8845405	2.2037569	2.5750827	3.0067975
33	1.9222314	2.2588508	2.6523352	3.1119423
34	1.9606760	2.3153221	2.7319053	3.2208603
35	1.9998895	2.3732051	2.8138624	3.3335904
36	2.0398873	2.4325353	2.8982783	3.4502661
37	2.0806850	2.4933487	2.9852266	3.5710254
38	2.1222987	2.5556824	3.0747834	3.6960113
39	2.1647447	2.6195744	3.1670269	3.8253717
40	2.2080396	2.6850638	3.2620377	3.9592597
41	2.2522004	2.7519004	3.3598989	4.0978338
42	2.2979444	2.8209952	3.4606958	4.2412579
43	2.3431893	2.8915500	3.5645167	4.3897020
44	2.3900531	2.9638080	3.6714522	4.5433416
45	2.4378542	3.0379032	3.7815958	4.7023585
46	2.4866112	3.1138508	3.8950437	4.8669411
47	2.5363435	3.1916971	4.0118950	5.0372840
48	2.5870703	3.2714895	4.1322518	5.2135889
49	2.6388117	3.3532768	4.2562194	5.3960645
50	2.6915880	3.4371087	4.3839060	5.5849268

T A B L E I.
The amount of 1 l. for years, compound interest.

Y.	2 per c.	2½ per c.	3 per c.	3½ per c.
1	1.0200000	1.0250000	1.0300000	1.0350000
2	1.0404000	1.0506250	1.0609000	1.0712250
3	1.0612080	1.0768906	1.0927270	1.1087178
4	1.0824321	1.1038128	1.1255088	1.1475230
5	1.1040808	1.1314082	1.1592740	1.1876863
6	1.1261624	1.1596934	1.1940523	1.2292553
7	1.1486856	1.1886857	1.2298733	1.2722792
8	1.1716593	1.2184029	1.2667700	1.3168089
9	1.1950925	1.2488629	1.3047731	1.3628973
10	1.2189944	1.2800845	1.3439163	1.4105987
11	1.2433743	1.3120866	1.3842338	1.4599697
12	1.2682417	1.3448888	1.4257608	1.5110686
13	1.2936066	1.3785110	1.4685337	1.5639560
14	1.3194787	1.4129738	1.5125897	1.6186945
15	1.3458683	1.4482981	1.5579674	1.6753488
16	1.3727857	1.4845056	1.6047064	1.7339860
17	1.4002414	1.5216182	1.6528476	1.7946755
18	1.4282462	1.5596587	1.7024230	1.8574892
19	1.4568111	1.5986501	1.7535060	1.9225013
20	1.4859474	1.6386164	1.8061112	1.9897888
21	1.5156663	1.6795818	1.8602945	2.0594314
22	1.5459796	1.7215714	1.9161034	2.1315115
23	1.5768992	1.7646106	1.9735865	2.2061144
24	1.6084372	1.8087259	2.0327941	2.2833284
25	1.6406059	1.8539441	2.0937779	2.3632449

T A B L E I.
The amount of 1 l. for years, compound interest.

T.	4 per c.	4½ per c.	5 per c.	6 per c.
26	2.7724697	3.1406792	3.5556727	4.5493829
27	2.8833685	3.2820095	3.7334563	4.8223459
28	2.9987033	3.4296999	3.9201291	5.1116866
29	3.1186514	3.5840364	4.1161356	5.4183878
30	3.2433975	3.7453181	4.3219424	5.7434911
31	3.3731334	3.9138574	4.5380395	6.0881006
32	3.5080587	4.0899810	4.7649415	6.4533866
33	3.6483811	4.2740301	5.0031885	6.8405898
34	3.7943163	4.4663615	5.2533480	7.2510252
35	3.9465889	4.6673478	5.5160154	7.6860867
36	4.1039325	4.8773784	5.7918161	8.1472519
37	4.2680898	5.0968604	6.0814069	8.6360870
38	4.4388134	5.3262192	6.3854773	9.1542523
39	4.6163659	5.5658990	6.7047511	9.7035074
40	4.8010206	5.8163645	7.0399887	10.2857178
41	4.9930614	6.0781009	7.3919881	10.9028609
42	5.1927839	6.3516154	7.7615875	11.5570326
43	5.4004952	6.6374381	8.1496669	12.2504545
44	5.6165150	6.9361229	8.5571503	12.9854818
45	5.8411756	7.2482484	8.9850078	13.7646107
46	6.0748227	7.5744196	9.4342582	14.5904873
47	6.3178156	7.9152684	9.9059711	15.4659166
48	6.5705282	8.2714555	10.4012696	16.3938716
49	6.8333493	8.6436710	10.9213331	17.3775039
50	7.1066833	9.0326362	11.4674000	18.4201541

T A B L E I.
The amount of 1 l. for years, compound interest.

T.	4 per c.	4½ per c.	5 per c.	6 per c.
1	1.0400000	1.0450000	1.0500000	1.0600000
2	1.0816000	1.0920250	1.1025000	1.1236000
3	1.1248640	1.1411661	1.1576250	1.1910160
4	1.1698586	1.1925186	1.2155063	1.2624769
5	1.2166529	1.2461819	1.2762816	1.3382256
6	1.2653190	1.3022621	1.3400956	1.4185191
7	1.3159318	1.3608618	1.4071034	1.5036303
8	1.3685691	1.4221006	1.4774554	1.5938481
9	1.4233118	1.4860951	1.5513282	1.6894790
10	1.4802443	1.5529694	1.6288946	1.7908477
11	1.5394541	1.6228530	1.7103393	1.8982980
12	1.6010322	1.6958814	1.7958563	2.0121965
13	1.6650735	1.7721961	1.8856491	2.1329283
14	1.7316764	1.8519449	1.9799316	2.2609039
15	1.8009435	1.9352824	2.0789332	2.3965582
16	1.8729812	2.0223721	2.1828746	2.5402517
17	1.9479005	2.1133768	2.2920183	2.6927728
18	2.0258165	2.2084787	2.4066192	2.8543392
19	2.1068492	2.3078623	2.5269502	3.0255995
20	2.1911231	2.4117140	2.6532977	3.2071355
21	2.2787681	2.5202411	2.7859626	3.3995636
22	2.3699188	2.6336520	2.9252607	3.6035374
23	2.4647155	2.7521663	3.0715238	3.8197497
24	2.5633042	2.8760138	3.2251000	4.0489346
25	2.6658363	3.0054344	3.3863549	4.2918707

T A B L E II.
The amount of 1 l. for days, compound interest.

D.	2 per c.	2½ per c.	3 per c.	3½ per c.
170	1.0092658	1.0115670	1.0138623	1.0161516
180	1.0098135	1.0122516	1.0146837	1.0171098
190	1.0103615	1.0129366	1.0155057	1.0180689
200	1.0109098	1.0136221	1.0163284	1.0190288
210	1.0114584	1.0143081	1.0171518	1.0199897
220	1.0120073	1.0149945	1.0179759	1.0209515
230	1.0125565	1.0156814	1.0188006	1.0219142
240	1.0131050	1.0163687	1.0196260	1.0228778
250	1.0136538	1.0170565	1.0204520	1.0238424
260	1.0142029	1.0177448	1.0212788	1.0248078
270	1.0147523	1.0184336	1.0221062	1.0257741
280	1.0153010	1.0191228	1.0229342	1.0267414
290	1.0158500	1.0198125	1.0237630	1.0277096
300	1.0164093	1.0205026	1.0245924	1.0286786
310	1.0169609	1.0211932	1.0254225	1.0296486
320	1.0175127	1.0218843	1.0262532	1.0306195
330	1.0180649	1.0225758	1.0270847	1.0315914
340	1.0186174	1.0232679	1.0279168	1.0325641
350	1.0191702	1.0239603	1.0287495	1.0335378
360	1.0197233	1.0246533	1.0295830	1.0345123
361	1.0197786	1.0247226	1.0296664	1.0346098
362	1.0198340	1.0247919	1.0297497	1.0347073
363	1.0198893	1.0248613	1.0298331	1.0348049
364	1.0199446	1.0249306	1.0299165	1.0349024
365	1.0200000	1.0250000	1.0300000	1.0350000

T A B L E II.
The amount of 1 l. for days, compound interest.

D.	2 per c.	2½ per c.	3 per c.	3½ per c.
1	1.0000542	1.0000676	1.0000809	1.0000942
2	1.0001085	1.0001353	1.0001619	1.0001885
3	1.0001627	1.0002029	1.0002429	1.0002827
4	1.0002170	1.0002706	1.0003240	1.0003770
5	1.0002713	1.0003383	1.0004050	1.0004713
6	1.0003255	1.0004059	1.0004860	1.0005656
7	1.0003798	1.0004736	1.0005670	1.0006600
8	1.0004341	1.0005412	1.0006480	1.0007542
9	1.0004884	1.0006090	1.0007291	1.0008486
10	1.0005426	1.0006767	1.0008101	1.0009429
20	1.0010856	1.0013539	1.0016209	1.0018867
30	1.0016289	1.0020315	1.0024324	1.0028315
40	1.0021725	1.0027097	1.0032445	1.0037771
50	1.0027163	1.0033882	1.0040573	1.0047236
60	1.0032605	1.0040673	1.0048708	1.0056710
70	1.0038049	1.0047468	1.0056849	1.0066193
80	1.0043497	1.0054267	1.0064996	1.0075685
90	1.0048947	1.0061071	1.0073151	1.0085186
100	1.0054401	1.0067880	1.0081311	1.0094696
110	1.0059857	1.0074693	1.0089479	1.0104214
120	1.0065316	1.0081511	1.0097653	1.0113742
130	1.0070779	1.0088334	1.0105834	1.0123279
140	1.0076244	1.0095161	1.0114021	1.0132825
150	1.0081712	1.0101993	1.0122215	1.0142379
160	1.0087183	1.0108829	1.0130415	1.0151943

T A B L E II.
The amount of 1 l. for days, compound interest.

D.	4 per c.	4½ per c.	5 per c.	6 per c.
170	1.0184350	1.0207126	1.0229843	1.0275105
180	1.0195299	1.0219442	1.0243527	1.0291522
190	1.0206261	1.0231774	1.0257228	1.0307964
200	1.0217233	1.0244120	1.0270949	1.0324433
210	1.0228218	1.0256481	1.0284687	1.0340928
220	1.0239215	1.0268858	1.0298444	1.0357450
230	1.0250233	1.0281249	1.0312219	1.0373998
240	1.0261243	1.0293655	1.0326013	1.0390572
250	1.0272275	1.0306076	1.0339825	1.0407173
260	1.0283319	1.0318512	1.0353656	1.0423800
270	1.0294375	1.0330963	1.0367505	1.0440454
280	1.0305443	1.0343429	1.0381373	1.0457135
290	1.0316522	1.0355910	1.0395259	1.0473842
300	1.0327614	1.0368406	1.0409164	1.0490576
310	1.0338717	1.0383917	1.0423087	1.0507336
320	1.0349832	1.0393444	1.0437029	1.0524124
330	1.0360960	1.0405985	1.0450990	1.0540938
340	1.0372099	1.0418542	1.0464969	1.0557779
350	1.0383250	1.0431114	1.0478967	1.0574647
360	1.0394413	1.0443700	1.0492984	1.0591542
361	1.0395530	1.0444960	1.0494387	1.0593233
362	1.0396648	1.0446220	1.0495790	1.0594924
363	1.0397765	1.0447479	1.0497193	1.0596616
364	1.0398882	1.0448739	1.0498596	1.0598308
365	1.0400000	1.0450000	1.0500000	1.0600000

T A B L E II.
The amount of 1 l. for days, compound interest.

D.	4 per c.	4½ per c.	5 per c.	6 per c.
1	1.0001074	1.0001206	1.0001336	1.0001596
2	1.0002149	1.0002412	1.0002673	1.0003193
3	1.0003224	1.0003618	1.0004011	1.0004790
4	1.0004299	1.0004824	1.0005348	1.0006387
5	1.0005374	1.0006031	1.0006685	1.0007985
6	1.0006449	1.0007238	1.0008023	1.0009583
7	1.0007524	1.0008445	1.0009361	1.0011181
8	1.0008600	1.0009652	1.0010699	1.0012779
9	1.0009675	1.0010859	1.0012037	1.0014378
10	1.0010751	1.0012066	1.0013376	1.0015976
20	1.0021513	1.0024148	1.0026770	1.0031979
30	1.0032288	1.0036243	1.0040182	1.0048007
40	1.0043074	1.0048354	1.0053611	1.0064060
50	1.0053871	1.0060479	1.0067059	1.0080139
60	1.0064680	1.0072618	1.0080525	1.0096244
70	1.0075501	1.0084773	1.0094009	1.0112375
80	1.0086333	1.0096942	1.0107511	1.0128531
90	1.0097177	1.0109125	1.0121031	1.0144713
100	1.0108033	1.0121324	1.0134569	1.0160921
110	1.0118900	1.0133537	1.0148125	1.0177155
120	1.0129779	1.0145765	1.0161699	1.0193415
130	1.0140670	1.0158007	1.0175291	1.0209701
140	1.0151572	1.0170265	1.0188902	1.0226013
150	1.0162487	1.0182537	1.0202531	1.0242351
160	1.0173412	1.0194824	1.0216178	1.0258715

T A B L E III.

The present worth of 1 l. for years, compound int.

T.	2 per c.	2½ per c.	3 per c.	3½ per c.
26	.5975793	.5262347	.4636947	.4088378
27	.5858620	.5133997	.4501891	.3950123
28	.5743746	.5008778	.4370768	.3816543
29	.5631123	.4886613	.4243464	.3687482
30	.5520709	.4767427	.4119868	.3562784
31	.5412460	.4651148	.3999871	.3442304
32	.5306333	.4537706	.3883370	.3325897
33	.5202287	.4427030	.3770263	.3213427
34	.5100282	.4319053	.3660449	.3104761
35	.5000276	.4213711	.3553834	.2999769
36	.4902232	.4110937	.3450324	.2898327
37	.4806109	.4010671	.3349829	.2800316
38	.4711872	.3912849	.3252262	.2705619
39	.4619482	.3817414	.3157536	.2614125
40	.4528904	.3724306	.3065568	.2525725
41	.4440102	.3633470	.2976280	.2440314
42	.4353041	.3544848	.2889592	.2357791
43	.4267688	.3458389	.2805429	.2278059
44	.4184008	.3374038	.2723718	.2201023
45	.4101968	.3291744	.2644386	.2126594
46	.4021537	.3211458	.2567365	.2054679
47	.3942684	.3133129	.2492588	.1985197
48	.3865376	.3056712	.2419988	.1918065
49	.3789584	.2982158	.2349503	.1853202
50	.3715279	.2909422	.2281071	.1790534

T A B L E III.

The present worth of 1 l. for years, compound int.

T.	2 per c.	2½ per c.	3 per c.	3½ per c.
1	.9803921	.9756097	.9708738	.9661836
2	.9611687	.9518144	.9425959	.9335107
3	.9423223	.9285994	.9151417	.9019427
4	.9238454	.9059506	.8884870	.8714422
5	.9057308	.8838542	.8626088	.8419732
6	.8879713	.8622968	.8374843	.8135006
7	.8705601	.8412654	.8130915	.7859910
8	.8534903	.8207465	.7894092	.7594116
9	.8367552	.8007283	.7664167	.7337310
10	.8203483	.7811984	.7440939	.7089188
11	.8042630	.7621447	.7224213	.6849457
12	.7884931	.7435558	.7013799	.6617833
13	.7730325	.7254203	.6809513	.6394041
14	.7578750	.7077272	.6611178	.6177818
15	.7430147	.6904655	.6418619	.5961906
16	.7284458	.6736249	.6231669	.5767059
17	.7141625	.6571950	.6050164	.5572038
18	.7001593	.6411659	.5873946	.5383611
19	.6864307	.6255277	.5702860	.5201557
20	.6729713	.6102709	.5536758	.5025659
21	.6597758	.5953862	.5375493	.4855709
22	.6468390	.5808646	.5218925	.4691506
23	.6341559	.5666972	.5066917	.4532856
24	.6217214	.5528753	.4919337	.4379571
25	.6095308	.5393905	.4776056	.4231470

T A B L E III. The present worth of 1 l. for years, compound int.				
T.	4 per c.	4½ per c.	5 per c.	6 per c.
26	.3606892	.3184025	.2812407	.2198100
27	.3468166	.3046914	.2678483	.2073685
28	.3334775	.2915707	.2550936	.1956301
29	.3206514	.2790150	.2429463	.1845567
30	.3083187	.2670000	.2313775	.1741101
31	.2964603	.2555024	.2203595	.1642548
32	.2850579	.2444999	.2098662	.1549574
33	.2740942	.2339712	.1998726	.1461862
34	.2635521	.2238959	.1903548	.1379115
35	.2534155	.2142544	.1812903	.1301052
36	.2436687	.2050282	.1726574	.1227408
37	.2342969	.1961992	.1644356	.1157932
38	.2252854	.1877504	.1566054	.1092389
39	.2166206	.1796655	.1491479	.1030555
40	.2082890	.1719287	.1420457	.0972222
41	.2002779	.1645251	.1352816	.0917191
42	.1925749	.1574403	.1288396	.0865274
43	.1851682	.1506605	.1227044	.0816296
44	.1180464	.1441728	.1168613	.0770091
45	.1711948	.1379644	.1112965	.0726501
46	.1646139	.1320233	.1059967	.0685378
47	.1582826	.1263381	.1009492	.0646583
48	.1521948	.1208977	.0961421	.0609984
49	.1463411	.1156916	.0915639	.0575457
50	.1407126	.1107097	.0872037	.0542884

T A B L E III. The present worth of 1 l. for years, compound int.				
T.	4 per c.	4½ per c.	5 per c.	6 per c.
1	.9615385	.9569378	.9523810	.9433962
2	.9245562	.9157299	.9070295	.8899964
3	.8889964	.8762966	.8638376	.8396193
4	.8548042	.8385613	.8227025	.7920937
5	.8219271	.8024511	.7835262	.7472582
6	.7903145	.7678957	.7462154	.7049605
7	.7599178	.7348285	.7106813	.6650571
8	.7306902	.7031851	.6768394	.6274124
9	.7025867	.6729044	.6446089	.5918985
10	.6755642	.6439277	.6139133	.5583948
11	.6495809	.6161987	.5846793	.5267875
12	.6245971	.5896639	.5568374	.4969694
13	.6005741	.5642716	.5303214	.4688390
14	.5774751	.5399729	.5050679	.4423010
15	.5552645	.5167204	.4810171	.4172651
16	.5339082	.4944693	.4581115	.3936463
17	.5133733	.4731764	.4362967	.3713644
18	.4936281	.4528004	.4155207	.3503438
19	.4746424	.4333018	.3957340	.3305130
20	.4563870	.4136429	.3768895	.3118047
21	.4388336	.3967874	.3589424	.2941554
22	.4219554	.3797009	.3418499	.2775051
23	.4057263	.3633501	.3255713	.2617973
24	.3901215	.3477035	.3100679	.2469786
25	.3751168	.3327306	.2953028	.2329986

Table I. shews the amount of 1 l. principal for years; and is constructed by the rule assigned in Prob. 1. *viz.* Multiply the amount of 1 l. for a year by itself continually. Thus, at 4 per cent. the amount of 1 l. is 1.04 for 1 year; and $1.04 \times 1.04 = 1.0816$ for 2 years; and $1.0816 \times 1.04 = 1.124864$ for 3 years, &c.

Table II. shews the amount of 1 l. for days; to construct which, find, by the directions given in the end of Prob. 1. the amount of 1 l. for one day; and this amount multiplied into itself continually, will give the amount for any number of days.

Example at 5 per cent.

1.00013368 the amount for 1 day.
1.00013368

Prod. 1.00026738 the amount for 2 days.
1.00013368

Prod. 1.00040110 the amount for 3 days.

Table III. shews the present worth of 1 l. principal for years; and is constructed by the rule assigned in Prob. 2. *viz.* Divide 1 l. by the amount of 1 l. for the given time and rate, as contained in Table I.

Example at 4 per cent.

1.04)1(.9615385 for 1 year.
1.0816)1(.9245562 for 2 years.
1.124864)1(.8889964 for 3 years, &c.

The table may, in this manner, be continued to any number of years whatever.

The use of Table I.

1. Principal, rate, and time, given, to find the amount.

Multiply the given principal by the tabular number corresponding to the rate and time.

Ex. What will 500 l. principal amount to in 15 years, at 5 per cent. compound interest?

$$\begin{array}{r} 2.0789282 \\ \times 500 \\ \hline 1039.4641000 = 1039 \text{ L. } 9 \text{ s. } 3\frac{1}{4} \text{ d.} \end{array}$$

If the given principal consist of pounds, shillings, pence, reduce the shillings and pence to a decimal, and then proceed as above.

2. Principal

2. Principal, amount, and time, given, to find the rate.

Divide the amount by the principal, and the quot will be the amount of 1 l.; which find in the table even with the given time, and you have the rate on the head.

Ex. At what rate of interest will 500 l. amount to 1039.4641 l. in 15 years?

$$500 \overline{) 1039.4641}$$

2.0789282 found under 5 per cent.

3. Principal, amount, and rate, given, to find the time.

Divide the amount by the principal, and the quot will be the amount of 1 l.; which find under the given rate, and on the side you have the time.

Ex. In what time will 500 l. amount to 1039.4641 l. at 5 per cent. compound interest?

$$500 \overline{) 1039.4641}$$

2.0789282 found opposite to 15 years.

4. Amount, rate, and time, given, to find the principal.

Divide the given amount by the tabular amount of 1 l. for the rate and time given, and the quot will be the principal.

Ex. What principal will amount to 1039.4641 l. in 15 years, at 5 per cent.?

$$2.0789282 \overline{) 1039.4641} (500 \text{ principal.}$$

The use of Table II.

This table shews the amount of 1 l. for days, and is used in the same manner as the former.

But if the given number of days be not in this table; or if the given number of years be not in Table I. work as follows:

Divide the given number of days or years into two such numbers as are found in the tables, multiply the tabular amounts into one another, and their product is the amount of 1 l. for the time given.

Ex. What will 500 l. amount to in 75 years and 184 days, at 5 per cent.?

Amount for 50 years, at 5 per cent. is	—	11.4674000
Amount for 25 years, at 5 per cent. is	—	3.3863549
Their product is the amount for 75 years	—	38.8326862

Amount

Amount for 180 days, is,	-	1.0243527
Amount for 4 days, is,	-	1.0005348
Their product is the amount for 184 days,		1.0249005
Amount for 75 years, is,	-	38.8326862
Amount for 184 days, is,	-	1.0249005
Their product is the amount of 1l. for 75 years and 184 days,		39.7996395
Multiply by the principal,		500
Amount sought,	-	19899.8197500

And so, 500l. in 75 years and 184 days, at 5 per cent. will amount to 19899l. 16s. 4½d. And in this manner may these two tables be extended to any number of years and days.

The use of Table III.

Amount, rate, and time given, to find the principal or present worth.

Multiply the tabular number answering to the rate and time, by the given amount.

Ex. What ready money will pay a debt of 562.432l. due 3 years hence, discounting at 4 per cent. compound interest?

$$.8889964 \times 562.432 = 500\text{l. the present worth sought.}$$

I shall here subjoin two or three practical questions.

Quest. 1. What will L. 136 : 15 : 6 amount to in 20 years, at 6 per cent. compound interest?

By Table I.

3.2071355 amount of 1l.
577.631 multiplier inverted.

32071355
9621406
1924281
224499
22449
1603

438.65593 amount sought.

Quest. 2.

Quest. 2. What will 5*l.* amount to in 400 years, at 5 *per cent.* compound interest?

By Table I.

The amount of 1*l.* is, 11.4674 for 50 years.

11.4674

131.5013 for 100 years.

131.5013

17292.5919 for 200 years.

17292.5919

297650327.2679 for 400 years.

5

Amount of 5*l.* is, 1488251636.3395 for 400 years.

The above example shows the rapid and surprising increase of numbers in geometrical proportion; and for the learner's amusement, it may not be improper to observe, that if a single farthing had been lent on compound interest, at 5 *per cent.* in the first year of the Christian æra, or at the birth of Christ, and had continued to this present year 1785, the amount would have been such an immense sum as to surpass in value some millions of globes of solid gold, each as large as this earth.

Quest. 3. A owes B several sums, *viz.* 180*l.* due 3 years hence; 200*l.* due 5 years hence; and 300*l.* due 7 years hence: What is the present worth of these debts, discounting at 4 *per cent.* compound interest?

By Table III.

.8889964 × 180 = 160.0193520

.8219271 × 200 = 164.3854200

.7599178 × 300 = 227.9753400

Present worth, 552.380112

C H A P. XIII.

A N N U I T I E S.

AN annuity is a sum of money, payable yearly, half-yearly, or quarterly, to continue a certain number of years, for ever, or for life.

L 1

An

An annuity is said to be in arrear, when it continues unpaid after it falls due. And an annuity is said to be in reversion, when the purchaser, upon paying the price, does not immediately enter upon possession; the annuity not commencing till some time after.

Interest on annuities may be computed either in the way of simple or compound interest. But compound interest being found most equitable, both for buyer and seller, the computation by simple interest is universally disused.

I. *Annuities for a certain Time.*

P R O B. -I.

Annuity, rate, and time, given, to find the amount, or sum of yearly payments, and interest.

R U L E.

Make 1 the first term of a geometrical series, and the amount of 1l. for a year the common ratio; continue this series to as many terms as there are years in the question; and the sum of this series is the amount of 1l. annuity for the given years; which multiplied by the given annuity, will produce the amount sought.

E X A M P L E.

An annuity of 40l. payable yearly is forborn and unpaid till the end of 5 years: What will then be due, reckoning compound interest at 5 per cent. on all the payments then in arrear?

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 \\ 1 : 1.05 : 1.1025 : 1.157625 : 1.21550625 ; \text{ whose sum is} \\ 5.525631251. ; \text{ and } 5.52563125 \times 40 = 221.02525 = 221\text{l. os. 6d.} \\ \text{the amount sought.} \end{array}$$

The amount may also be found thus: Multiply the given annuity by the amount of 1l. for a year; to the product add the given annuity, and the sum is the amount in two years; which multiply by the amount of 1l. for a year; to the product add the given annuity, and the sum is the amount in 3 years, &c. The former question wrought in this manner follows:

40 amount in 1 year.	126.1 amount in 3 years.
<u>1.05</u>	<u>1.05</u>
42.00	132.405
40	40
82 amount in 2 years.	172.405 amount in 4 years.
<u>1.05</u>	<u>1.05</u>
86.10	181.02525
40	40
126.1 amount in 3 years.	221.02525 amount in 5 years.

If the given time be years and quarters, find the amount for the whole years, as above; then find the amount of 1 l. for the given quarters; by which multiply the amount for the whole years; and to the product add such a part of the annuity as the given quarters are of a year.

If the given annuity be payable half-yearly, or quarterly, find the amount of 1 l. for half a year or a quarter; by which find the amount for the several half-years or quarters, in the same manner as the amount for the several years is found above.

P R O B. II.

Annuity, rate, and time, given, to find the present worth, or sum of money that will purchase the annuity.

R U L E.

Find the amount of the given annuity by the former problem; and then, by Prob. 2. in compound interest, find the present worth of this amount, as a sum due at the end of the given time.

E X A M P L E.

What is the present worth of an annuity of 40 l. to continue 5 years, discounting at 5 *per cent.* compound interest?

By the former problem, the amount of the given annuity for 5 years, at 5 *per cent.* is 221.02525; and by Prob. 1. in compound interest, the amount of 1 l. for 5 years, at 5 *per cent.* is 1.2762815625.

And, $1.2762815625 \times 221.02525000 = 173.179 = 173 \text{ l. } 3 \text{ s. } 7 \text{ d.}$ the present worth sought.

The present worth may also be found thus: By Prob. 2. of compound interest, find the present worth of each year by itself, and the sum of these is the present worth sought. The former example done in this way follows:

L 1 2

1.2762815625

1.2762815625	40.0000000000	(31.3410
1.21550625	40.00000000	(32.9080
1.157625	40.000000	(34.5535
1.1025	40.0000	(36.2811
1.05	40.0	(38.0952

Present worth, 173.1788

A third way of finding the present worth, take as follows, *viz.* Find a principal sum whereof the given annuity is one year's interest: then find the present worth of this principal, as a sum due at the end of the given time; subtract this present worth from its principal; and the remainder is the present worth sought. The former question done in this manner follows:

As .05 : 1 :: 40 : 800l. principal.

$$1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$$

And 1.2762815625)800.00000000(626.8208

800

626.8208

173.1792 present worth sought.

If the annuity to be purchased be in reversion, find first the present worth of the annuity, as commencing immediately, by any of the three methods taught above; and then by Prob. 2. of compound interest, find the present worth of that present worth, rebating for the time in reversion; and this last present worth is the answer.

Ex. What is the present worth of a yearly pension or rent of 75l. to continue 4 years, but not to commence till 3 years hence, discounting at 5 per cent.?

$$.05 : 1 :: 75 : 1500$$

$$1.05 \times 1.05 \times 1.05 \times 1.05 = 1.21550625$$

$$1.21550625)1500.000000(1234.05371$$

1500

1234.05371

265.94629 present worth of the annuity, if it was to commence immediately.

$$1.05 \times 1.05 \times 1.05 = 1.157625 \quad L. \quad s. \quad d.$$

$$1.157625)265.94629(229.7344 = 229 \quad 14 \quad 8\frac{1}{4}$$

P R O B. III.

Present worth, rate, and time, given, to find the annuity.

R U L E.

R U L E.

By the preceding problem, find the present worth of 1 l. annuity for the rate and time given; and then say, As the present worth thus found to 1 l. annuity, so the present worth given to its annuity; that is, divide the given present worth by that of 1 l. annuity.

E X A M P L E.

What annuity, to continue 5 years, will 173 l. 3 s. 7 d. purchase, allowing compound interest at 5 per cent.?

$$.05 : 1 :: 1 : 201.$$

$$1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$$

$$1.2762815625)20.000000000(15.6705$$

20

15.6705

4.3295 present worth of 1 l. annuity.

4.329)173.179(40 l. annuity. *Ans.*

The operations in annuities, as well as in compound interest, turn out heavy and troublesome; and on that account tables are universally used. I shall here annex three tables of that sort; and to the tables shall subjoin a brief illustration of their uses. And in regard there will sometimes be occasion for the promiscuous use of these tables on annuities, and the three former tables on compound interest, the following tables shall be numbered Table IV. V. VI.; by which means any table to be used will be easily referred to.

Table IV. shows the amount of 1 l. annuity, and may be constructed from Prob. I.; or rather, thus: To 1 l. the first year of this table, add the first year of Table I. and the sum is the second year of this table; to which add the second year of Table I. and the sum will be the third year of this table, &c. The reason is obvious.

Table V. shews the present worth of 1 l. annuity; and may be constructed from Prob. II. or more easily from Table III.; thus: The first year in Table III. and V. is the same; the sum of the first and second years in Table III. make the second year in Table V.; and the sum of the second and third years in Table III. make the third year in Table V. &c.

x first,

Table VI. shews the annuity which 1 l. will purchase; and may be constructed from Prob. III.; but more readily thus: Divide 1 by the first year of Table V. and the quot is the first year of Table VI. Again, divide 1 by the second year of Table V. and the quot is the second year of Table VI. &c.

L 13

T A B L E

T A B L E IV.

The amount of 1 l. annuity, compound interest.

<i>T.</i>	2 per c.	2½ per c.	3 per c.	3½ per c.
26	33.6709057	36.0117080	38.5530422	41.3131017
27	35.3442238	37.9120007	40.7096335	43.7590602
28	37.0512103	39.8598008	42.9309225	46.2906273
29	38.7922345	41.8562958	45.2188522	48.9107993
30	40.5680792	43.9027032	47.5754157	51.6226773
31	42.3794408	46.0002707	50.0026782	54.4294710
32	44.2270296	48.1502775	52.5027585	57.3345025
33	46.1115702	50.3540345	55.0778413	60.3412101
34	48.0338016	52.6128653	57.7301765	63.4531524
35	49.9944776	54.9282074	60.4620818	66.6740127
36	51.9943672	57.3014126	63.2759443	70.0076032
37	54.0342545	59.7339479	66.1742226	73.4578693
38	56.1149396	62.2272966	69.1594493	77.0288947
39	58.2372384	64.7829791	72.2342327	80.7249060
40	60.4019832	67.4025535	75.4012597	84.5502778
41	62.6100228	70.0876174	78.6632975	88.5095375
42	64.8622233	72.8398078	82.0231964	92.6073713
43	67.1594678	75.6608030	85.4838923	96.8486293
44	69.5026511	78.5523231	89.0484191	101.2383313
45	71.8927103	81.5161312	92.7198614	105.7816729
46	74.3305645	84.5540344	96.5014172	110.4840315
47	76.8171758	87.6678853	100.3965009	115.3509726
48	79.3535193	90.8595824	104.4083960	120.3882566
49	81.9405697	94.1310729	108.5406479	125.6018456
50	84.5794015	97.4843488	112.7968673	130.9979102

T A B L E IV.

The amount of 1 l. annuity, compound interest.

<i>T.</i>	2 per c.	2½ per c.	3 per c.	3½ per c.
1	1.0000000	1.0000000	1.0000000	1.0000000
2	2.0200000	2.0250000	2.0300000	2.0350000
3	3.0604000	3.0756230	3.0909000	3.1062250
4	4.1216080	4.1525156	4.1836270	4.2149429
5	5.2040402	5.2563285	5.3091358	5.3624659
6	6.3081210	6.3877367	6.4684099	6.5501522
7	7.4342834	7.5474322	7.6624622	7.7794075
8	8.5829691	8.7361159	8.8923360	9.0516866
9	9.7546284	9.9545188	10.1591061	10.3684958
10	10.9497210	11.2033318	11.4638793	11.7313931
11	12.1687154	12.4834663	12.8077957	13.1419919
12	13.4120897	13.7955530	14.1920296	14.6019616
13	14.6803315	15.1404418	15.6177904	16.1130303
14	15.9739381	16.5189528	17.0863242	17.6769864
15	17.2934169	17.9319267	18.5989139	19.2956809
16	18.6392853	19.3802248	20.1568813	20.9710297
17	20.0120719	20.8647304	21.7615877	22.7050158
18	21.4123124	22.3863487	23.4144354	24.4996913
19	22.8405586	23.9460074	25.1168684	26.3571805
20	24.2973698	25.5446576	26.8703745	28.2796818
21	25.7833172	27.1832740	28.6764857	30.2694707
22	27.2989835	28.8628559	30.5367803	32.3289022
23	28.8449632	30.5844273	32.4528837	34.4604137
24	30.4218625	32.3490379	34.4264702	36.6665282
25	32.0302297	34.1577639	36.4592643	38.9498567

T A B L E IV.

The amount of 1 l. annuity, compound interest.

T.	4 per c.	4½ per c.	5 per c.	6 per c.
26	44.3117446	47.5706446	51.1134538	59.1563827
27	47.0842144	50.7113236	54.6691265	63.7057657
28	49.9675830	53.9933332	58.4025828	68.5281116
29	52.9662863	57.4230332	62.3227119	73.6397983
30	56.0849377	61.0070698	66.4388475	79.0581862
31	59.3283352	64.7523878	70.7607899	84.8016774
32	62.7014687	68.6662452	75.2988294	90.8897780
33	66.2095274	72.7562263	80.0637738	97.3431647
34	69.8579085	77.0302565	85.0669594	104.1837546
35	73.6522248	81.496618	90.3203773	111.4347799
36	77.5983138	86.1639658	95.8363227	119.1208667
37	81.7022464	91.0413443	101.6281388	127.2681187
38	85.9703362	96.1382248	107.7095158	135.9042058
39	90.4091497	101.4644240	114.0950231	145.0584581
40	95.0255157	107.0303231	120.7997742	154.7619656
41	99.8265363	112.8466876	127.8397829	165.0476836
42	104.8195978	118.9247885	135.2317511	175.9505446
43	110.0123817	125.2764040	142.9933386	187.5075772
44	115.4128169	131.9138422	151.1430056	199.7580319
45	121.0293920	138.8499651	159.7001559	212.7435138
46	126.8705677	146.0982135	168.6851637	226.5081246
47	132.9453904	153.6726331	178.1194218	241.0986121
48	139.2632063	161.5879016	188.0253929	256.5645388
49	145.8337342	169.8593572	198.4266626	272.9584006
50	152.6670836	178.5030282	209.3479957	290.3359046

T A B L E IV.

The amount of 1 l. annuity, compound interest.

T.	4 per c.	4½ per c.	5 per c.	6 per c.
1	1.0000000	1.0000000	1.0000000	1.0000000
2	2.0400000	2.0400000	2.0500000	2.0600000
3	3.1216000	3.1370250	3.1525000	3.1836000
4	4.2464640	4.2781911	4.3101250	4.3746016
5	5.4163226	5.4707097	5.5256312	5.6370930
6	6.6329755	6.7168917	6.8019128	6.9753187
7	7.8982945	8.0191518	8.1420084	8.3938378
8	9.2142263	9.3800136	9.5491089	9.8974681
9	10.5827953	10.8021142	11.0265643	11.4913162
10	12.0061071	12.2882094	12.5778925	13.1807958
11	13.4863514	13.8411788	14.2067871	14.9716435
12	15.0258055	15.4640318	15.9171265	16.8699420
13	16.6268377	17.1599133	17.7129828	18.8821385
14	18.2919112	18.9321094	19.5986320	21.0150667
15	20.0235876	20.7840543	21.5785636	23.2759707
16	21.8245311	22.7193367	23.6574918	25.6725289
17	23.6975124	24.7417069	25.8403664	28.2128806
18	25.6454129	26.8550837	28.1323847	30.9056534
19	27.6712294	29.0635625	30.5390039	33.7599925
20	29.7780786	31.3714228	33.0659541	36.7855920
21	31.9692017	33.7831368	35.7192518	39.9927275
22	34.2479698	36.3033779	38.5052144	43.3922911
23	36.6178886	38.9370299	41.4304751	46.9958285
24	39.0826041	41.6791962	44.5019989	50.8155782
25	41.6459083	44.5652101	47.7270988	54.8645128

T A B L E V.

The present worth of 1 l. annuity, compound int.

1.	2 per c.	2½ per c.	3 per c.	3½ per c.
26	20.1210358	18.9506115	17.8768429	16.8903523
27	20.7068978	19.4640109	18.3270315	17.2853645
28	21.2812724	19.9648887	18.7641082	17.6670188
29	21.8443847	20.4535499	19.1884546	18.0357670
30	22.3964556	20.9302926	19.6004415	18.3920454
31	22.9377015	21.3954074	20.0004285	18.7362758
32	23.4683348	21.8491780	20.3887655	19.0688656
33	23.9885636	22.2918809	20.7657918	19.3902082
34	24.4985917	22.7237863	21.1318367	19.7006842
35	24.9986193	23.1451573	21.4872200	20.0006612
36	25.4888425	23.5562511	21.8322525	20.2904938
37	25.9694534	23.9573181	22.1672354	20.5705254
38	26.4406406	24.3486030	22.4924616	20.8410874
39	26.9025888	24.7303444	22.8082151	21.1024999
40	27.3554792	25.1027751	23.1147719	21.3550723
41	27.7994895	25.4661220	23.4123999	21.5991037
42	28.2347936	25.8200688	23.7013592	21.8348828
43	28.6615623	26.1664457	23.9819021	22.0626887
44	29.0799631	26.5038495	24.2542739	22.2827910
45	29.4901599	26.8330239	24.5187125	22.4954503
46	29.8923136	27.1541696	24.7754490	22.7009183
47	30.2865820	27.4674826	25.0247078	22.8994378
48	30.6731196	27.7731537	25.2667066	23.0912443
49	31.0520780	28.0713695	25.5016569	23.2765645
50	31.4236059	28.3623117	25.7297640	23.4556179

T A B L E V.

The present worth of 1 l. annuity, compound int.

1.	2 per c.	2½ per c.	3 per c.	3½ per c.
1	0.9803922	0.9756098	0.9708738	0.9661836
2	1.9415609	1.9274242	1.9134697	1.8996943
3	2.8838833	2.8560236	2.8286114	2.8016371
4	3.8077287	3.7619742	3.7170984	3.6730792
5	4.7134595	4.6458285	4.5797072	4.5150524
6	5.6014309	5.5081254	5.4171914	5.3285530
7	6.4719911	6.3493906	6.2302829	6.1145439
8	7.3254814	7.1701372	7.0196922	6.8739555
9	8.1622367	7.9708655	7.7861089	7.6276865
10	8.9825850	8.7520639	8.5302028	8.3166053
11	9.7868480	9.5142087	9.2526241	9.0015510
12	10.5753412	10.2577646	9.9540040	9.6633343
13	11.3483737	10.9831839	10.6349553	10.3027385
14	12.1062487	11.6909122	11.2960731	10.9205203
15	12.8492635	12.3813777	11.9379351	11.5174109
16	13.5777093	13.0550027	12.5611020	12.0941168
17	14.2918719	13.7121977	13.1661185	12.6513206
18	14.9920313	14.3533636	13.7535131	13.1890817
19	15.6784620	14.9788913	14.3237991	13.7098374
20	16.3514333	15.5891623	14.8774748	14.2124033
21	17.0112092	16.1845486	15.4150241	14.6979742
22	17.6580482	16.7654132	15.9369166	15.1671248
23	18.2922041	17.3321105	16.4436084	15.6204105
24	18.9139256	17.8849858	16.9355421	16.0583676
25	19.5234565	18.4243764	17.4131477	16.4815146

T A B L E V.
The present worth of 1 l. annuity, compound int.

i.	4 per c.	4½ per c.	5 per c.	6 per c.
26	15.9827678	15.1466115	14.3751853	13.0031663
27	16.3295844	15.4513028	14.6430336	13.2105342
28	16.6630618	15.7428735	14.8981272	13.4061644
29	16.9837132	16.0218885	15.1410735	13.5907211
30	17.2920318	16.2888885	15.3724510	13.7648312
31	17.5884921	16.5443909	15.5928104	13.9290861
32	17.8735500	16.7888909	15.8026766	14.0840435
33	18.1476441	17.0228621	16.0025491	14.2302297
34	18.4111962	17.2467580	16.1929039	14.3681412
35	18.6646116	17.4610124	16.3741942	14.4982465
36	18.9082803	17.6660406	16.5468516	14.6209872
37	19.1425771	17.8622398	16.7112872	14.7367804
38	19.3678625	18.0499902	16.8678926	14.8460192
39	19.5844831	18.2296557	17.0170406	14.9490747
40	19.7927721	18.4015844	17.1590862	15.0462969
41	19.9930500	18.5661095	17.2943678	15.1380160
42	20.1856250	18.7235498	17.4232074	15.2245434
43	20.3707931	18.8742103	17.5459118	15.3061730
44	20.5488395	19.0183831	17.6627732	15.3831821
45	20.7200378	19.1563474	17.7740697	15.4558321
46	20.8846517	19.2883707	17.8800663	15.5243699
47	21.0429342	19.4147088	17.9610155	15.5890282
48	21.1951289	19.5356066	18.0771576	15.6500266
49	21.3414700	19.6512981	18.1687215	15.7075723
50	21.4821826	19.7620078	18.2559253	15.7618610

T A B L E V.
The present worth of 1 l. annuity, compound int.

i.	4 per c.	4½ per c.	5 per c.	6 per c.
1	0.9615385	0.9569378	0.9523809	0.9433962
2	1.8860947	1.8726678	1.8594104	1.8333926
3	2.775910	2.7489644	2.7232480	2.6730119
4	3.6298952	3.5875257	3.5459505	3.4651056
5	4.4518223	4.3899767	4.3294767	4.2123638
6	5.2421369	5.1578725	5.0756921	4.9173244
7	6.0220547	5.8927009	5.7863734	5.5823815
8	6.7327448	6.5958861	6.4632128	6.2097939
9	7.4353314	7.2687905	7.1078217	6.8016923
10	8.1108955	7.9127182	7.7217349	7.3600871
11	8.7604763	8.5289169	8.3064142	7.8868747
12	9.3850733	9.1185808	8.8632516	8.3838440
13	9.9856473	9.6828524	9.3935730	8.8526831
14	10.5631223	10.2228253	9.8986409	9.2949840
15	11.1183868	10.7395457	10.3796580	9.7122491
16	11.6522949	11.2340151	10.8377695	10.1058953
17	12.1656680	11.7071914	11.2740662	10.4772597
18	12.6592961	12.1599918	11.6895869	10.8276035
19	13.1339385	12.5932936	12.0853108	11.1581165
20	13.5903253	13.0079365	12.4622103	11.4699213
21	14.0291589	13.4047239	12.8211527	11.7640767
22	14.4511142	13.7844248	13.1630026	12.0415818
23	14.8568405	14.1477749	13.4885739	12.3033790
24	15.2469619	14.4954784	13.7986418	12.5503576
25	15.6220787	14.8282089	14.0939445	12.7833562

T A B L E VI.
The ann. which 1 l. will purchase, comp. int.

L.	2 per c.	2½ per c.	3 per c.	3½ per c.
26.	.046092	.0527688	.0559383	.0592054
27.	.0482931	.0513769	.0545642	.0578524
28.	.0469897	.0500879	.0532932	.0566027
29.	.0457784	.0488913	.0521147	.0554454
30.	.0446499	.0477776	.0510193	.0543713
31.	.0435964	.0467490	.0499989	.0533724
32.	.0426106	.0457683	.0490466	.0524415
33.	.0416865	.0448591	.0481561	.0515724
34.	.0408187	.0440068	.0473220	.0507597
35.	.0400022	.0432056	.0465393	.0499984
36.	.0392329	.0424516	.0458038	.0492842
37.	.0385068	.0417409	.0451116	.0486133
38.	.0378206	.0410701	.0444593	.0479821
39.	.0371711	.0404362	.0438439	.0473878
40.	.0365558	.0398362	.0432624	.0468273
41.	.0359719	.0392679	.0427124	.0462982
42.	.0354173	.0387288	.0421917	.0457983
43.	.0348899	.0382169	.0416981	.0453254
44.	.0343879	.0377304	.0412299	.0448777
45.	.0339096	.0372675	.0407852	.0444534
46.	.0334534	.0368268	.0403625	.0440511
47.	.0330179	.0364067	.0399605	.0436692
48.	.0326018	.0360060	.0395778	.0433065
49.	.0322040	.0356235	.0392131	.0429617
50.	.0318032	.0352581	.0388655	.0426337

T A B L E VI.
The ann. which 1 l. will purchase, comp. int.

L.	2 per c.	2½ per c.	3 per c.	3½ per c.
1.	.0200000	.0250000	.0300000	.0350000
2.	.5150495	.5188272	.5226108	.5264005
3.	.3467547	.3501372	.3535304	.3569342
4.	.2626238	.2658179	.2690271	.2722511
5.	.2121584	.2152469	.2183546	.2214814
6.	.1785258	.1815499	.1845975	.1876682
7.	.1545120	.1574954	.1605064	.1635445
8.	.1365098	.1394674	.1424564	.1454767
9.	.1225154	.1254569	.1284339	.1314460
10.	.1113265	.1142588	.1172305	.1202414
11.	.1021779	.1051060	.1080775	.1110920
12.	.0945596	.0974871	.1004621	.1034840
13.	.0881183	.0910483	.0940295	.0970616
14.	.0826020	.0855365	.0885263	.0915707
15.	.0778255	.0807665	.0837666	.0868251
16.	.0736501	.0765990	.0796109	.0826848
17.	.0699698	.0729278	.0759525	.0790431
18.	.0667021	.0696701	.0727087	.0758168
19.	.0637818	.0667606	.0698139	.0729403
20.	.0611567	.0641471	.0672157	.0703610
21.	.0587847	.0617873	.0648718	.0680366
22.	.0566314	.0596466	.0627474	.0659321
23.	.0546681	.0576964	.0608139	.0640188
24.	.0528511	.0559128	.0590474	.0622728
25.	.0512204	.0542759	.0574279	.0606740

T A B L E VI.

The ann. which 1 l. will purchase, com. int.

T.	4 per c.	4½ per c.	5 per c.	6 per c.
26	.0625674	.0660214	.0695643	.0769044
27	.0612385	.0647195	.0682919	.0756972
28	.0600130	.0635208	.0671225	.0745926
29	.0588799	.0624146	.0660455	.0735796
30	.0578301	.0613915	.0650514	.0726489
31	.0568554	.0604435	.0641321	.0717522
32	.0559486	.0595632	.0632804	.0710023
33	.0551036	.0587445	.0624900	.0702729
34	.0543148	.0579819	.0617554	.0695984
35	.0535773	.0572705	.0610717	.0689739
36	.0528869	.0566058	.0604345	.0683948
37	.0522396	.0559840	.0598398	.0678574
38	.0516319	.0554017	.0592842	.0673581
39	.0510608	.0548557	.0587640	.0668938
40	.0505235	.0543431	.0582782	.0664915
41	.0500174	.0538616	.0578223	.0660589
42	.0495402	.0534087	.0573947	.0656834
43	.0490899	.0529824	.0569933	.0653331
44	.0486645	.0525807	.0566163	.0650061
45	.0482625	.0522020	.0562617	.0647005
46	.0478821	.0518447	.0559282	.0644149
47	.0475219	.0515073	.0556142	.0641477
48	.0471807	.0511886	.0553184	.0638977
49	.0468571	.0508872	.0550397	.0636636
50	.0465502	.0506021	.0547767	.0634443

T A B L E VI.

The ann. which 1 l. will purchase, comp. int.

T.	4 per c.	4½ per c.	5 per c.	6 per c.
1	1.0400000	1.0450000	1.0500000	1.0600000
2	.531961	.5339976	.5378049	.5454369
3	.3623485	.3637734	.3672086	.3741098
4	.2754901	.2787437	.2820118	.2885915
5	.2246271	.2277916	.2309748	.2373964
6	.1907619	.1938784	.1970157	.2033362
7	.1666096	.1697015	.1728198	.1791350
8	.1485279	.1516097	.1547218	.1610359
9	.134493	.1375745	.1406901	.1470222
10	.1232909	.1263788	.1295246	.1358680
11	.1141490	.1172482	.1203890	.1267929
12	.1065522	.1096662	.1128254	.1192770
13	.1001437	.1032754	.1064558	.1129601
14	.0946690	.0978203	.1010240	.1075849
15	.0899411	.0931138	.0963423	.1029628
16	.0858200	.0890154	.0922699	.0989521
17	.0821985	.0854176	.0886991	.0954448
18	.0789933	.0822369	.0855462	.0923565
19	.0761386	.0794073	.0827450	.0896209
20	.0735818	.0768761	.0802426	.0871846
21	.0712801	.0746006	.0779961	.0850046
22	.0691988	.0725157	.0759705	.0830456
23	.0673091	.0706825	.0741368	.0812785
24	.0655868	.0689870	.0724709	.0796790
25	.0640121	.0674390	.0709525	.0782267

The Use of Table IV.

1. Annuity, rate, and time, given, to find the amount.

Multiply the amount of 1 l. annuity, for the rate and time given, by the given annuity, and the product is the amount sought.

Ex. What will an annuity of 85 l. 10 s. amount to in 20 years, at 5 per cent. compound interest?

$$\begin{array}{r}
 33.0659541 \text{ amount of 1 l. annuity.} \\
 85.5 \\
 \hline
 1653297705 \\
 1653297705 \\
 2645276328 \\
 \hline
 2827.13907555 \text{ amount sought.}
 \end{array}$$

2. Annuity, time, and amount, given, to find the rate.

Divide the amount by the annuity, and the quot will be the amount of 1 l.; which find in the table even with the given time, and on the head you have the rate.

Ex. At what rate per cent. will 85.5 amount to 2827.13907555?

$$\begin{array}{r}
 85.5)2827.13907555 \\
 \hline
 33.0659541 \text{ under 5 per cent.}
 \end{array}$$

3. Annuity, rate, and amount, given, to find the time.

Divide the amount by the annuity, and the quot will be the amount of 1 l.; which find under the given rate, and on the side you have the time.

Ex. In what time will an annuity of 85.5 l. amount to 2827.13907555 l. at 5 per cent.?

$$\begin{array}{r}
 85.5)2827.13907555 \\
 \hline
 33.0659541 \text{ opposite to 20 years.}
 \end{array}$$

4. Amount, rate, and time, given, to find the annuity.

Divide the given amount by the amount of 1 l. for the rate and time given, and the quot is the annuity.

Ex. What annuity will amount to 2827.13907555 l. in 20 years, at 5 per cent.?

$$\begin{array}{r}
 33.0659541)2827.13907555 \\
 \hline
 85.5 \text{ annuity.}
 \end{array}$$

The Use of Table V.

1. Annuity, rate, and time, given, to find the present worth.

Multiply

Multiply the present worth of 1l. annuity, for the rate and time given, by the given annuity, and the product is the present worth sought.

Ex. What is the present worth of an annuity of 100l. to continue 15 years, discounting at 5 per cent. compound interest?

$$10.3796580 \times 100 = 1037.9658 = 1037 \text{ } 19 \text{ } 3\frac{3}{4} \text{ Ans.}$$

2. Annuity, time, and present worth, given, to find the rate.

Divide the present worth by the annuity, and the quot is the present worth of 1l. annuity; which find opposite to the time, and on the head you have the rate.

Ex. The present worth of an annuity of 100l. to continue 15 years, is 1037.9658l.: At what rate is interest computed?

$$100)1037.9658(10.379658 \text{ found under 5 per cent.}$$

3. Annuity, rate, and present worth, given, to find the time.

Divide the present worth by the annuity, and the quot is the present worth of 1l. annuity; which find under the given rate, and you have the time on the side.

Ex. The present worth of an annuity of 100l. discounting at 5 per cent. is 1037.9658l.: How many years will the annuity continue?

$$100)1037.9658(10.379658 \text{ found opposite to 15 years.}$$

4. Present worth, rate, and time, given, to find the annuity.

Divide the given present worth by the tabular number answering to the rate and time, and the quot is the annuity.

Ex. What annuity, to continue 15 years, will 1037.9658l. purchase, computing compound interest at 5 per cent.?

$$10.379658)1037.965800(100 \text{ l. annuity.}$$

The Use of Table VI.

Price, or present worth, rate, and time, given, to find the annuity.

Multiply the price by the tabular number answering to the rate and time, and the product is the annuity sought.

Ex. What annuity, to continue 15 years, will 1037.9658l. purchase, reckoning at 5 per cent. compound interest?

$$1037.9658 \times .0963423 = 100 \text{ l. annuity.}$$

Having thus shown the use of each table separately, or by itself, I shall now subjoin a few practical questions; in solving several of which the promiscuous use of the tables will be necessary.

Quest. 1.

Quest. 1. A pays 2000*l.* for an annuity of 100*l.* to continue 50 years; B puts 2000*l.* out at interest: Which of them will amount to the greatest sum at the end of the 50 years, at the rate of $4\frac{1}{2}$ per cent. compound interest?

By Table I. the amount of 2000 <i>l.</i> for 50 years,	}	L.
at $4\frac{1}{2}$ per cent. is,		
By Table IV. the amount of 100 <i>l.</i> annuity, for	}	18065.27240
that time and rate, is,		
B's amount exceeds that of A by		17850.30282
		214.96958

By Table III. the present worth of 214.96958*l.* is 23*l.* 15*s.* 11 $\frac{3}{4}$ *d.* discounting for 50 years, at $4\frac{1}{2}$ per cent.; and so much was B's case better than that of A.

Quest. 2. A owes B 600*l.* to be paid in 12 years, viz. 100*l.* at the end of every 2 years; but agrees with B to pay him the whole in 6 years, by equal yearly payments, reckoning compound interest at 6 per cent.: What must the annual payment be?

By Table III. find the present worths of the 6 payments that were to be made in 12 years, and their sum will be 406.98272*l.*

By Table VI. find what annuity for 6 years the said sum will purchase, viz. $406.98272 \times .2033626 = 82.7651$. the annual payment.

Quest. 3. Which is most advantageous, a term of fifteen years in an estate of 500*l.* per annum, or the reversion of the same estate for ever, after the expiration of the said 15 years, reckoning compound interest at 5 per cent.?

An estate of 500 <i>l.</i> per annum for ever, or, as it is	}	L.
usually expressed, in fee-simple, at 5 per cent. is		
worth a sum whose interest is 500 <i>l.</i> yearly, viz.		10000
By Table V. The present worth of the same e-	}	5189.829
state, for a term of 15 years, at 5 per cent.		
is,		4810.171
And so the reversion is worth		

From the value of the term, viz.	-	-	5189.829
Subtract the value of the reversion,	-	-	4810.171

The term is better than the reversion by

379.658

Quest. 4. A has a term of 7 years in an estate of 100*l.* per annum; B has a term of 14 years in the same estate in reversion, after the expiration of the 7 years; and C has a further term of 21 years in reversion, after A and B: Required the present worths of the several terms, discounting at $4\frac{1}{2}$ per cent. compound interest?

By Table V. find the present worth of the yearly rent or annuity,

nuity, for 42 years, for 21 years, for 7 years; subtract these present worths from each other, and the remainders are the answers, as under.

$$\begin{array}{rcl}
 & L. & s. & d. \\
 42 \text{ years, } 18.7235498 \times 100 = 1872.35498 = & 1872 & 7 & 1 \\
 21 \text{ years, } 13.4047239 \times 100 = 1340.47239 = & 1340 & 9 & 5\frac{1}{2} \\
 7 \text{ years, } 5.8927009 \times 100 = 589.27009 = & 589 & 5 & 4\frac{3}{4}
 \end{array}$$

$$\begin{array}{rcl}
 L. & s. & d. \\
 589 & 5 & 4\frac{3}{4} \\
 \text{A's term.}
 \end{array}$$

$$\begin{array}{rcl}
 L. & s. & d. \\
 1340 & 9 & 5\frac{1}{2} \\
 589 & 5 & 4\frac{3}{4}
 \end{array}$$

$$\begin{array}{rcl}
 751 & 4 & 0\frac{1}{2} \\
 \text{B's term.}
 \end{array}$$

$$\begin{array}{rcl}
 L. & s. & d. \\
 1872 & 7 & 1 \\
 1340 & 9 & 5\frac{1}{2}
 \end{array}$$

$$\begin{array}{rcl}
 531 & 17 & 7\frac{1}{2} \\
 \text{C's term.}
 \end{array}$$

Quest. 5. A person having 9 years to run, in a lease for 49 years, of an estate of 80 l. *per annum*, would know what sum he ought to pay presently, in order to have the lease renewed or completed, by having 40 years added thereto, reckoning at 6 *per cent.* compound interest?

$$\begin{array}{rcl}
 \text{By Table V. the present worth of 1 l. annuity, at 6 } & \left. \begin{array}{l} L. \\ \text{per cent. for 49 years, is,} \end{array} \right\} & 15.7075723 \\
 \text{By the same table, the value of 1 l. annuity, at 6 } & \left. \begin{array}{l} \\ \text{per cent. for 9 years, is,} \end{array} \right\} & 6.8016923 \\
 \text{Their difference is,} & - & 8.9058800 \\
 \text{Which multiplied by} & - & 80 \\
 \hline
 \text{Gives for answer,} & - & 712.4704
 \end{array}$$

Quest. 6. A farmer gets a lease of 21 years renewed, for a fine or grassum of 400 l. and it is put in his option, either to pay the fine presently, or to pay yearly an additional rent equivalent thereto: Required the additional rent, reckoning compound interest at 5 *per cent.*?

$$\begin{array}{rcl}
 \text{By Table VI. the annuity for 21 years that 1 l. will } & \left. \begin{array}{l} L. \\ \text{purchase, at 5 per cent. is,} \end{array} \right\} & .0779961 \\
 \text{Which multiplied by} & - & 400 \\
 \hline
 \text{Gives the additional rent} & & 31.1984400
 \end{array}$$

Quest. 7. What annuity to continue 19 years, and to commence presently, may be purchased by a bill of 2000 l. payable 3 years hence, reckoning at 6 *per cent.* compound interest?

$$\begin{array}{rcl}
 \text{By Table III. the present worth of 2000 l. due 3 } & \left. \begin{array}{l} L. \\ \text{years hence, at 6 per cent. is,} \end{array} \right\} & 1679.238600 \\
 \text{By Table VI. the annuity which 1679.2386 l. will } & \left. \begin{array}{l} \\ \text{purchase for 19 years, at 6 per cent. is,} \end{array} \right\} & 150.494874
 \end{array}$$

Quest. 8.

Quest. 8. What annuity would be sufficient to pay off a debt of 120 millions, in the space of 30 years, at 4 *per cent.* compound interest?

By Table VI. the annuity for 30 years that 1*l.* will
purchase at 4 *per cent.* is, } .0578301
Which multiply by the debt - 120000000

The product is the annuity sought, *viz.* - L. 6939612
From which subtract the yearly interest of 120 mil- }
lions, at 4 *per cent.* *viz.* 4800000

The remainder, if we suppose the 120 millions to be
the national debt, will be a sinking fund sufficient } L. 2139612
to clear the whole in 30 years, -

From the above example, appears the conveniency, or rather
the necessity, of continuing the decimals in the tables to seven pla-
ces at least.

Quest. 9. For a certain lease to continue 7 years, A offers 650*l.*
fine, and 200*l.* *per annum*; B offers 150*l.* fine, and 300*l.* *per*
annum; and C offers 1800*l.* fine, without any yearly rent: Which
of these three offers is the best, reckoning at 5 *per cent.* compound
interest?

By Table V. the present worth of 200*l.* *per annum* } L.
for 7 years, at 5 *per cent.* is } 1157.27468
To which add the fine - 650

Value of A's offer - 1807.27468

By Table V. the present worth of 300*l.* *per annum* }
for 7 years, at 5 *per cent.* is, } 1735.91202
To which add the fine - 150

Value of B's offer - 1885.91202

C's offer is 1800

By which it appears that B's offer is the best; and that, ne-
glecting the decimal, he offers 78*l.* more than A, and 85*l.* more
than C.

Quest. 10. A lease of an estate, to continue 14 years, is offered
for 250*l.* fine, and 44*l.* yearly rent; but the tenant wants to re-
duce the rent to 20*l.* *per annum*: What ought the fine to be
reckoning compound interest at 6 *per cent.*?

The difference between 44 and 20, the two rents, is 24*l.*

By

By Table V. the present worth of 24l. for 14 years, } $\overset{L.}{223.0796160}$
 at 6 per cent. is,
 To which add the first fine $\underset{250}{}$

The fine sought $\underline{473.0796160}$

II. *Annuities for ever, or freehold Estates.*

In freehold estates, commonly called *annuities in fee-simple*, the things chiefly to be considered are, 1. The annuity or yearly rent. 2. The price or present worth. 3. The rate of interest. The questions that usually occur on this head will fall under one or other of the following problems:

Prob. 1. Annuity and rate of interest given, to find the price.

As the rate of 1l. to 1l. so the rent to the price.

Ex. The yearly rent of a small estate is 40l.: What is it worth in ready money, computing interest at $3\frac{1}{2}$ per cent.?

$\overset{L.}{} \quad \overset{s.}{} \quad \overset{d.}{}$

As .035 : 1 :: 40 : 1142.857142 = 1142 $\overset{17}{} \overset{1\frac{1}{2}}{}$

Prob. 2. Price and rate of interest given, to find the rent or annuity.

As 1l. to its rate, so the price to the rent.

Ex. A gentleman purchases an estate for 4000l. and has $4\frac{1}{2}$ per cent. for his money: Required the rent.

As 1 : .045 :: 4000 : 180l. rent sought.

Prob. 3. Price and rent given, to find the rate of interest.

As the price to the rent, so 1 to the rate.

Ex. An estate of 180l. yearly rent is bought for 4000l.: What rate of interest has the purchaser for his money?

As 4000 : 180 :: 1 : .045 rate sought.

Prob. 4. The rate of interest given, to find how many years purchase an estate is worth.

Divide 1 by the rate, and the quot is the number of years purchase the estate is worth.

Ex. A gentleman is willing to purchase an estate, provided he can have $2\frac{1}{2}$ per cent. for his money: How many years purchase may he offer?

.025)1.000(40 years purchase. *Ans.*

Prob. 5. The number of years purchase at which an estate is bought or sold, given, to find the rate of interest.

M m

Divide

Divide 1 by the number of years purchase, and the quot is the rate of interest.

Ex. A gentleman gives 40 years purchase for an estate: What interest has he for his money?

40)1.000(.025 rate sought.

The computations hitherto are all performed by a single division or multiplication, and the learner will scarcely perceive that the operations are conducted by the rules of compound interest; but when a reversion occurs, there will be occasion for the tables, as follows:

Prob. 6. The rate of interest, and the rent of a freehold estate in reversion, given, to find the present worth or value of the reversion.

By *Prob. 1.* find the price or present worth of the estate, as if possession was to commence presently; and then, by *Table V.* find the present value of the given annuity, or rent, for the years prior to the commencement, subtract this value from the former value, and the remainder is the value of the reversion.

Ex. A has the possession of an estate of 130 l. *per annum*, to continue 20 years; B has the reversion of the same estate from that time for ever: What is the value of the estate, what the value of the 20 years possession, and what the value of the reversion, reckoning compound interest at 6 *per cent.*?

By *Prob. 1.* .06)130.00(2166.6666 value of the estate.

By *Table V.* 1491.0896 value of the possession.

675.5770 value of the reversion.

Prob. 7. The price or value of a reversion, the time prior to the commencement, and rate of interest, given, to find the annuity or rent.

By *Table I.* find the amount of the price of the reversion for the years prior to the commencement; and then, by *Prob. 2.* find the annuity which that amount will purchase.

Ex. The reversion of a freehold estate, to commence 20 years hence, is bought for 675.577 l. compound interest being allowed at 6 *per cent.*: Required the annuity or rent.

By *Table I.* the amount of 675.577 l. for $\left. \begin{array}{l} L. \\ 20 \text{ years, at } 6 \text{ per cent. is} \end{array} \right\} 2166.6$

By *Prob. 2.* $2166.6 \times .06 = 130.0$ rent sought.

III. *Life Annuities.*

The value of annuities for life is determined from observations made on the bills of mortality. Dr Halley, Mr Simpson, and Monf. de Moivre, are gentlemen of distinguished merit in calculations of this kind.

Dr Halley had recourse to the bills of mortality at Breslaw, the capital of Silesia, as a proper standard for the other parts of Europe, being a place pretty central, at a distance from the sea, and not much crowded with traffickers or foreigners. He pitches upon 1000 persons all born in one year, and observes how many of these were alive every year, from their birth to the extinction of the last, and consequently how many died each year, as in the following table :

Dr Halley's table on the bills of mortality at Breslaw.

<i>Age.</i>	<i>Perf. liv.</i>	<i>A.</i>	<i>Perf. liv.</i>	<i>A.</i>	<i>Perf. liv.</i>	<i>A.</i>	<i>Perf. liv.</i>
1	1000	24	573	47	377	70	142
2	855	25	567	48	367	71	131
3	798	26	560	49	357	72	120
4	760	27	553	50	346	73	109
5	732	28	546	51	335	74	98
6	710	29	539	52	324	75	88
7	692	30	531	53	313	76	78
8	680	31	523	54	302	77	68
9	670	32	515	55	292	78	58
10	661	33	507	56	282	79	49
11	653	34	499	57	272	80	41
12	646	35	490	58	262	81	34
13	640	36	481	59	252	82	28
14	634	37	472	60	242	83	23
15	628	38	463	61	232	84	20
16	622	39	454	62	222	85	15
17	616	40	445	63	212	86	11
18	610	41	436	64	202	87	8
19	604	42	427	65	192	88	5
20	598	43	417	66	182	89	3
21	592	44	407	67	172	90	1
22	586	45	397	68	162	91	0
23	579	46	387	69	152		

The above table is well adapted to Europe in general; but in the city of London, there is observed to be a greater disparity in

the births and burials than in any other place, owing probably to the vast resort of people thither, in the way of commerce, from all parts of the known world. Mr Simpson, therefore, in order to have a table particularly suited to this populous city, pitches upon 1280 persons all born the same year, and records the number remaining alive each year, till none were in life; as in the following table:

Mr Simpson's table on the bills of mortality at London.

<i>Age.</i>	<i>Perf. liv.</i>	<i>A.</i>	<i>Perf. liv.</i>	<i>A.</i>	<i>Perf. liv.</i>	<i>A.</i>	<i>Perf. liv.</i>
0	1280	24	434	48	220	72	59
1	870	25	426	49	212	73	54
2	700	26	418	50	204	74	49
3	635	27	410	51	196	75	45
4	600	28	402	52	188	76	41
5	580	29	394	53	180	77	38
6	564	30	385	54	172	78	35
7	551	31	376	55	165	79	32
8	541	32	367	56	158	80	29
9	532	33	358	57	151	81	26
10	524	34	349	58	144	82	23
11	517	35	340	59	137	83	20
12	510	36	331	60	130	84	17
13	504	37	322	61	123	85	14
14	498	38	313	62	117	86	12
15	492	39	304	63	111	87	10
16	486	40	294	64	105	88	8
17	480	41	284	65	99	89	6
18	474	42	274	66	93	90	5
19	468	43	264	67	87	91	4
20	462	44	255	68	81	92	3
21	455	45	246	69	75	93	2
22	448	46	237	70	69	94	1
23	441	47	228	71	64	95	0

It may not be improper in this place to observe, that however perfect tables of this sort may be in themselves, and however well adapted to any particular climate, yet the conclusions deduced from them must always be uncertain, being nothing more than probabilities, or conjectures drawn from the usual period of human life. And the modish practice of buying and selling annuities on lives, by rules founded on such principles, may be justly considered as a sort of lottery or chance work, in which the parties concerned must often be deceived. But as estimates and computations

putations of this kind are now become fashionable, I shall here give some brief account of such as appear most material.

From the above tables, the probability of the continuance or extinction of human life, is estimated as follows:

1. The probability that a person of a given age shall live a certain number of years, is measured by the proportion which the number of persons living at the proposed age has to the difference between the said number and the number of persons living at the given age.

Thus, if it be demanded, what chance a person of 40 years has to live 7 years longer? from 445, the number of persons living at 40 years of age in Dr Halley's table, subtract 377, the number of persons living at 47 years of age, and the remainder, 68, is the number of persons that died during these 7 years; and the probability or chance that the person in the question shall live these 7 years is as 377 to 68, or nearly as $5\frac{1}{2}$ to 1. But by Mr Simpson's table the chance is something less than that of 4 to 1.

2. If the year to which a person of a given age has an equal chance of arriving before he dies, be required, it may be found thus: Find half the number of persons living at the given age in the tables, and in the column of age you have the year required.

Thus, if the question be put with respect to a person of 30 years of age, the number of that age in Dr Halley's table is 531, the half whereof is 265, which is found in the table between 57 and 58 years; so that a person of 30 years has an equal chance of living between 27 and 28 years longer.

3. By the tables, the premium of insurance upon lives may in some measure be regulated.

Thus, the chance that a person of 25 years has to live another year, is, by Dr Halley's table, as 80 to 1; but the chance that a person of 50 years has to live a year longer is only 30 to 1. And, consequently, the premium for insuring the former ought to be to the premium for insuring the latter for one year, as 30 to 80, or as 3 to 8.

4. By the tables may be computed the value of an annuity for life.

Thus, if it be required to find the value of an annuity of 1 l. for the life of a person of 30 years of age, interest 5 per cent. by Table V. in *Annuities certain*, find the value of 1 l. annuity for 1 year, at 5 per cent. viz. .952. Now, it is obvious, that this .952 would be the true value of the first year's annuity, provided the purchaser was sure of receiving payment: but the person may

M m 3

die;

die; and therefore this value must be lessened in proportion to the risk, by the following analogy from Dr Halley's table, viz. as 531 to 523, so .952 to .9375, the true value of the first year's annuity. In like manner may the value of the second year's annuity be found: for, by Table V. the value of 1 l. annuity for 2 years, at 5 per cent. is 1.8594; from which deduce the value of 1 year's annuity, viz. .9523, and there remains for the separate value of the second year's annuity .9071. Then, from the table, say, As 531 to 515, so .9071 to .8797, the true value of the second year's annuity. And by proceeding in this manner, the separate value of every year's annuity, to the utmost extent of life, may be found; the sum of all which will be the value of the annuity sought.

But the above method being tedious, and therefore unfit for practice, I shall here lay down another method, much shorter and easier, and shall deliver what seems necessary to be said on life-annuities in the way of problems. Previous to which it will be proper to observe, that in calculations of this kind, the age of 86 years is esteemed the utmost extent or limit of life; and the difference between the given age and 86 is called the *complement of life*.

P R O B. I.

To find the value of an annuity of 1 l. for the life of a single person of any given age.

Monf. de Moivre, by observing the decrease of the probabilities of life, as exhibited in the table, composed an algebraic theorem or canon, for computing the value of an annuity for life; which canon I shall here lay down by way of

R U L E.

Find the complement of life; and, by Table V. in *Annuities Certain*, find the value of 1 l. annuity for the years denoted by the said complement; multiply this value by the amount of 1 l. for a year, and divide the product by the complement of life; then subtract the quot from 1; divide the remainder by the interest of 1 l. for a year; and this last quot will be the value of the annuity sought; or, in other words, the number of years purchase the annuity is worth.

Ex. What is the value of an annuity of 1 l. for an age of 50 years, interest at 5 per cent.

86

50 age given.

—
36 complement of life.

By

By Table V. the value is, 16.5468
Amount of 1l. for a year, 1.05

		827340
		165468
Complement of life	36)	17.374140(.482615
From unity, viz.	1.000000	
Subtract		.482615

Complement of life $36)17.374140(.482615$

From unity, viz. 1.000000

Subtract 482615

Interest of 1l. .05), 517385 (10.3477 value fought.

By the preceding problem is constructed the following table.

28.51	19.51	20.51	00.51	12.51	10.51	1
02.51	00.51	14.51	13.51	17.51	15.51	2
12.51	10.51	14.51	11.51	20.51	18.51	3
21.51	17.51	14.51	17.51	13.51	21.51	4
01.51	13.51	14.51	12.51	00.51	00.51	5
11.51	12.51	14.51	13.51	00.51	11.51	6
20.51	19.51	14.51	12.51	10.51	10.51	7
00.51	11.51	14.51	11.51	11.51	02.51	8
00.51	18.51	14.51	12.51	11.51	23.51	9
17.51	18.51	14.51	10.51	10.51	01.51	10
20.51	00.51	14.51	10.51	10.51	10.51	11
02.51	02.51	14.51	10.51	10.51	08.51	12
03.51	10.51	14.51	12.51	10.51	20.51	13
04.51	10.51	14.51	14.51	10.51	14.51	14
12.51	10.51	14.51	14.51	10.51	20.51	15
22.51	10.51	14.51	14.51	10.51	20.51	16
11.51	02.51	14.51	14.51	10.51	00.51	17
11.51	20.51	14.51	14.51	10.51	10.51	18
20.51	20.51	14.51	14.51	10.51	10.51	19
00.51	10.51	14.51	14.51	10.51	04.51	20
00.51	00.51	14.51	14.51	10.51	00.51	21
00.51	00.51	14.51	14.51	10.51	20.51	22
02.51	10.51	14.51	14.51	10.51	10.51	23
22.51	10.51	14.51	14.51	10.51	10.51	24
24.51	10.51	14.51	14.51	10.51	10.51	25
26.51	10.51	14.51	14.51	10.51	10.51	26
20.51	00.51	14.51	14.51	10.51	10.51	27
00.51	10.51	14.51	14.51	10.51	10.51	28

The

The value of 1l. annuity for a single life.

Age.	3 per c.	3½ per c.	4 per c.	4½ per c.	5 per c.	6 per c.
9 = 10	19.87	18.27	16.88	15.67	14.60	12.80
8 = 11	19.74	18.16	16.79	15.59	14.53	12.75
7 = 12	19.60	18.05	16.64	15.51	14.47	12.70
13	19.47	17.94	16.60	15.43	14.41	12.65
6 = 14	19.33	17.82	16.50	15.35	14.34	12.60
15	19.19	17.71	16.41	15.27	14.27	12.55
16	19.05	17.59	16.31	15.19	14.20	12.50
5 = 17	18.90	17.46	16.21	15.10	14.12	12.45
18	18.76	17.33	16.10	15.01	14.05	12.40
19	18.61	17.21	15.99	14.92	13.97	12.35
4 = 20	18.46	17.09	15.89	14.83	13.89	12.30
21	18.30	16.96	15.78	14.73	13.81	12.20
22	18.15	16.83	15.67	14.64	13.72	12.15
23	17.99	16.69	15.55	14.54	13.64	12.10
3 = 24	17.83	16.56	15.43	14.44	13.55	12.00
25	17.66	16.42	15.31	14.34	13.46	11.95
26	17.50	16.28	15.19	14.23	13.37	11.90
27	17.33	16.13	15.04	14.12	13.28	11.80
28	17.16	15.98	14.94	14.02	13.18	11.75
29	16.98	15.83	14.81	13.90	13.09	11.65
30	16.80	15.68	14.68	13.79	12.99	11.60
2 = 31	16.62	15.53	14.54	13.67	12.88	11.50
32	16.44	15.37	14.41	13.55	12.78	11.40
33	16.25	15.21	14.27	13.43	12.67	11.35
34	16.06	15.05	14.12	13.30	12.56	11.25
35	15.86	14.89	13.98	13.17	12.45	11.15
36	15.67	14.71	13.82	13.04	12.33	11.05
37	15.46	14.52	13.67	12.90	12.21	11.00
38	15.29	14.34	13.52	12.77	12.09	10.90
1 = 39	15.05	14.16	13.36	12.63	11.96	10.80
40	14.84	13.98	13.20	12.48	11.83	10.70
41	14.63	13.79	13.02	12.33	11.70	10.55
42	14.41	13.59	12.85	12.18	11.57	10.45
43	14.19	13.40	12.68	12.02	11.43	10.35
44	13.96	13.20	12.50	11.87	11.29	10.25
45	13.73	12.99	12.32	11.70	11.14	10.10

The

The value of 1l. annuity for a single life.

A.	3 per c.	3½ per c.	4 per c.	4½ per c.	5 per c.	6 per c.
46	13.49	12.78	12.13	11.54	10.99	10.00
47	13.25	12.56	11.94	11.37	10.84	9.85
48	13.01	12.36	11.74	11.19	10.68	9.75
49	12.76	12.14	11.54	11.00	10.51	9.60
50	12.51	11.92	11.34	10.82	10.35	9.45
51	12.26	11.69	11.13	10.64	10.17	9.30
52	12.00	11.45	10.92	10.44	9.99	9.20
53	11.73	11.20	10.70	10.24	9.82	9.00
54	11.46	10.95	10.47	10.04	9.63	8.85
55	11.18	10.69	10.24	9.82	9.44	8.70
56	10.90	10.44	10.01	9.61	9.24	8.55
57	10.61	10.18	9.77	9.39	9.04	8.35
58	10.32	9.91	9.52	9.16	8.83	8.20
59	10.03	9.64	9.27	8.93	8.61	8.00
60	9.73	9.36	9.01	8.69	8.39	7.80
61	9.42	9.08	8.75	8.44	8.16	7.60
62	9.11	8.79	8.48	8.19	7.93	7.40
63	8.79	8.49	8.20	7.94	7.68	7.20
64	8.46	8.19	7.92	7.67	7.43	6.95
65	8.13	7.88	7.63	7.39	7.18	6.75
66	7.79	7.56	7.33	7.12	6.91	6.50
67	7.45	7.24	7.02	6.83	6.64	6.25
68	7.10	6.91	6.75	6.54	6.36	6.00
69	6.75	6.57	6.39	6.23	6.07	5.75
70	6.38	6.22	6.06	5.92	5.77	5.50
71	6.01	5.87	5.72	5.59	5.47	5.20
72	5.63	5.51	5.38	5.26	5.15	4.90
73	5.25	5.14	5.02	4.92	4.82	4.60
74	4.85	4.77	4.66	4.57	4.49	4.30
75	4.45	4.38	4.29	4.22	4.14	4.00
76	4.05	3.98	3.91	3.84	3.78	3.65
77	3.63	3.57	3.52	3.47	3.41	3.30
78	3.21	3.16	3.11	3.07	3.03	2.95
79	2.78	2.74	2.70	2.67	2.64	2.55
80	2.34	2.31	2.28	2.26	2.23	2.15

The

The value of 1l. annuity for a single life.

Age.	3 per c.	3½ per c.	4 per c.	4½ per c.	5 per c.	6 per c.
9 = 10	19.87	18.27	16.88	15.67	14.60	12.80
8 = 11	19.74	18.16	16.79	15.59	14.53	12.75
7 = 12	19.60	18.05	16.64	15.51	14.47	12.70
13	19.47	17.94	16.60	15.43	14.41	12.65
6 = 14	19.33	17.82	16.50	15.35	14.34	12.60
15	19.19	17.71	16.41	15.27	14.27	12.55
16	19.05	17.59	16.31	15.19	14.20	12.50
5 = 17	18.90	17.46	16.21	15.10	14.12	12.45
18	18.76	17.33	16.10	15.01	14.05	12.40
19	18.61	17.21	15.99	14.92	13.97	12.35
4 = 20	18.46	17.09	15.89	14.83	13.89	12.30
21	18.30	16.96	15.78	14.73	13.81	12.20
22	18.15	16.83	15.67	14.64	13.72	12.15
23	17.99	16.69	15.55	14.54	13.64	12.10
3 = 24	17.83	16.56	15.43	14.44	13.55	12.00
25	17.66	16.42	15.31	14.34	13.46	11.95
26	17.50	16.28	15.19	14.23	13.37	11.90
27	17.33	16.13	15.04	14.12	13.28	11.80
28	17.16	15.98	14.94	14.02	13.18	11.75
29	16.98	15.83	14.81	13.90	13.09	11.65
30	16.80	15.68	14.68	13.79	12.99	11.60
2 = 31	16.62	15.53	14.54	13.67	12.88	11.50
32	16.44	15.37	14.41	13.55	12.78	11.40
33	16.25	15.21	14.27	13.43	12.67	11.35
34	16.06	15.05	14.12	13.30	12.56	11.25
35	15.86	14.89	13.98	13.17	12.45	11.15
36	15.67	14.71	13.82	13.04	12.33	11.05
37	15.46	14.52	13.67	12.90	12.21	11.00
38	15.29	14.34	13.52	12.77	12.09	10.90
1 = 39	15.05	14.16	13.36	12.63	11.96	10.80
40	14.84	13.98	13.20	12.48	11.83	10.70
41	14.63	13.79	13.02	12.33	11.70	10.55
42	14.41	13.59	12.85	12.18	11.57	10.45
43	14.19	13.40	12.68	12.02	11.43	10.35
44	13.96	13.20	12.50	11.87	11.29	10.25
45	13.73	12.99	12.32	11.70	11.14	10.10

The

The value of 1l. annuity for a single life.

A.	3 per c.	3½ per c.	4 per c.	4½ per c.	5 per c.	6 per c.
46	13.49	12.78	12.13	11.54	10.99	10.00
47	13.25	12.56	11.94	11.37	10.84	9.85
48	13.01	12.36	11.74	11.19	10.68	9.75
49	12.76	12.14	11.54	11.00	10.51	9.60
50	12.51	11.92	11.34	10.82	10.35	9.45
51	12.26	11.69	11.13	10.64	10.17	9.30
52	12.00	11.45	10.92	10.44	9.99	9.20
53	11.73	11.20	10.70	10.24	9.82	9.00
54	11.46	10.95	10.47	10.04	9.63	8.85
55	11.18	10.69	10.24	9.82	9.44	8.70
56	10.90	10.44	10.01	9.61	9.24	8.55
57	10.61	10.18	9.77	9.39	9.04	8.35
58	10.32	9.91	9.52	9.16	8.83	8.20
59	10.03	9.64	9.27	8.93	8.61	8.00
60	9.73	9.36	9.01	8.69	8.39	7.80
61	9.42	9.08	8.75	8.44	8.16	7.60
62	9.11	8.79	8.48	8.19	7.93	7.40
63	8.79	8.49	8.20	7.94	7.68	7.20
64	8.46	8.19	7.92	7.67	7.43	6.95
65	8.13	7.88	7.63	7.39	7.18	6.75
66	7.79	7.56	7.33	7.12	6.91	6.50
67	7.45	7.24	7.02	6.83	6.64	6.25
68	7.10	6.91	6.75	6.54	6.36	6.00
69	6.75	6.57	6.39	6.23	6.07	5.75
70	6.38	6.22	6.06	5.92	5.77	5.50
71	6.01	5.87	5.72	5.59	5.47	5.20
72	5.63	5.51	5.38	5.26	5.15	4.90
73	5.25	5.14	5.02	4.92	4.82	4.60
74	4.85	4.77	4.66	4.57	4.49	4.30
75	4.45	4.38	4.29	4.22	4.14	4.00
76	4.05	3.98	3.91	3.84	3.78	3.65
77	3.63	3.57	3.52	3.47	3.41	3.30
78	3.21	3.16	3.11	3.07	3.03	2.95
79	2.78	2.74	2.70	2.67	2.64	2.55
80	2.34	2.31	2.28	2.26	2.23	2.15

The

The above table shews the value of an annuity of one pound for a single life, at all the current rates of interest; and is esteemed the best table of this kind extant, and preferable to any other of a different construction. But yet those who sell annuities have generally one and a half or two years more value, than specified in the table, from purchasers whose age is 20 years or upwards.

Annuities of this sort are commonly bought or sold at so many years purchase; and the value assigned in the table may be so reckoned. Thus the value of an annuity of one pound for an age of 50 years, at 3 per cent. interest, is 12.51; that is, 12l. 10s. or twelve and a half years purchase. The marginal figures on the left of the column of age serve to shorten the table, and signify, that the value of an annuity for the age denoted by them, is the same with the value of an annuity for the age denoted by the numbers before which they stand. Thus the value of an annuity for the age of 9 and 10 years is the same; and the value of an annuity for the age of 6 and 14, for the age of 3 and 24, &c. is the same. The further use of the table will appear in the questions and problems following.

Quest. 1. A person of 50 years would purchase an annuity for life of 200l.: What ready money ought he to pay, reckoning interest at $4\frac{1}{2}$ per cent.?

L.
By the table the value of 1l. is 10.82
Multiply by 200

Value to be paid in ready money 2164.00 *Ans.*

Quest. 2. A young merchant marries a widow lady of 40 years of age, with a jointure of 300l. a-year, and wants to dispose of the jointure for ready money: What sum ought he to receive, reckoning interest at $3\frac{1}{2}$ per cent.?

L.
By the table the value of 1l. is 13.98
300

Value to be received in ready money 4194.00 *Ans.*

If an annuity for life, together with a reversion for a term of years, or the reversion by itself, is offered; in this case reduce the years purchase found in the table, to years certain, by Table V.; and to the years certain add the years in reversion; and the present worth corresponding to the sum of the years in Table V. will be the value of the life and reversion; from which, if the value of the life be subtracted, there will remain the value of the reversion.

Quest. 3. What is the present worth or value of an estate of 80l. yearly rent, together with a reversion of 20 years after the death of the present possessor, supposed to be 40 years of age? and what

is

is the value of the reversion by itself, reckoning interest at 5 per cent.

The value of the life by the above table is 11.83; and the nearest value to this in Table V. under 5 per cent. is 11.6895869; opposite to which is 18 years, and $18 + 28 = 38$ years; and by Table V. the value of 1 l. annuity for 38 years at 5 per cent. is 16.8678926; which multiplied by 80 gives 1349.4314080 l. for the value of the life and reversion.

To find the value of the reversion.

	From	16.8678926
	Subtract	11.6895869
Value of 1 l. reversion		5.1783057
	Multiply by	80
Value of 80 l. reversion		414.2644560

If the value of a reversion in fee-simple, or for ever, after a life of a given age, be required; from the value of the fee-simple, or perpetuity, subtract the value of the life in possession; and the remainder will be the value of the reversion.

Quest. 4. A widow lady of 60 years of age is in possession for life of an estate of L. 500 per annum; What is the value of the reversion in fee-simple, interest at 5 per cent?

	L.
Value of 1 l. in fee, is,	20
Value of the life by the table, is,	8.39
Value of 1 l. in reversion,	11.61
	Multiply by 500
Value of 500 l. in reversion,	5805.00

If the reversion depends on two joint lives, or on the longest of two lives, find the value of the joint lives, or of the longest of the two lives, by Prob. II. or Prob. III. following, and subtract the value so found from the value of the perpetuity, or estate in fee-simple.

P R O B. II.

To find the value of an annuity for the joint continuance of two lives; one life failing, the annuity to cease.

Here there are two cases, according as the ages of the two persons are equal or unequal.

1. If the two persons be of the same age, work by the following

R U L E.

R U L E.

Take the value of any one of the lives from the table, multiply this value by the interest of 1 l. for a year, subtract the product from 2, divide the foresaid value by the remainder, and the quot will be the value of 1 l. annuity, or the number of years purchase sought.

Ex. What is the value of 100 l. annuity for the joint lives of two persons, of the age of 30 years each, reckoning interest at 4 per cent.?

By the table, one life of 30 years, is,	-	14.68
Multiply by	-	.04
Subtract the product	-	.5872
From	-	2.0000
Remains	-	1.4128

And $1.4128 \times 14.68 = 10.39$ value of 1 l. annuity,

And $10.39 \times 100 = 1039$ the value sought.

2. If the two persons are of different ages, work as directed in the following

R U L E.

Take the values of the two lives from the table, multiply them into one another, calling the result the first product; then multiply the said first product by the interest of 1 l. for a year, calling the result the second product; add the values of the two lives, and from their sum subtract the second product; divide the first product by the remainder, and the quot will be the value of 1 l. annuity, or the number of years purchase sought.

Ex. What is the value of 70 l. annuity for the joint lives of two persons, whereof one is 40, and the other 50 years of age, reckoning interest at 5 per cent.?

By the table, the value of 40 years is,	-	11.83
And the value of 50 years is,	-	10.35
First product,	-	122.4405
Multiply by	-	.05
Second product,	-	6.122025
Sum of the two lives,	-	22.180000
Second product deduct,	-	6.122025
Remainder,	-	16.057975

And

And $16.057975 \times 122.4405$ (7.62 value of 1 l. annuity.

70

533.40 value sought.

P R O B. III.

To find the value of an annuity upon the longest of two lives ; that is, to continue so long as either of the persons is in life.

R U L E.

From the sum of the values of the single lives, subtract the value of the joint lives, and the remainder will be the value sought.

Ex. What is the value of an annuity of 1 l. upon the longest of two lives, the one person being 30, and the other 40 years of age, interest at 4 per cent. ?

By the table, 30 years is,	-	-	-	14.68
40 years is,	-	-	-	13.20

27.88

Value of their joint lives, by Prob. II. Case 2. is,	9.62
--	------

Value sought,	-	-	-	-	18.26
---------------	---	---	---	---	-------

If the annuity be any other than 1 l. multiply the answer found as above by the given annuity.

If the two persons be of equal age, find the value of their joint lives by Case 1. of Prob. II.

P R O B. IV.

To find the value of the next presentation to a living.

R U L E.

From the value of the successor's life, subtract the joint value of his and the incumbent's life, and the remainder will be the value of 1 l. annuity ; which multiplied by the yearly income, will give the sum to be paid for the next presentation.

Ex. A enjoys a living of 100 l. per annum, and B would purchase the said living for his life after A's death : The question is, What he ought to pay for it, reckoning interest at 5 per cent. A being 60, and B 25 years of age ?

By the table, B's life is,	-	-	-	L.
Joint value of both lives, by Prob. II. is,	-	-	-	13.46

6.97

The value of 1 l. annuity,	-	-	-	6.49
Multiply by	-	-	-	100

Value of next presentation,	-	-	-	649.00
-----------------------------	---	---	---	--------

The

The value of a direct presentation is the same as that of any other annuity for life, and is found by 1 l. by the table; which being multiplied by the yearly income, gives the value sought.

P R O B. V.

To find the value of a reversion for ever, after two successive lives; or to find the value of a living after the death of the present incumbent and his successor.

R U L E.

By Prob. III. find the value of the longest of the two lives, and subtract that value from the value of the perpetuity, and the remainder will be the value sought.

Ex. A, aged 50, enjoys an estate or living of 100 l. *per annum*; B, aged 30, is entitled to his lifetime of the same estate after A's death; and it is proposed to sell the estate just now with the burden of A and B's lives on it: What is the reversion worth, reckoning interest at 4 *per cent.*?

By the table, A's life of 50 is,	-	-	L!	11.34
B's life of 30 is,	-	-	-	14.68
			Sum,	26.02
Value of their joint lives, found by Prob. II.	}			8.60
Case 2. is,				17.42 sub.
Value of the longest life,	-	-	-	25.00
From the value of the perpetuity,	-	-	-	7.58
Remains the value of 1 l. reversion,	-	-	-	100
Multiplied by	-	-	-	758.00
Value of the reversion,	-	-	-	

P R O B. VI.

To find the value of the joint continuance of three lives, one life failing, the annuity to cease.

R U L E.

Find the single values of the three lives from the table; multiply these single values continually, calling the result the product of the three lives; multiply that product by the interest of 1 l. and that product again by 2, calling the result the double product; then, from the sum of the several products of the lives, taken two and two, subtract the double product; divide the product of the three lives by the remainder, and the quot will be the value of the three joint lives.

Ex.

Ex. A is 18 years of age, B 34, and C 56 : What is the value of their joint lives, reckoning interest at 4 per cent. ?

By the table, the value of A's life is 16.1, of B's 14.12, and of C's 10.01.

$$16.1 \times 14.12 \times 10.01 = 2275.6 \text{ product of the 3 lives.}$$

$$\begin{array}{r} .04 \\ \hline 91.024 \\ 2 \end{array}$$

182.048 double product.

$$\text{Product of A and B, } 16.1 \times 14.12 = 227.33$$

$$\text{A and C, } 16.1 \times 10.01 = 161.16$$

$$\text{B and C, } 14.12 \times 10.01 = 141.34$$

$$\begin{array}{r} \text{Sum of all, two and two,} \quad - \quad 529.83 \\ \text{Double product subtract,} \quad - \quad 182.048 \\ \hline \end{array}$$

$$\text{Remainder,} \quad - \quad 347.782$$

And $347.782 \div 2275.600 = 6.54$ value sought.

P R O B. VII.

To find the value of an annuity upon the longest of three lives.

R U L E.

From the sum of the values of the three single lives taken from the table, subtract the sum of all the joint lives taken two and two, as found by Prob. II. and to the remainder add the value of the three joint lives, as found by Prob. VI. and that sum will be the value of the longest life sought.

Ex. A is 18 years of age, B 34, and C 56 : What is the value of the longest of these three lives, interest at 4 per cent. ?

$$\begin{array}{r} \text{By the table, the single value of A's life is,} \quad 16.1 \\ \text{single value of B's life is,} \quad 14.12 \\ \text{single value of C's life is,} \quad 10.01 \\ \hline \text{Sum of the single values,} \quad 40.23 \end{array}$$

$$\begin{array}{r} \text{By Prob. II. the joint value of A and B is,} \quad 10.76 \\ \text{joint value of A and C is,} \quad 8.19 \\ \text{joint value of B and C is,} \quad 7.65 \\ \hline \text{Sum of the joint lives,} \quad 26.60 \end{array}$$

$$\text{Remainder,} \quad - \quad 13.63$$

$$\text{By Prob. VI. the value of the 3 joint lives is,} \quad 6.54$$

$$\text{Value of the longest of the three lives,} \quad - \quad 20.17$$

Other

Other problems might be added, but these adduced are the more useful, and sufficient for most purposes. The reader probably may wish that the reason of the rules, which, it must be owned, are intricate, had been assigned; but this could not be done without entering deeper into the subject than was practicable in this place. They who want further satisfaction on this head may have recourse to Mons. de Moivre's Doctrine of Chances, and other authors on this subject.

I shall now conclude *Life-Annuities* by presenting the reader with a table, shewing how many years of an annuity, or annual income, will reimburse the purchaser or annuitant of the price or sum paid for the annuity, with the compound interest arising from the price as a capital or principal sum; that is, the table shews how many years and days possession will indemnify the purchaser, or save him from being a loser.

The possession that will reimburse the annuitant.

Years purch.	3 per c.		3½ perc.		4 per c.		4½ perc.		5 per c.		6 per c.	
	Y.	da.	Y.	da.	Y.	da.	Y.	da.	Y.	da.	Y.	da.
5	5	182	5	216	5	252	5	289	5	327	6	44
5½	6	37	6	79	6	122	6	168	6	216	6	319
6	6	261	6	311	6	364	7	55	7	113	7	241
6½	7	124	7	184	7	247	7	314	8	20	8	176
7	7	356	8	62	8	137	8	217	8	303	9	127
7½	8	227	8	311	9	34	9	129	9	231	10	95
8	9	104	9	200	9	304	10	51	10	172	11	81
8½	9	350	10	97	10	217	10	348	11	125	12	88
9	10	236	11		11	138	11	290	12	92	13	119
9½	11	128	11	274	12	69	12	245	13	75	14	177
10	12	24	12	191	13	9	13	212	14	75	15	265
10½	12	292	13	115	13	324	14	194	15	94	17	23
11	13	200	14	48	14	286	15	190	16	134	18	188
11½	14	115	14	354	15	259	16	203	17	196	20	36
12	15	36	15	305	16	246	17	234	18	285	21	309
12½	15	329	16	265	17	246	18	285	20	38	23	289
13	16	264	17	235	18	261	19	358	21	189	25	360
13½	17	206	18	216	19	292	21	90	23	13	28	183
14	18	156	19	209	20	340	22	215	24	247	31	164
14½	19	115	20	215	22	43	24	5	26	168	35	5
15	20	82	21	234	23	132	25	195	28	151	39	189
15½	21	59	22	267	24	245	27	60	30	209	45	233
16	22	45	23	316	26	18	28	336	32	360	55	88
16½		41	25	16	27	185	30	300	35	264	79	12

The use of the Table.

The left-hand column contains the number of years purchase paid for the annuity, or the number of pounds paid in ready money for each pound of the annuity; and the other columns show how long the annuitant must be in possession before he come to be reimbursed of the price paid, and the interest thence arising, reckoned at all the usual rates; the use whereof will appear in the solution of the following, or like questions.

Quest. 1. A person buys an annuity for 10 years purchase: How long must he enjoy it, in order to be reimbursed of the price, reckoning interest at $3\frac{1}{2}$ per cent.?

Under $3\frac{1}{2}$ per cent. and opposite to 10 on the left, you have the answer, 12 years and 191 days.

Quest. 2. How many years possession will reimburse an annuitant, who pays 16 $\frac{1}{2}$ years purchase, interest at 5 per cent.? *Ans.* 35 years 264 days.

C H A P. XIV.

M E N S U R A T I O N.

EVERY magnitude is measured by some magnitude of the same kind. A line by a lineal foot, yard, &c. A superficies, or surface, by a square foot, yard, &c. A solid by a cubic foot, yard, &c.

The lineal or running measure is known to all, and needs neither direction nor example. There remains then the superficial and solid measure to be explained.

Superficial measure may be conceived, by imagining a floor paved with tiles, each a foot square; for then the number of tiles will be equal to the number of square feet in that floor. Now, if the floor be just one foot broad, the number of tiles, or square feet, will be equal to the number of lineal feet in the length of the floor. If the floor be two or three feet broad, the number of tiles will be twice or thrice as many. And universally, the number of feet in length multiplied by the number of feet in breadth will give the number of tiles or square feet in the floor.

Solid measure may be conceived by imagining a wall built with stones, each a cubic foot; for then the number of stones will be equal to the number of cubic feet in the wall. First then, if the wall be one foot thick, and one foot high, the number of stones, or cubic feet, will be equal to the number of lineal feet in the length of the wall. Secondly, if the wall be of the same length and height, as before, *viz.* one foot, but two or three feet thick, then the number of stones, or cubic feet, will be twice or thrice as many as in the first supposition. Thirdly, if the length and thickness be the same as in the last supposition, but the height two or

N n

three

three feet, then the number of stones, or cubic feet, in the wall, will accordingly be twice or thrice as many as in the last supposition. And universally, the number of feet in length multiplied by the number of feet in thickness, and that product by the number of feet in height, will give the number of stones, or cubic feet, in the wall.

In treating of mensuration, I shall enumerate the various sorts of surfaces and solids, give short descriptions of them, with such draughts or figures as will in some measure show the reason of the rules. In the examples, I shall use absolute or abstract numbers, that may represent any sort of measure; as, inches, feet, yards, &c. After this general sketch of mensuration, I shall apply the doctrine to surveying, to the measuring of artificers work, to the measuring of board and timber, and to gauging.

I. Superficial Measure.

Problems, shewing how to find the superficial content, or area.

I. A square, or four-sided figure, whose angles are right, and sides equal; as, A B C D. Plate 1.

Rule. Multiply the side into itself, the product is the area, or content.

Ex. Multiply the side A B = 8
by itself 8

Area or content 64

II. A rectangle, oblong, or rectangular parallelogram; viz. a four-sided figure, whose angles are right, and whose opposite sides only are equal; as, A B C D. Pl. 1.

Rule. Multiply the length by the breadth, and the product is the area.

Ex. Multiply the length A B = 24
by the breadth B D = 8

Area 192

III. A rhombus, or four-sided figure, whose sides are equal, but the angles not right; as A B C D. Pl. 1.

Rule. Multiply one side into the perpendicular height, the product is the area.

Ex. Multiply the side A B = 9.5
by the height A E or B G = 7.8

760
665

Area 74.10

IV. A

IV. A rhomboides, or parallelogram not rectangular; viz. a four-sided figure, whose angles are not right, and whose opposite sides only are equal; as A B C D. Pl. 1.

Rule. Multiply the length by the perpendicular height or breadth, the product is the area.

Ex. Multiply the length A B = 12.6
by the height A E or B G = 4.3

$$\begin{array}{r} 50.6 \\ \frac{1}{3} = 4.2 \end{array}$$

Area 54.8

V. A right-angled triangle, or triangle wherein one of the angles is right; as A B C. Pl. 1.

Rule. Multiply the perpendicular into the base, half the product is the area.

Or, Multiply half the perpendicular into the base, or half the base into the perpendicular, the product is the area.

Ex. Multiply the perpendicular A B = 6.4
by the base B C = 8.5

$$\begin{array}{r} 320 \\ 512 \end{array}$$

$$2)54.40$$

27.2 area.

Or, Mult. half A B = 3.2
by the base B C = 8.5

$$\begin{array}{r} 160 \\ 256 \end{array}$$

27.20 area.

Or, Mult. half B C = 4.25
by A B = 6.4

$$\begin{array}{r} 1700 \\ 2550 \end{array}$$

27.200 area.

Note. In a right-angled triangle, if you add the squares of the base and perpendicular, the square root of that sum will be the hypotenuse, or longest side. *Euclid* I. 47.

VI. An oblique-angled triangle, or triangle wherein none of the angles is right; as, A B C. Pl. 1.

Rule. Drop a perpendicular from any one of the angles upon the base, produced, if need be, as in fig. 2.; then proceed as in right-angled triangles, viz. multiply the perpendicular into the base, half the product is the area.

N n 2

Or,

Or, Multiply half the perpendicular into the base, or half the base into the perpendicular, the product is the area.

Ex. Multiply the base $AB = 14.8$
by the perpendicular $CD = 5.4$

$$\begin{array}{r} 592 \\ 740 \\ \hline \end{array}$$

$$2)79.92$$

39.96 area.

Or, Multiply the base $AB = 14.8$
by half the perpendicular $CD = 2.7$

$$\begin{array}{r} 1036 \\ 296 \\ \hline \end{array}$$

39.96 area.

Or, Multiply half the base $AB = 7.4$
by the perpendicular $CD = 5.4$

$$\begin{array}{r} 296 \\ 370 \\ \hline \end{array}$$

39.96 area.

VII. The three sides of any triangle being given, to find the area, without having recourse to the perpendicular. Pl. I.

Rule. Add the three given sides, and from their half-sum subtract the three sides severally; multiply the half-sum and the three remainders into one another continually; the square root of the last product is the area.

Ex. $AB = 21$

$BC = 17$

$AC = 10$

$$2)48$$

24 half-sum.

24

10

14 rem.

24

17

7 rem.

24

21

3 rem.

$$\text{And } 24 \times 14 \times 7 \times 3 = 7056.$$

$$\text{And } \sqrt{7056} (84 \text{ root} = \text{area.})$$

64

$$\begin{array}{r} 164)656 \\ 656 \\ \hline \end{array}$$

VIII. A

VIII. A trapezium, *viz.* any four-sided figure that is neither a square, rectangle, rhombus, nor rhomboides; as, A B C D, Pl. 1.

Rule. Multiply the diagonal into the half-sum of the two perpendiculars, or the contrary; the product is the area.

Ex. Multiply the diagonal B D = 28.4
by half of A a 7 + half of C a 6 = 6.5

1420
1704

Area 184.60

IX. A parallelopleuron, or trapezium that has two of its sides parallel, being a segment of a triangle cut by a line drawn parallel to the base; as, A B C D. Pl. 1.

Rule. Multiply the half-sum of the two parallel sides by the perpendicular height.

Ex. Multiply half A B 12 + half C D 22 = 17
by the perpendicular B E = 13

51
17

Area 221

Note. The parallelopleuron may also be a segment of a right-angled triangle; and in this case B D is the perpendicular, which multiplied into half the sum of A B and C D, will give the area.

X. A polygram, or irregular polygon, *viz.* a figure bounded by five or more unequal sides; as, A B C D E F. Pl. 1.

Rule. Divide all such many-sided, multangular, and irregular figures, into trapeziums and triangles, then measure them by Prob. 6. and 8.

Ex. Divide the polygram A B C D E F into the trapezium A B C F, and the two triangles C D F and F D E; drop the perpendiculars A a, C a, D r, and D m; then measure the trapezium and triangles, as taught in Prob. 8. and 6.; and the sum of their areas will be the area of the given polygram.

XI. A regular polygon, or ordinate figure whose sides and angles are all equal; as, A B C D E. Pl. 1.

Such polygons take their names from the number of their sides, or rather angles. Thus, if the polygon has 3 sides, or angles, it is called a *trigon*, or *equilateral triangle*; if the polygon have 4 sides, it is called a *tetragon*, or *square*; if 5, a *pentagon*; if 6, a *hexagon*; if 7, a *heptagon*; if 8, an *octagon*; if 9, an *enneagon*; if 10, a *decagon*; if 11, an *endecagon*; if 12, a *dodecagon*, &c.

N n 3

Rule

Rule. Bisect any two adjacent angles, and from the point where the bisecting lines meet, drop a perpendicular upon the base, multiply half the sum of the sides into the perpendicular, and the product is the area of the polygon.

Ex. The side of a pentagon is $= 16.4$
Multiply by $\frac{1}{2}$ the number of sides 2.5

820
328

Half sum of the sides 41.00
Multiply by the perpend. m n 11.3

123
41
41

Area of the pentagon 463.3

Hence every polygon may be resolved into as many equal triangles as it has sides, the sum of whose areas will be equal to the area of a right-angled triangle, one of whose legs is the perimeter or sum of the sides, and the other the radius or semidiameter of the inscribed circle.

If the side of the several regular polygons be 1, and their respective areas be computed by trigonometry, these areas may be inserted in a table, and will be so many multipliers for the ready finding the area of any other like polygon. The table of areas or multipliers follows :

<i>Names.</i>	<i>Multipliers.</i>	<i>Names.</i>	<i>Multipliers.</i>
Trigon	.433013	Octagon	4.828427
Tetragon	1.000000	Enneagon	6.181827
Pentagon	1.720475	Decagon	7.694209
Hexagon	2.598076	Endecagon	8.514250
Heptagon	3.633959	Dodecagon	9.330125

The areas of like polygons are to one another as the squares of their homologous or like sides, Euclid VI. 20 ; but the side of the polygons in the table is 1, whose square is 1 ; and therefore the square of the side of any given polygon multiplied into the tabular area of the like polygon will produce the area thereof.

Ex. Required the area of a hexagon whose side is 10.

The square of 10, or $10 \times 10 = 100$; and $100 \times 2.598076 = 259.8076$ the area sought ; and in like manner may the area of any other regular polygon be found.

XII. A circle, the most capacious of all figures, is bounded by one curve line, called the *circumference*, or *periphery*, to which all lines drawn from the middle point or centre are equal; as A D B E. Pl. 1. fig. 1.

The line A B passing through the centre C, and terminated on both sides by the circumference, is called the *diameter*; and half that line, *viz.* A C, drawn from the centre to the circumference, is called the *semidiameter* or *radius*. The diameter divides the circle into two equal parts called *semicircles*; as A D B and A B E.

Rule. Multiply the radius into half the periphery, or the periphery into half the radius; the product is the area.

The reason is obvious: for a circle differs not from, or is the same with, a regular polygon of an infinite number of sides.

Ex. Required the area of a circle whose diameter is 20, and periphery 62.8318.

Half the diameter or radius is 10; and half the periphery is 31.4159; and $31.4159 \times 10 = 314.159$, the area sought.

Or, half the radius is 5, and $62.8318 \times 5 = 314.159$, as before.

If the area of a semicircle, as A B D, be required, multiply the radius A C into half the semicircular arch A D B; or, multiply the square of the diameter A B into .3927; either of these products is the area; or, find the area of the whole circle, and then take its half.

If the area of a quadrant, as A C E, be demanded, multiply the radius A C into half the quadrantal arch A E; or, multiply the square of the diameter A B into .19635; either of these products is the area: or, find the area of the whole circle, and then take one-fourth of it.

A great many other questions relative to the diameter, periphery, and area of circles, may occur; most of which may be solved by one or other of the proportions following:

1. As 7 to 22, nearly; or, as 113 to 355, more nearly; or, as 1 to 3.14159, still more nearly; so the diameter of a circle to the circumference.

2. As 22 to 7; or, as 355 to 113; or as 3.14159 to 1; or, as 1 to .31831; so the circumference to the diameter.

3. As 1 to 3.14159, so the square of the radius to the area of the circle.

4. As 1 to .0795775, so the square of the periphery to the area of the circle.

5. As 14 to 11; or, as 452 to 355; or, as 1 to .785398, or, for practice, to .7854; so the square of the diameter, *viz.* the area of the circumscribed square A B C D, fig. 2. to the area of the circle.

Note. .7854 is the area of a circle whose diameter is unity, and circles are to one another as the squares of their diameters, Euclid XII. 2.

6. As 1 to .7071, so the diameter of a circle to the side of the inscribed square m n o p. Or, Multiply the square of the semidiameter

N n 4

diameter

diameter by 2, and the square root of the product is the side of the inscribed square.

7. As 1 to .2251, so the periphery of a circle to the side of the inscribed square.

8. As 1 to 1.2732, so the area of a circle to the square of the diameter.

9. As 1 to 12.56637; or, as .0795775 to 1; so the area of a circle to the square of the periphery.

10. As 1 to .6366, so the area of a circle to the area of the inscribed square.

11. As 1 to 1.4142, so the side of a given square to the diameter of the circumscribing circle.

12. As 1 to 4.443, so the side of a square to the periphery of the circumscribing circle.

13. As 1 to .8862, so $m n$, the diameter of a circle, to $A B$, the side of a square, equal to the circle in area; fig. 3.

14. As 1 to .2821, so the periphery of a circle to the side of a square equal in area.

15. As 1 to 1.128, so the side of a square to the diameter of a circle equal to the square in area.

16. As 1 to 3.545, so the side of a square to the periphery of a circle equal in area.

XIII. A sector of a circle; as $A C B E$, or $A C B D$. Pl. 2. fig. 1.

Rule. Multiply the radius $A C$ into half the arc $A E B$, the product is the area of the sector $A C B E$; which subtracted from the area of the whole circle, leaves the area of the sector $A C B D$.

Ex. Suppose the radius $A C$ to be 10, and half the arc $A E B$ to be 10.47198; then $10 \times 10.47198 = 104.7198$, the area of the sector $A C B E$; which subtracted from the area of the whole circle, found by Prob. 12. viz. 314.159, leaves 209.4393, the area of the sector $A C B D$.

If the area of the segment $A B E$, or $A B D$, be required, from the area of the sector $A C B E$, found above, subtract the area of the triangle $A C B$, and there will remain the area of the segment $A B E$; which subtracted from the area of the whole circle, will leave the area of the segment $A B D$.

The length of the arch line of any sector or segment, as $A C B$, fig. 2. may be found thus:

Divide the chord-line $A B$ into four equal parts, and set one of these parts from A to D , and from B to C ; join $D C$; which line $D C$ will be equal to half the arch line $A C B$.

Or, if the arch line be given in degrees, say, As 360, the periphery in degrees, to the periphery in measure, found by Prob. 12, so the degrees of the arch line to the arch line in measure.

If $A B$, the chord of an arc $A C B$, fig. 3. be given, and also the

the versed sine D C, the diameter of the whole circle may be found thus :

As D C : D A :: D A : D E.

And D E + D C = C E, the diameter.

Again, D E : D A :: D A : D C.

And the square root of D C $\times$ D E = D A, or D B; Euclid III. 31. and VI. 8. Cor.

XIV. A lune, or space bounded by arcs of two circles of a different radius, like the falcated moon ; as, A D B E. Pl. 2.

Rule. Find C and K, the centres of the two circles A E B L and A D B M, and complete the circles.

Then, by Prob. 12. or 13. find the area of the semicircle or segment A C B E, and also the area of the segment A C B D; subtract the one from the other, and the remainder is the area of the lune.

XV. A ring, or space included between the circumferences of two concentric circles ; as, a b c d, Pl. 2. fig. 1.

Rule. Find the area of both circles, by Prob. 12. ; subtract the lesser from the greater ; the remainder is the area of the ring.

If such a portion of the ring as d be required, find the area of both sectors, and subtract the lesser from the greater.

If a portion of the ring, as b, be demanded, find the area of both segments, and subtract the lesser from the greater.

If the area of a mixed or compound figure be required, viz. a figure bounded partly by right lines, and partly by circular arcs, as A B C D, fig. 2. resolve such a figure into triangles, trapezia, and into sectors or segments ; then find the area of these severally ; and their sum will be the area of the compound figure sought.

XVI. An ellipse or oval, viz. any thorough plane section of a cone, not parallel to the base ; as A B C D, Pl. 2. fig. 1.

Rule. Multiply the longer axis or transverse diameter A B into the shorter axis or conjugate diameter C D ; then multiply that product by .7854 ; and this last product is the area.

Ex. Suppose the transverse diameter A B to be 26, and the conjugate D C 20, required the area.

$26 \times 20 = 520$, and $520 \times .7854 = 408.4080$, the area of the given ellipse.

The conic writers demonstrate the following proportions, viz.

The area of any ellipse is a mean proportional between the area of the circumscribing and inscribed circle, viz. a circle on the transverse, and another on the conjugate diameter ; as A C B D and c m d n ; fig. 2.

2. As the longer axis to the shorter, that is, as C D (= A B) to

to $c d$, so any chord-line of the circle $b b$ (parallel to $c d$) to $r r$. And as $m n (= c d)$ to $A B$, so $o o$ (parallel to $A B$) to $r t$.

3. As $C D$ to $c d$, or as $b b$ to $r r$, so the area of the circular segment $b A b$ to the area of the elliptical segment $r A r$.

4. As $m n$ to $A B$, or as $o o$ to $r t$, so the area of the circular segment $o d o$ to the area of the elliptical segment $r d t$.

If the elliptical area included between two parallels be required, by the third or fourth of the above proportions, find the area of both segments, and subtract the lesser from the greater.

XVII. A parabola, *viz.* any section of a cone parallel to a plane, applied to the outside of the cone; as $A C B$. Pl. 2.

Rule. Multiply the base, or greatest ordinate, $A B$, into the perpendicular height $D C$, and two thirds of the product is the area, the parabola being equal to two thirds of the circumscribing rectangle $A E G B$.

Ex. Suppose $A B$ to be 18.5, and $D C$ 19; required the area of the parabola.

$$\begin{array}{r}
 18.5 \\
 19 \\
 \hline
 1665 \\
 185 \\
 \hline
 3)351.5 \\
 \hline
 117.1\bar{6} \\
 117.1\bar{6} \\
 \hline
 234.3 \text{ area.}
 \end{array}$$

Or, Multiply 351.5 by $\frac{2}{3} = \frac{6}{9}$.

$$\begin{array}{r}
 351.5 \\
 6 \\
 \hline
 9)2109.0 \\
 \hline
 234.3 \text{ area.}
 \end{array}$$

If the parabolic area included between two parallels $A B$ and $m n$, be required, find the area both of the parabola $A C B$ and $m C n$, and subtract the lesser from the greater.

If a line be drawn from C to A , and another from C to B , the triangle $A C B$ will be three fourths of the parabola.

XVIII. A cycloid; as $A D B E$. Pl. 2.

Rule. Find, by Prob. 12. the area of the circle C , described on the axis $D E$, multiply that area by 3, and the product is the area of the cycloid.

Note. The cycloid is formed by applying any circle, as C , to the line $A B$, so as the point n may coincide with the point A ; and then, rolling the circle, as a wheel, along the line $A B$, till the point n complete a revolution, and coincide with B . The path described by the point n is the cycloidal curve; $A B$ the base, $D E$ the axis, the point E the vertex, and C the generating circle.

XIX. The

XIX. The surface of a cylinder, or solid, like a rolling stone, formed by the revolution of a rectangle round one of its sides; as $A B C D$. Pl. 2.

The surface of a cylinder, excluding the bases, when spread out, becomes a rectangle, one of whose sides is the periphery of the circular base, and the other the same with the height or length of the cylinder.

Rule. Multiply the periphery of the base into the height, and to the product add twice the area of the base; the sum is the area sought.

XX. The surface of a cone, or solid like a sugar-loaf, formed by the revolution of a right-angled triangle round one of its legs; as $A B C$. Pl. 2.

The surface of a cone, excluding the circular base, when spread out, becomes a sector of a circle, whose terminating arc is the periphery of the base, and whose radius is $A B$ or $A C$, the length of the outside of the cone, or hypotenuse of the generating triangle. The point A is called the *vertex*, and $A o$ the *axis*, of the cone.

Rule. Multiply the periphery of the base into $A B$, the length of the side; and to half the product add the area of the base; the sum is the area sought. See Prob. 12. and 13.

If the surface of the frustum of a cone, cut by a plane parallel to the base, as $m n B C$, be required, find the curve surface of the whole cone $A B C$, and also of the cone $A m n$, cut off; from the former subtract the latter; to the remainder add the area of the two bases; and the sum is the area of the frustum.

The axis $A o$, or height of the whole cone, is found by saying, As the semidifference of the diameters of the greater and lesser base to the perpendicular height of the frustum; so the semidiameter of the greater base to $A o$, from which subtract the height of the frustum, and the remainder will be the height of the cone cut off; whence $A m$ or $A n$ may be found by the note in Prob. 5.

Or, $A m$ may be directly found by saying, As the semidifference of the two diameters to $B m$, so the semidiameter of the greater base to $A B$; from which subtract $B m$, and the remainder will be $A m$.

The area of the outside-surface of the frustum of a cone may be found more readily thus: Add the peripheries of the two bases, and multiply half their sum by the outside-height $B m$, and to the product add the area of the two bases.

XXI. The surface of a sphere or globe, *viz.* a round body formed by revolving a semicircle round its diameter; as $A B C D$. Pl. 2.

Rule. Multiply the periphery of the globe into the axis or diameter $A B$, the product is the area.

Or,

Or, Find the area of a circle whose diameter is the same with that of the globe, and multiply the area so found by 4.

Example. Suppose the diameter A B to be 20, then by Prob. 12. the periphery will be 62.8318; and $62.8318 \times 20 = 1256.636$, the area of the surface of the globe.

Or, the area of a circle whose diameter is 20, by Prob. 12. is 314.159; and $314.159 \times 4 = 1256.636$, as before.

If the area of the surface of a segment, as m C n, be required, say,

As the axis or diameter of the sphere C D,
To the area of the whole globe;
So the height of the segment C r,
To the area of the segment, exclusive of the base.

If the diameter of the globe be wanted, divide the square of m r, the semidiameter of the frustum's base by C r its height, and the quot will give r D the height of the other frustum, and the sum of C r and r D is the diameter sought. See Prob. 13.

The area of the segment m C n may also be found thus: Square m r and r C, add these squares, and the square root of their sum is C m; then find the area of a circle whose radius is C m, and this will be equal to the area of the given segment, exclusive of the circular base.

If the area of that part of the surface of a sphere that lies between two parallel planes, as m n, and a o, be demanded, it may be found thus: Find, as above directed, the area of the two segments m C n and a C o, subtract the lesser area from the greater, and the remainder is the area sought.

XXII. A spherical triangle, or triangle formed on the surface of a globe by three great circles, *viz.* circles whose centre is the same with that of the globe; as, A B C. Pl. 2.

Rule. From the sum of the three angles subtract 180 degrees, multiply the area of the whole surface of the globe by the remainder, divide the product by 720, the quot is the area of the triangle.

E X A M P L E.

Suppose the sum of the angles $A + B + C = 225 \quad 10$
Subtract 180

Rem. $45 \quad 10$

Suppose again the diameter of the given sphere or globe to be 20, then the area of the whole surface, as found above, will be 1256.636.

Multiply

Multiply 1256.636
by the rem. 45.10

1256636
6283180
5026544

56674.2836
418.87866
418.87866

720)57512.04093(79.877 area of the triangle.

The surfaces of prisms, pyramids, and other regular bodies, consist of triangles, parallelograms, or polygons; and so the way of measuring them is taught in the preceding problems.

II. Mensuration of Solids.

Problems, shewing how to find the Solid Content or Solidity.

XXIII. A cube, or solid contained under six equal squares; as, A B C D. Pl. 3.

Rule. Multiply the side of the cube into itself, and that product again by the side; this last product is the solid content or solidity of the cube.

Ex. Multiply the side A D = 4
by itself - = 4

Again, Multiply this product 16
by 4

Solidity of the cube 64

XXIV. A prism, or solid, whose bases are equal similar parallel triangles, squares, parallelograms, or polygons, and the sides all parallelograms.

A prism whose bases are squares or parallelograms, is usually called a *parallelopiped*.

When the bases are triangles, the solid is called a *triangular prism*, and if the bases are polygons, it is called *polygonal prism*.

Rule. Find the area of the base as taught in superficial measure, and multiply that by the length of the prism or parallelopiped.

Example. Required the solid content of the triangular prism A B C D. Pl. 3.

Multiply

$$\begin{array}{r}
 \text{Multiply } B C \quad \quad \quad = 18 \\
 \text{by half the per. } m n = 6 \\
 \hline
 \text{Area of the base } 108 \\
 \text{Multiply by the length } A B = 40 \\
 \hline
 \text{Solidity of the prism } 4320
 \end{array}$$

XXV. A cylinder. See a description of this solid. Prob. 19. Pl. 3. fig. 1.

Rule. The solidity of a cylinder is found the same way as that of a prism or parallelopiped, viz. Find the area of a circular base by Prob. 12. multiply that by the length or height, the product is the solid content.

Ex. Required the solidity of a cylinder, the diameter of whose base is 9, and length 15.

$$9 \times 9 \times .7854 \times 15 = 954.261 \text{ the solidity.}$$

If it be required to find how much liquor is contained in a cylindrical vessel that is in part empty, lying with its axis parallel to the horizon; such as a vessel whose base is the circle A B C D, fig. 2. the empty part being the segment A B D; find by Prob. 13. the area of the segment B C D, and multiply this area by the length.

XXVI. A pyramid, or solid, whose base is a triangle, square, parallelogram, or polygon, and its sides all plane triangles, terminating in a point; as, A D B. Pl. 3.

Rule. Find the area of the base, and multiply that into one third of the altitude or height; the product is the solid content, a pyramid being equal to one third of the circumscribing prism. Euclid xii. 7.

Ex. Suppose the area of the base A B to be 4.25
Multiply that by $\frac{1}{3}$ of D C = 5.5

$$\begin{array}{r}
 2125 \\
 2125 \\
 \hline
 \end{array}$$

Solidity of the pyramid 23.375

If the pyramid be cut by a plane m n parallel to the base A B, and the solidity of the frustum A m n B be required; from the solidity of the whole pyramid A D B subtract the solidity of the pyramid m D n cut off, the remainder is the solidity of the frustum.

To find the height of the whole pyramid, say, As the difference between one of the sides of the greater base and the correspondent side

side of the lesser base, to the height of the frustum, so the side of the greater base to the height of the whole pyramid; from which subtract the height of the frustum, and the remainder will be the height of the pyramid cut off.

The solidity of the frustum of a pyramid may also be found by one or other of the three rules following:

Rule 1. To the product of any two correspondent sides of the two bases, add the squares of these sides; multiply that sum by one third of the frustum's height, and the product will be the solidity, if the bases are squares.

Rule 2. Multiply the areas of the two bases together, and to the square root of the product add these two areas; multiply that sum by one third of the frustum's height, and the product is the solidity of the frustum, whether the bases be triangles, squares, or polygons.

Rule 3. To the product of any two correspondent sides of the two bases, add one third part of the square of their difference; and that sum will be the area of a mean base; which multiplied into the height, gives the solidity of the frustum, whether the bases be triangular, square, or multangular.

A solid, resembling the frustum of a pyramid, and having parallel bases, and these bases both rectangles, or the one a rectangle and the other a square, but disproportional, the sides of the one base not having the same proportion to one another as the correspondent side of the other, is called a *prismoid*; as, A B C. Pl. 3. fig. 2. And its solidity may be found as follows:

To the longest side of the greater base add half the longest side of the lesser base, and multiply the sum by the breadth of the greater base, and reserve the product.

Again, to the longest side of the lesser base add half the longest side of the greater base; and multiply the sum by the breadth of the lesser base; and to this product add the product formerly reserved; and multiply the sum by a third part of the height, and the product is the solidity of the prismoid.

If the area of the outside surface of the prismoid be required, multiply half the sum of the perimeters of both bases by the outside height, and to the product add the area of the two bases.

XXVII. A cone. See this solid described, Prob. 20.

A cone is one third of the circumscribing cylinder; or, A cone is the same with a pyramid of an infinite number of sides: and the solidity is found the same way, *viz.* Multiply the area of the base by one third of the altitude or perpendicular height; Euclid XII. 10.

If the cone be cut by a plane *m n* parallel to the base, the solidity of the frustum *m n B C* is also found by subtracting the solidity of the cone cut off *A m n* from the solidity of the whole cone. A B C. Pl. 3. See Prob. 20.

The

The solidity of the frustum of a cone may also be found by any of the three rules following :

Rule 1. To the product of the diameters of the two bases add the squares of the said two diameters, and multiply the sum by .7854, the product will be the triple area of a mean base ; which multiplied by one third of the perpendicular height, will give the solidity of the frustum.

Rule 2. Multiply the area of the greater base into the area of the lesser ; extract the square root of the product ; to this root add the areas of the two bases ; and multiply the sum by one third of the frustum's height, and the product is the solidity.

Rule 3. To the product of the greater and lesser diameters add one third part of the square of their difference ; and multiply the sum by .7854 ; the product is the area of a mean base ; which multiplied by the perpendicular height, the product is the solidity.

If the base of a cone, or of a cylinder, be an ellipse at right angles to the axis, find the area of the base by Prob. 16. ; and then proceed to find the solidity, as taught above. The solidity of the frustum of such a cone may also be found as above ; only find the area of the elliptical bases by Prob. 16.

A solid, resembling the frustum of a cone, and having parallel bases, and these bases both elliptical, or the one base an ellipse and the other a circle, but disproportional, the diameters of the one base not having the same proportion to one another as the correspondent diameters of the other base, is called a *cylindroid* ; as, A B C. Pl. 3. fig. 2. And its solidity may be found thus :

To the longest diameter of the greater base add half the longest diameter of the lesser base, and multiply the sum by the shortest diameter of the greater base, and reserve the product.

Again, to the longest diameter of the lesser base, add half the longest diameter of the greater base, and multiply the sum by the shortest diameter of the lesser base ; and to this product add the product formerly reserved, and that sum will be the triple square of a mean diameter ; which multiply by .7854 ; and again multiply that product by a third part of the height ; and this last product is the solidity of the cylindroid.

If the area of the outside surface of the cylindroid be required, measure the periphery of both bases, and multiply half their sum by the outside height, and to the product add the area of the two bases.

XXVIII. A sphere or globe. See the description of this solid, Prob. 21. Pl. 3.

Rule. A sphere or globe is equal to two thirds of the circumscribed cylinder ; that is, a cylinder of equal diameter and altitude with the globe ; and therefore find the area of a great circle ; multiply this by the diameter ; two thirds of the product is the solid content.

Ex.

Ex. Suppose the diameter of a sphere or globe to be 20, then, by Prob. 12.

The area of the circle will be	-	-	314.159
Multiply by the diameter,			20
			3)6283.180
			2094.393
			2094.393
Solidity of the globe,			4188.786

A sphere may be considered as composed of an infinite number of cones, having their bases in the surface, and their common vertex in the centre; and so the solidity of a sphere may also be found thus: Multiply the diameter into the circumference; the product is the superficial content or area: then multiply this area by one sixth of the diameter, or by one third of the radius; the product is the solid content.

Ex. Suppose the diameter of a globe to be 20; then, by Prob. 12.

The periphery will be	-	-	62.8318
Multiply by the diameter,			20
			3)1256.636
Multiply by $\frac{1}{6}$ diameter,		=	3.3
			3769.908
			418.8786
Solidity, as before,			4188.7866

The solidity of a sphere may be found nearly, by multiplying the cube of the diameter into .5236, or, for greater accuracy, into .523598, the solidity of a sphere whose diameter is unity.

Thus, the cube of the diameter 20 is $20 \times 20 \times 20 = 8000$; and $.5236 \times 8000 = 4188.8$ the solidity nearly; or, $.523598 \times 8000 = 4188.784$, more nearly.

XXIX. The polar segment, or frustum of a sphere; as, A C B D. Pl. 3.

Rule. From the triple product of the diameter of the sphere C E into the square of the frustum's height C D, subtract twice the cube of the segment's height C D; then divide the remainder by 1.91; or, multiply by .52356, or, for practice, by .5236, the quot or product is the solid content.

O o

Ex.

Ex. Suppose $CE = 20$; and $CD = 5$.

Then $5 \times 5 \times 20 \times 3 - 5 \times 5 \times 5 \times 2 = 1250$.

And $1.91)1250(654.45$, the solidity sought.

Or, $1250 \times .52356 = 654.45$, as before.

The segment $A C B D$ subtracted from the whole sphere, leaves the greater segment $A E B D$.

XXX. The middle segment or zone of a sphere; as, $A B C D$. Pl. 3.

Rule. Multiply the square of the sphere's diameter EG by 2, and to the product add the square of the diameter of the base AB or CD ; divide this sum by 3.82; or, for greater accuracy, by 3.8197; and multiply the quot by the zone's thickness, or height mn , the product is the solid content.

Ex. Suppose $EG = 20$, AB or $CD = 17.32$, and $mn = 10$.

Then $20 \times 20 \times 2 + 17.32 \times 17.32 = 1099.9824$

And $3.82)1099.9824(287.95$

And $287.95 \times 10 = 2879.5$ the solidity of the zone.

The solidity of the zone $A B C D$ may also be found by subtracting twice the solidity of the polar segment $A E B m$ from the solidity of the whole sphere.

Hence may be found the solidity of any segment of a sphere included between two parallel planes; as, $a o B A$; viz. Find the solidity of the segment $A E B$, and also that of the segment $a E o$, and subtract the lesser from the greater.

XXXI. A sector of a sphere; as, $A B D C$. Pl. 3.

Rule. A spherical sector, as already observed, consists of an infinite number of cones, having their bases in the surface of the sphere $B D C$, and their common vertex in the centre A : Wherefore, by Prob. 21. find the area of the segment $B D C$; and multiply this area into the third part of AB , the radius of the sphere.

Hence we have another method of finding the solidity of a spherical segment, such as $B D C$, viz. From the sector $A B D C$ subtract the cone $A B C$, and there will remain the solidity of the segment. But if the spherical segment be greater than a hemisphere, the cone must be added to the sector.

XXXII. A spheroid, or solid generated by revolving a semi-ellipse round its axis, as, $A B C D$. Pl. 3.

If the semiellipse be revolved round the longer axis AB , the solid thence generated is called an *oblong spheroid*, and resembles an egg.

If the revolution be round the shorter axis CD , the solid thence arising is called an *oblate spheroid*, being somewhat like a bias-bowl.

Rule.

Rule. Multiply the axis round which the semiellipse was revolved into the square of the other axis; then multiply this product by .5236; this last product is the solidity of the spheroid, whether oblong or oblate.

Example. Suppose $CD = 5.5$; and $AB = 9$.

Then $5.5 \times 5.5 \times 9 \times .5236 = 142.5501$ the solidity of the spheroid.

A spheroid is two thirds of a cylinder whose height is the axis round which the semiellipse was revolved, and whose diameter is the other axis; and accordingly the solidity of the above spheroid may also be found thus:

$5.5 \times 5.5 \times .7854 \times 9 = 213.82515$; whereof two thirds is 142.5501, as before.

If the solidity of a segment, as, $A m n$, or of the middle frustum $m n o p$, be required, imagine a sphere described on the axis AB , and cut into segments similar to those of the spheroid, by extending the planes $m n$ and $o p$; and then say, As the square of the axis AB to the square of the axis CD ; so the solidity of the spherical segment or frustum found by Prob. 29. or 30. to the solidity of the segment or frustum of the spheroid. *N. B.* Twice the segment $A m n$ subtracted from the whole spheroid, leaves the middle frustum or zone $m n o p$.

Note. The middle frustum of the spheroid $m n o p$ differs little from the like frustum of a sphere described on the axis CD , and may have its solidity computed in the same manner, *viz.* To twice the square of CD add the square of $m n$ or $o p$, divide the sum by 3.82, and multiply the quot by $a r$, the height.

XXXIII. A parabolic conoid, or solid, found by revolving a semiparabola $A n C$ round its axis $A o$; as, ABC . Pl. 3.

Rule. A parabolic conoid is one half of the circumscribed cylinder; multiply therefore the area of the circular base BC into one half of the altitude $A O$, the product is the solid content.

Example. Suppose the diameter of the base $B o C$ to be 12, and the height $A o$ to be 18: Required the solid content.

$12 \times 12 \times .7854 \times 9 = 1017.8784$ solidity sought.

If the parabolic conoid be cut by a plane $m n$ parallel to the base BC , the solidity of the frustum $m n B C$ may be found thus: To the square of the diameter of the greater base BC add the square of the diameter of the lesser base $m n$; divide the sum by 2.5464; or, multiply by .3927; and then multiply the result by the height of the frustum; the product is the solid content. The

O o 2

frustum

frustum A m n cut off is a complete conoid, and its solidity is found as taught above.

XXXIV. A parabolic spindle, or solid generated by revolving a parabola A B C round its ordinate or base A C; as, A B C D. Pl. 3.

Rule. The parabolic spindle is equal to eight fifteenths of the circumscribing cylinder; find therefore the solidity of a cylinder, the diameter of whose base is B D, the greatest diameter of the spindle, and its height A C, the length of the spindle, or ordinate of the parabola, and $\frac{8}{15}$ of this is the solid content of the spindle.

Example. Suppose B D = 6.5, and A C = 8.

Then $6.5 \times 6.5 \times .7854 \times 8 \times \frac{8}{15} = 141.58144$ solidity.

Or, Multiply the square of B D by A C, and that product by $.41888 = \frac{8}{15}$ of .7854.

Thus, $6.5 \times 6.5 \times 8 \times .41888 = 141.58144$ as before..

The solidity of the middle frustum of the spindle, *viz.* m n o p is found thus: To twice the square of the greatest diameter B D add the square of the diameter of the base, *viz.* m n or o p, and from the sum subtract four tenths of the square of the difference between these two diameters; divide the remainder by 3.82; or, for greater accuracy, by 3.8197; and multiply the quot by t r, the height of the frustum, the product is the solidity.

Or thus, Multiply the square of the greatest diameter by 1.5708; and multiply the square of the lesser diameter by .7854; and multiply the square of the difference of these diameters by .31416; from the sum of the two former products subtract the latter product; and multiply the remainder by one third part of the height; and that product will be the solidity of the middle frustum required.

If the solidity of the middle frustum be subtracted from that of the whole spindle, half the remainder will be the solidity of the frustum A m n or C o p.

XXXV. The five regular or Platonic bodies, Pl. 4. *viz.*

1. A tetraedron, or solid contained under four equal equilateral triangles.

This solid is a pyramid on a triangular base, and its solidity may be found by Prob. 26.

2. An hexaedron, or cube, *viz.* a solid contained under six equal squares.

This solid is a prism; and its solidity may be found by Prob. 23. or 24.

3. An octaedron, or solid contained under eight equal equilateral triangles.

This

This solid consists of two pyramids on the same square base, and of the same height; and consequently its solidity may be found by Prob. 26.

4. A dodecaedron, or solid contained under twelve equal equilateral pentagons.

This solid consists of twelve equal pyramids, having pentagonal bases, whose common vertex is in the middle point or centre; and so its solidity may be found by Prob. 26.

5. An icosaedron, or solid contained under twenty equal equilateral triangles.

This solid consists of twenty equal pyramids having triangular bases, and their common vertex in the centre; and so its solidity may likewise be found by Prob. 26.

But the area, or superficial content, as well as the solidity of any of these bodies, may easily and readily be found from the following table:

<i>Names.</i>	<i>Area.</i>	<i>Solidity.</i>
Tetraedron	1.732051	0.1178511
Hexaedron	6.000000	1.0000000
Octaedron	3.464102	0.4714045
Dodecaedron	20.645729	7.663119
Icosaedron	8.660254	2.181695

The table exhibits the area and the solidity of any of the above bodies, the side being unity. In using the table observe the following rules:

1. Multiply the proper tabular number by the square of the given side; the product is the area, or superficial content.

2. Multiply the proper tabular number by the cube of the given side; the product is the solidity.

Ex. Required the area and solidity of a dodecaedron, the side being 3.

$$3 \times 3 \times 20.645729 = 185.811561 \text{ the area.}$$

$$3 \times 3 \times 3 \times 7.663119 = 206.904213 \text{ the solidity.}$$

XXXVI. Any irregular body, such as, the bones in a horse's head, a thorn or whin bush, &c.

Rule. Take any vessel of a regular form, such as that of a prism or cylinder, and therein put the irregular solid; then pour in as much water as will cover the solid: this being done, take out the solid, and observe how far the water sinks or falls on the side of the vessel, and compute the solidity by Prob. 24. or 25.

Or, Take any sort of vessel, fill it with water to the brim; then immerse the irregular body; receive the water that runs over, and pour it into some vessel of a regular form; and then proceed as above.

III. *Surveying.*

A surveyor, or person who measures land, ought to be provided with a theodolite and plain table, a case of instruments, with a set of plotting scales, and a gunter's chain.

The gunter's chain is divided into 100 equal links, and is in length equal to 4 rods or poles; and 160 square rods, or 10 square chains, make an acre.

The English rod or pole is $16\frac{1}{2}$ feet; and so the length of the English chain is 66 feet, or 22 yards. The Scotch rod, pole, or fall, is $18\frac{1}{2}$ feet, and the length of the chain 74 feet. And 4 Scotch acres are nearly equal to 5 English acres.

My design in this place is not to teach surveying at any great length, but only to give a general notion of the art, so far as to enable the reader to measure any park or plain field, and that with accuracy. For this purpose, it will be necessary to show what lines are to be measured; how the measures ought to be taken; and in what manner the number of acres is computed.

If the ground to be measured lie in the form of a rectangle, you have only to multiply the length into the breadth: only observe, that if the two ends differ somewhat in breadth, you may add them, and take half their sum for a mean breadth; and do the same with respect to the sides, if they happen not to be equal.

But most fields lie in the form of some irregular polygon; and in this case you must go round the field, and erect a pole at every angle. Then make out an eye-draught, *viz.* a figure on paper as near the shape of the field as can be done by the guess of the eye, and divide this draught into triangles, trapezia, or other figures, as the shape of the draught and field directs. Draw also, in the triangles and trapezia, lines to represent the perpendiculars. Now, the lines to be measured are the diagonals and perpendiculars. But before they be measured, the points in the field where they intersect one another must be found as follows:

Provide yourself with a theodolite, or, in lieu thereof, with a cross, represented in Pl. 4. fig. 1. *viz.* a square board, with four pins or pegs erected in the corners, so that A B may be perpendicular, or at right angles, to C D; and place the cross horizontally on the top of a staff F O fastened into a hole in the middle of the board at O. By means of this cross find the points in the field where the perpendiculars cut the diagonals, in the manner following.

Supposing the polygram A B C D E F G in Pl. 4. fig. 2. to represent the field, you must step in a straight line from G to B; and when you come to the place where you imagine the perpendicular A m will cut the diagonal G B, set up the cross directing C D to B; and, if now B A on the cross point directly to A in the field, you have guessed right; but if it do not, go further on toward B, or return back toward G, till you hit upon the place, and there
fix

fix a pole. In like manner, find the points where all the other perpendiculars cut the diagonals, and mark the places thus found by fixing poles in them.

Then measure the several segments into which the diagonals are divided, and also the perpendiculars; and set down the measures so taken in a field-book, or on the eye-draught described above. When you come home, delineate the whole on paper, from a scale of equal parts, draw the outlines A B, B C, &c. and cast up the content, as follows.

The area of a triangle is found by Prob. 6. and that of a trapezium, by Prob. 8. as follows.

For the triangle A B G,

$$\text{Multiply the base } G B = 11 + 17.20 = 28.2$$

$$\text{by half the perpendicular } A m = 9$$

$$\text{Area } 253.8$$

For the trapezium B C F G,

$$\text{Multiply the sum of the perpendiculars } B n + F r = 34.54$$

$$\text{by half the diagonal } G C = 25$$

$$G C = 19 + 5 + 26 = 50$$

$$17270$$

$$6908$$

$$\text{Area } 863.50$$

For the trapezium F C D E,

$$\text{Multiply the sum of the perpendiculars } D o + F p = 31.86$$

$$\text{by half the diagonal } E C = 23.02$$

$$E C = 13.5 + 14 + 18.54 = 46.04$$

$$6372$$

$$9558$$

$$6372$$

Area

$$733.4172$$

$$\text{To which add } \left\{ \begin{array}{l} B C F G \ 863.5 \\ A B G \ 253.8 \end{array} \right.$$

$$\text{Area of the field, } - \ 1850.7172$$

The area of the field thus computed is 1850 square chains, and 7172 square links; and, because 10 square chains make an acre, the square chains are reduced to acres by dividing by 10; that is, by moving the decimal point one place toward the left; and the figures on the right are decimal parts of an acre, as follows :

The decimal is reduced to value by multiplying by 4, by 40, by $30\frac{1}{4}$, and by 9, as the table of land-measure directs.

Acres 185.07172

4

Roods .28688

40

Poles or Roods 11.47520

$30\frac{1}{4}$

14.256

.1188

Square yards 14.3748

9

Square feet 3.3732

Some set down the measures taken in the field without inserting any decimal point between the chains and the links; and in this way the area of the above field will be 18507172 square links: which are easily reduced to acres; for the chain consists of 100 links, and consequently the square chain will contain 10000 square links, and 10 square chains make an acre; therefore the acre will contain 100000 square links. Divide then the area given in square links by 100000, and the quot will be acres; that is, cut off by a decimal point five places on the right, the figures on the left are so many acres, and the figures on the right are decimal parts of an acre. Thus, 18507172 square links becomes 185.07172 acres, as above.

It is usual to measure gardens, or small inclosures, with a rod-pole, *viz.* a rod one pole long; and in this case the rod-pole should be divided into 100 equal parts, that the parts in any measured line above a just number of rods may be decimal parts of a rod. Some use a half-rod-pole divided into 50 equal parts.

If we suppose the above field to have been measured by a rod-pole, the area will be 1850.7172 square rods, which are reduced to acres by dividing by 160, the number of square rods in an acre; or you may divide the square rods by 40, and the quot will be roods; and then divide the roods by 4, and this last quot will be acres, as under.

Acres,

$$\begin{array}{r} \text{Acres.} \\ 160)1850.7172(11.5669825 \\ \underline{4} \end{array}$$

$$\begin{array}{r} \text{Roods } 2.2679300 \\ \underline{40} \end{array}$$

$$\begin{array}{r} \text{Poles } 10.7172 \\ \underline{30\frac{1}{4}} \end{array}$$

$$\begin{array}{r} 21.516 \\ .1793 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Sq. yards } 21.6953 \\ \underline{9} \end{array}$$

$$\text{Sq. feet } 6.2577$$

$$\begin{array}{r} \text{Or thus:} \\ 40)1850.7172(\end{array}$$

$$\begin{array}{r} \underline{4)46.26793} \end{array}$$

$$\text{Acres } 11.5669825$$

If a piece of ground lie in the form of a ridge, take the breadth at several places, add these several breadths, and divide their sum by the number of breadths taken, the quot will be a mean breadth, which multiplied into the length, will give the area.

If the measures of a field be taken in feet, the area will be square feet; which you may reduce into English acres by dividing by 43560, the number of square feet in an English acre; or it may be reduced to Scotch acres by dividing by 54760, the number of square feet in a Scotch acre.

If the measures of a field be taken in yards, the area will be square yards; and these may be reduced to English acres by dividing by 4840, the number of square yards in an English acre.

If the measures of an island, kingdom, or empire, be taken in English miles, the area will be square miles; and these may be reduced to English acres by multiplying by 640, the number of acres in a square mile.

And if the number of English acres on the whole surface of the terraqueous globe be required, find by Prob. 21. the area of the surface of a globe, whose diameter is 8000 English miles, and multiply the area in square miles so found by 640.

IV. Artificers Work.

1. Carpenters Work.

Carpenters measure their work by the square of 100 feet, viz. a square whose side is 10 feet. Their measurable work consists chiefly of flooring, partitioning, and roofing. See Prob. 2.

Ex. 1. If a floor be 38 feet 6 inches long, and 17 feet 9 inches broad, how many squares are in that floor?

Decimally.

Decimally.

$$\begin{array}{r}
 17.75 \\
 38.5 \\
 \hline
 8875 \\
 14200 \\
 5325 \\
 \hline
 \end{array}$$

$$100)6.83375$$

Anf. 6 squares 83 feet.

Or thus :

$$\begin{array}{r}
 F. \text{ in.} \\
 38 \text{ } 6 \\
 17 \text{ } 9 \\
 \hline
 654 \text{ } 6 \\
 28 \text{ } 10 \text{ } 6 \\
 \hline
 \end{array}$$

$$100)6|83 \text{ } 4 \text{ } 6$$

Example 2. How many planks or deals, that are 9 feet long and 8 inches broad, will floor a house that is 47 feet 6 inches long, and 24 feet 4 inches wide ?

Inches.

$$\begin{array}{r}
 8 = 8 \\
 \text{Length } 9 \\
 \hline
 \text{Product } 6.
 \end{array}$$

$$\begin{array}{r}
 24.3 \\
 47.5 \\
 \hline
 1216 \\
 17033 \\
 97333 \\
 \hline
 6)1155.83(192.638 \\
 9 \\
 \hline
 5.750
 \end{array}$$

Anf. 192 planks, $5\frac{3}{4}$ feet.

Or, Reduce the dimensions to inches, then multiply the length of the house into the breadth, and divide the product by the square inches in one plank.

Example 3. How many squares are contained in a partition that is 64 feet 6 inches long, and 12 feet 3 inches high ?

$$\begin{array}{r}
 12.25 \\
 64.5 \\
 \hline
 6125 \\
 4900 \\
 7350 \\
 \hline
 \end{array}$$

$$100)7.90125$$

Anf. 7 squares, 90 feet.*F. in.*

$$\begin{array}{r}
 64 \text{ } 6 \\
 12 \text{ } 3 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 774 \\
 16 \text{ } 1 \text{ } 6 \\
 \hline
 \end{array}$$

$$100)7|90 \text{ } 1 \text{ } 6$$

Example 4. How many squares of roofing will cover a house, whose length within the walls is 48 feet 6 inches, and breadth 18 feet 3 inches ?

It is a received rule amongst workmen, That the flat and half-flat

flat of any house, taken within the walls, is equal to the roof, though the measure of the roof varies a little, as the roof falls below or rises above the true pitch; that is, as the angle at which the two sides of the roof meets, is more or less than a right angle.

$\begin{array}{r} 18.25 \\ 48.5 \\ \hline 9125 \\ 14600 \\ 7300 \\ \hline \end{array}$	$\begin{array}{r} \text{F. in.} \\ 48 \quad 6 \\ 18 \quad 3 \\ \hline 873 \quad 0 \\ 12 \quad 1 \quad 6 \\ \hline \end{array}$
Flat 885.125 Half 442	Flat 885 1 6 Half 442
$100)13.27$	$100)13 27$

Ans. 13 squares, 27 feet.

2. Bricklayers Work.

Bricklayers work consists chiefly of tiling; which is measured by the square of 100 feet; and walling or chimney work, which are measured by the square rod of 272.25 square feet, the side of the square being 16.5 feet. But, in practice, the integer 272 is generally esteemed sufficiently accurate.

In roofs covered with tile, it is usual to reckon the eaves double, which is done by adding the depth of the eaves to the whole depth of the roof.

Ex. 1. There is a roof covered with tiles, whose depth on both sides, the eaves being reckoned double, is 35 feet 3 inches, and the length 48 feet: How many squares of tiling are in it?

$\begin{array}{r} 35.25 \\ 48 \\ \hline 28200 \\ 14100 \\ \hline 100)16.9200 \end{array}$	$\begin{array}{r} \text{F. in.} \\ 35 \quad 3 \\ 48 \quad 0 \\ \hline 12 \\ 280 \\ 140 \\ \hline 100)16 92 \end{array}$
---	---

Ans. 16 squares 92 feet.

Ex. 2. A brick-wall is 598 feet long, 9 feet high, and $1\frac{1}{2}$ brick in thickness: How many rods of work are in it?

$$\begin{array}{r}
 598 \\
 9 \\
 \hline
 272.25)5382.00(19 \text{ rods.} \\
 27225 \\
 \hline
 265950 \\
 245025 \\
 \hline
 68.06)209.25(3 \text{ quarters of a rod.} \\
 20418 \\
 \hline
 5.07
 \end{array}$$

Ans. 19 rods 3 quarters 5 feet.

Brick and half is reckoned standard thickness; and if the thickness of a wall be any other than brick and half, it may be reduced to standard thickness by the following

R U L E.

Multiply the number of superficial feet contained in the wall by the number of half bricks in the thickness, and one third of the product will be the content reduced to standard thickness.

Ex. 3. A brick-wall is 84 feet 6 inches long, and 17 feet 3 inches high, and 5 bricks and a half thick: How many rods of brick-work will be therein, when reduced to standard thickness?

$$\begin{array}{r}
 17.25 \\
 84.5 \\
 \hline
 8625 \\
 6900 \\
 13800 \\
 \hline
 1457.625 \\
 11 \\
 \hline
 1457625 \\
 1457625 \\
 \hline
 3)16033.875 \\
 272.25)5344.625(19 \text{ rods.} \\
 27225 \\
 \hline
 262212 \\
 245025 \\
 \hline
 68)171.875(2 \text{ quarters.} \\
 136 \\
 \hline
 35.875
 \end{array}$$

Ans. 19 rods 2 quarters 35 feet.

But

But the operation may be shortened, by constructing a table of divisors, suited to the number of half-bricks in the thickness, as follows :

Divide 3, the number of half-bricks in $1\frac{1}{2}$, by the number of half-bricks in the thickness, and the quot will be a divisor for giving the answer in feet. And, to obtain a divisor that will give the answer in rods, multiply 272.25 by the divisors found for feet. The table follows :

<i>Thickness of the wall in bricks and half-bricks.</i>	<i>Divisors giving the answer in feet.</i>	<i>Divisors giving the answer in rods.</i>
1	1.5	408.375
$1\frac{1}{2}$	1.	272.25
2	.75	204.1875
$2\frac{1}{2}$	.6	163.35
3	.5	136.125
$3\frac{1}{2}$	.4285	116.659
4	.375	102.0937
$4\frac{1}{2}$	.3	90.75
5	.3	81.675
$5\frac{1}{2}$	.27,	74.25
6	.25	68.09375

The former example done by the table follows :

74.25)1457.625(- - - - 19.63 rods.

7425

4

71512

2.52 quarters.

66825

68

46875

416

44550

312

23250

35.36 feet.

22275

975

If a brick chimney stand alone by itself, it must be girt round; and this is to be esteemed the length; the height of the story is the breadth; and the thickness is the same with that of the jambs. Every story is measured by itself, and the dimensions taken in the same manner; and the shaft or stalk above the roof is also measured by itself, and the content found by multiplying the girt into the height.

Ex. 4.

Ex. 4. The girt of a brick chimney in a low story is 32 feet 9 inches, and the height of the story 12 feet 6 inches: How many square feet of work are in the same?

$ \begin{array}{r} 32.75 \\ 12.5 \\ \hline 16375 \\ 6550 \\ \hline 13275 \end{array} $	$ \begin{array}{r} \text{F. in.} \\ 32 \ 9 \\ 12 \ 6 \\ \hline 393 \\ 16 \ 4 \ 6 \\ \hline \text{Feet } 409 \ 4 \ 6 \end{array} $
Feet 409.375	Feet 409 4 6

If it be required to reduce the square feet thus found to feet or rods of standard thickness, divide by the divisor that suits the thickness of the jambs. And if the chimney be esteemed double work, as most chimneys are, double the result.

If a chimney have two or more funnels, the fenders or bridges that separate the funnels must be measured, and their content found by multiplying the length into the breadth.

If a chimney do not stand by itself, but be placed in a gavel or side-wall, the back of the chimney in this case is not measured, but accounted part of the gavel or side-wall; and the length of the chimney is found by adding the depth of the two jambs to the horizontal extent of the breast.

3. Plaster-work.

Plaster-work is either on the roof, called *ceiling*, or on the partitions or walls of a building; and is all measured by the square yard, containing 9 square feet.

Example 1. If a ceiling be 54 feet 9 inches long, and 22 feet 6 inches broad, how many yards are in it?

$ \begin{array}{r} 54.75 \\ 22.5 \\ \hline 27375 \\ 10950 \\ \hline 10950 \\ 9)1231.875 \\ \hline \text{Yards } 136.875 \end{array} $	$ \begin{array}{r} \text{F. in.} \\ 54 \ 9 \\ 22 \ 6 \\ \hline 16 \ 6 \\ 108 \\ 108 \\ \hline 27 \ 4 \ 6 \\ \hline 1231 \ 10 \ 6 \end{array} $
--	--

Example 2. The length of a partition plastered on both sides is 44 feet 6 inches, and the height of the room 9 feet 3 inches: How many yards of plaster-work are in it?

44.5
9.25
222.5
890
4005
9)411.625
Yards 45.736

F. in.
44 6
9 3
4 6
396
11 1 6
411 7 6

4. Joiners Work.

Joiners measure their work also by the square yard; but in taking the height of any room, they use a small cord, with which they gird over the mouldings, panels, &c. alleging they ought to measure where their plane touches; but in measuring round the room they take it as it is upon the floor.

Doors, window-shutters, and such work as is wrought on both sides, are generally esteemed work and half-work.

Example. If the wainscoting of a room, girt downward over the mouldings, be 12 feet 9 inches high, and 118 feet 3 inches in compass, how many square yards of work are in it?

118.25
12.75
59125
82775
23650
11825
9)1507.6875
Yards 167.52083

F. in.
118 3
12 9
1419
59 1 6
29 6 9
1507 8 3

The doors and windows of the above room must again be measured by themselves, and the half of their content must be added to the content of the whole room found above.

The content of chimneys, and other void places, must also be measured by themselves, and their content is to be deduced.

5. Painters Work.

Painters work is also measured by the square yard, and the height is likewise taken by girding over the mouldings and panels with a small cord.

Example 1. A room is painted, whose height, taken by girding over the mouldings, is 14 feet 6 inches, and the compass of the room 84 feet 6 inches: How many yards of painted work are in it?

84.5

84.5	F. in.
14.5	84 6
	14 6
4225	
3380	343
845	84
	42 3
9)1225.25	
	1225 3
Yards 136.138	

Example 2. A round pillar is painted, whose height is 18 feet 4 inches, and the girt round 10 feet 6 inches: How many yards of work are in it?

18.3	F. in.
10.5	18 4
	10 6
916	
18333	183 4
	9 2
9)192.50	
	192 6
Yards 21.38	

6. Glasiers Work.

Glasiers measure their work by the foot square, and take their dimensions to a quarter of an inch, the inch being supposed to be divided into 12 equal parts.

But it is more usual to measure by a foot decimally divided, and then the length or breadth consists of so many feet and decimal parts of a foot.

Round or oval windows, half-rounds, &c. are always measured at their full height and breadth, in consideration of the waste of glass and trouble such work is attended with.

Example 1. Suppose 6 panes of glass, each 4 feet 7 inches 3 quarters long, and 1 foot 5 inches 1 quarter broad, how many feet of glass are in the said 6 panes?

4.646	F. in. p.
1.437	4 7 9
	1 5 3
32522	
13938	1 1 11 3
18584	1 11 2 9
4646	4 7 9
6.676302	6 8 1 8 3
6	6
Feet 40.057812	40 0 10 1 6

Example 2. Suppose 20 panes of glaſs, which meaſured by a decimal foot are each 2.458 feet long, and 1.75 feet broad, how many feet of glaſs do they contain?

$$\begin{array}{r}
 2.458 \\
 1.75 \\
 \hline
 12290 \\
 17206 \\
 2458 \\
 \hline
 430150 \\
 20 \\
 \hline
 \text{Feet } 86.030
 \end{array}$$

7. *Masons Work.*

Masons meaſure their work, ſometimes by the foot ſolid, ſometimes by the ſquare foot or yard, and ſometimes by the rod, that is, 21 feet long and 3 feet high, viz. 63 ſquare feet. But the rod differs in different places.

Example 1. A wall of hewn ſtone is 24 feet 6 inches long, 9 feet 6 inches high, and 2 feet 3 inches thick: How many ſolid feet are in that wall?

24.5	F. in.
9.5	24 6
1225	9 6
2205	220 6
232.75	12 3
2.25	232 9
116375	2 3
46550	465 6
46550	58 2 3
Feet 523.6875	523 8 3

Example 2. If a wall be 218 feet 9 inches long, and 12 feet 6 inches high, how many ſquare feet, yards, and rods, are in it?

P p

218.75

84.5	F. in.
14.5	84 6
	14 6
4225	
3380	343
845	84
	42 3
9)1225.25	
	1225 3
Yards 136.138	

Example 2. A round pillar is painted, whose height is 18 feet 4 inches, and the girt round 10 feet 6 inches: How many yards of work are in it?

18.3	F. in.
10.5	18 4
	10 6
916	
18333	183 4
	9 2
9)192.50	
	192 6
Yards 21.38	

6. Glasiers Work.

Glasiers measure their work by the foot square, and take their dimensions to a quarter of an inch, the inch being supposed to be divided into 12 equal parts.

But it is more usual to measure by a foot decimally divided, and then the length or breadth consists of so many feet and decimal parts of a foot.

Round or oval windows, half-rounds, &c. are always measured at their full height and breadth, in consideration of the waste of glass and trouble such work is attended with.

Example 1. Suppose 6 panes of glass, each 4 feet 7 inches 3 quarters long, and 1 foot 5 inches 1 quarter broad, how many feet of glass are in the said 6 panes?

4.646	F. in. p.
1.437	4 7 9
	1 5 3
32522	
13938	I I II 3
18584	I II 2 9
4646	4 7 9
6.676302	6 8 I 8 3
6	6
Feet 40.057812	40 0 10 I 6

Example 2. Suppose 20 panes of glass, which measured by a decimal foot are each 2.458 feet long, and 1.75 feet broad, how many feet of glass do they contain?

$$\begin{array}{r}
 2.458 \\
 1.75 \\
 \hline
 12290 \\
 17206 \\
 2458 \\
 \hline
 4.30150 \\
 20 \\
 \hline
 \end{array}$$

Feet 86.030

7. Masons Work.

Masons measure their work, sometimes by the foot solid, sometimes by the square foot or yard, and sometimes by the rod, that is, 21 feet long and 3 feet high, viz. 63 square feet. But the rod differs in different places.

Example 1. A wall of hewn stone is 24 feet 6 inches long, 9 feet 6 inches high, and 2 feet 3 inches thick: How many solid feet are in that wall?

24.5	F. in.
9.5	24 6
	9 6
1225	
2205	220 6
	12 3
232.75	
2.25	232 9
	2 3
116375	
46550	465 6
46550	58 2 3
Feet 523.6875	523 8 3

Example 2. If a wall be 218 feet 9 inches long, and 12 feet 6 inches high, how many square feet, yards, and rods, are in it?

P p

218.75

218.75	F. in.
12.5	218 9
	12 6
109375	
43750	2625
21875	109 4 6
9)2734.375 feet.	2734 4 6
7)303.8194 yards.	
43.4027 rods.	

8. Paving.

Paving and caufewaying are measured by the square yards, of 9 square feet.

Ex. 1. A paved walk is 92 feet 6 inches long, and 8 feet 4 inches broad: How many yards does it contain?

92.5	F. in.
8.3	92 6
	8 4
740.0	
30.83	740
	30 10
9)770.83	770 10
Yards 85.6,481,	

Ex. 2. A stable, 19 feet 6 inches long, and 12 feet 6 inches broad, is to be floored or caufewayed with hard bricks, called *clinkers*, each 6 inches long, and 3 inches wide: How many clinkers will it require?

F. in. in.
19 6 = 234
12 6 = 150
1170
234
6 x 3 = 18)35100(1950 clinkers. <i>Ans.</i>
18...
171
162
90
90
—

V. Board and Timber Measure.

1. Board.

To measure board or plank is to find the superficial content or area of a rectangle. See Prob. 2.

R U L E.

Multiply the length in feet by the breadth in inches, divide the product by 12, and the quot is square feet, every remainder being one twelfth of a square foot.

Or, Reduce the length to inches; then multiply by the inches in the breadth; divide the product by 144, and the quot is square feet, the remainder being square inches.

Ex. 1. In a plank 14 feet long, and 8 inches broad, how many square feet?

Or thus :

$$\begin{array}{r} 14 \\ 8 \\ \hline 12 \overline{) 112} \quad (9 \frac{4}{12} = 9 \frac{1}{3} \text{ Ans.} \\ 108 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 14 \\ 12 \\ \hline 168 \\ 8 \\ \hline 144 \overline{) 1344} \quad (9 \frac{48}{144} = 9 \frac{1}{3} \\ 1296 \\ \hline 48 \end{array}$$

By Gunter's Scale.

Extend the compasses from 12 to the breadth 8; and that extent, set the same way, will reach from the length 14 feet to $9 \frac{1}{3}$.

By the sliding Rule.

Set the breadth 8 inches on the slip or slider to 12 on the rule, and against the length 14 feet on the rule you have on the slip $9 \frac{1}{3}$.

Ex. 2. In a plank 18 feet 6 inches long, and 15 inches broad, how many square feet?

Or thus:

$$\begin{array}{r}
 18.5 \\
 15 \\
 \hline
 925 \\
 185 \\
 \hline
 12)277.5 \\
 \hline
 23.125 = 23\frac{1}{8} \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 18.6 \\
 12 \\
 \hline
 222 \\
 15 \\
 \hline
 1110 \\
 222 \\
 \hline
 144)3330(23\frac{1}{4} = 23\frac{1}{4} \\
 288 \\
 \hline
 450 \\
 432 \\
 \hline
 18
 \end{array}$$

The operation by Gunter's scale and the sliding rule is the same as above.

If the two ends of a plank differ in breadth, add them, and take half their sum for a mean breadth.

If there be a number of planks of the same dimensions, measure one of them, and multiply the content by the number of planks.

Ex. 3. If a plank be 9 inches broad, how many inches in length will make a foot square?

$$9)144(16 \text{ inches in length. } \text{Ans.}$$

Hence is constructed the following table, shewing how many inches in length make a square foot, at any breadth, not exceeding 40 inches.

B.	L.	B.	L.	B.	L.	B.	L.	B.	L.
1	144	9	16	17	8.47	25	5.76	33	4.36
2	72	10	14.4	18	8	26	5.54	34	4.23
3	48	11	13.09	19	7.58	27	5.3	35	4.01
4	36	12	12	20	7.2	28	5.14	36	4
5	28.8	13	11.07	21	6.85	29	4.96	37	3.89
6	24	14	10.28	22	6.54	30	4.8	38	3.79
7	20.57	15	9.6	23	6.26	31	4.64	39	3.69
8	18	16	9	24	6	32	4.5	40	3.6

The use of the table is obvious. Look for the breadth of the board or plank under B, and under L you have the inches in length that make a square foot: Take this length in the compasses

ses from a scale of inches, and run that extent along the board, from end to end, and you have the number of square feet that board contains; or you may, by this means, cut off from that board any number of square feet required.

MORE EXAMPLES.

Length. Feet.	Breadth. Inches.	Content. Sq. feet.
14	18	21
9	$17\frac{1}{2}$	$13\frac{1}{8}$
$11\frac{1}{4}$	$7\frac{3}{4}$	$7\frac{1}{4}$
$9\frac{3}{4}$	$13\frac{1}{4}$	$10\frac{3}{4}$
$8\frac{1}{4}$	22	$15\frac{1}{8}$
$14\frac{1}{2}$	20	$24\frac{1}{6}$

2. Timber.

To measure timber is to find the solid content of a cube, parallelopiped, prism, or cylinder. See Prob. 23. 24. and 25.

Equal squared Timber.

By equal squared timber is meant, such as is in the form of a parallelopiped, whose bases are equal squares or rectangles; and its solidity is found by multiplying the area of the base by the length.

Example 1. In a piece of squared timber 18 feet long, the side of the square base being 15 inches, how many solid feet?

	F. in.
15	1 3
15	1 3
—	—
75	1 3
15	3 9
—	—
225	1 6 9
18	9
—	—
1800	14 0 9
225	2
—	—
144)4050(28.125	28 1 6

Anf. 28 feet, and half a quarter.

Instead of dividing by 144, you may divide by the component parts 12×12 . Or you may reduce the length to inches, and then divide the product by 1728, or by $12 \times 12 \times 12$.

In working by feet and inches, instead of multiplying by 18, I multiply by the component parts 9×2 .

By Gunter's scale.

Extend the compasses from 12 to 15 inches, the side of the square; and that extent twice set will reach from the length 18 feet to the answer 28 feet, and something more.

By the sliding Rule.

Set the length in feet on the slip to 12 on the girt line, and against the side of the square 15 inches, on the girt line, you have the answer on the slip, 28 feet, and somewhat more.

Example 2. In a piece of squared timber 32 inches broad, 18 inches thick, and 14 feet 6 inches long, how many solid feet?

32	F. in.
18	2 8
—	1 6
256	—
32	2 8
—	1 4
576	—
14.5	4 0
—	14 6
2880	—
2304	56
576	2
—	—
144)8352.0(58 feet.	58 feet. Ans.

The operation by Gunter's scale and the sliding rule is the same as above, only the side of a square equal to the area of the base must be found, by extracting the square root of $576 = 32 \times 18$, viz. 24; or it may be found by dividing the extent between 18 and 32 on the scale into two equal parts, and the middle point will be 24, the side of the square sought. But on the sliding rule, set the lines C and D even at the ends; and against any square number on C you have its square root on D.

Some persons, of inferior skill, add the depth and breadth together, and take half their sum for the side of the square. But this method is false; and if the depth and breadth differ widely, the error thence resulting will be considerable. In the above example $32 + 18 = 50$, and half of 50 is 25 for the side of the square; and $25 \times 25 = 625$ for the area of the base; which multiplied into the length, would give a product by far too great.

Divide 1728, the cubical inches in a solid foot, by the area of the

the base in inches, and the quot will be the inches in length that make a solid foot.

The above rule is general, and extends to timber of all sorts, that is of equal breadth and thickness from end to end, whether the base be square, triangular, multangular, or round.

Hence is constructed the following table, which shews how many inches in length make a solid foot, the side of the square, equal to the area of the base, not exceeding 30 inches.

B.	Length.	B.	L.	B.	L.	B.	L.	B.	L.
1	1728	7	35.26	13	10.22	19	4.78	25	2.76
2	432	8	27	14	8.81	20	4.32	26	2.55
3	192	9	21.3	15	7.68	21	3.92	27	2.37
4	108	10	17.28	16	6.75	22	3.57	28	2.2
5	69.12	11	14.28	17	5.94	23	3.26	29	2.05
6	48	12	12	18	5.3	24	3	30	1.92

The use of the table is plain. Seek the side of the square, equal to the area of the base, under B; and under L you have the number of inches in length that make a solid foot. Take this length from a scale of inches, and with it run the piece of timber from end to end, and you have the number of solid feet it contains. By this means too you may cut off any number of solid feet desired.

Unequal squared timber.

By unequal squared timber, is meant any piece of squared timber whose bases are unequal, whether they be squares or rectangles; and such are most timber trees, when hewn and brought to their squares, the root-end being generally grosser than the other.

The usual customary way of measuring such timber is to take the square or rectangle at the middle of the piece for a mean base, and then proceed as taught above.

Example 1. In a piece of squared timber 20 feet long, the side of the square base at the greater end is 25 inches, and at the lesser end 9 inches: How many feet of timber are in that tree?

$$\begin{array}{r}
 25 \\
 9 \\
 \hline
 2)34 \\
 \hline
 17 \text{ the side of the square} \\
 \text{in the middle.}
 \end{array}
 \qquad
 \begin{array}{r}
 17 \\
 17 \\
 \hline
 119 \\
 17 \\
 \hline
 289 \\
 20 \\
 \hline
 \text{Feet.}
 \end{array}$$

144)5780(40.13 *Anf.*

Example 2. In a piece of timber 18 feet long, the base at the greater end being 32 inches by 20, and at the lesser end 16 inches by 10 : How many feet of timber ?

32	20	24
16	10	15
—	—	—
2)48	2)30	120
—	—	24
24	15	—
		360 area of mean base.
		18
		—
		288
		36
		— Feet.
		144)6480(45 <i>Ans.</i>

This customary way is erroneous ; and the greater the difference between the two bases is, the greater will the error be. But notwithstanding this, universal custom prevails, and purchasers will accept of no other sort of measure. I shall, however, here observe, that a piece of unequal squared timber is the frustum of a pyramid ; and the content may be accurately computed by Prob. 26. The former example done in this way follows :

32	16	320
20	10	640
—	—	160
640	160	—
160		3)1120(373.3
—		18
384		—
64		29868
—		37333
102400(320		—
9		144)6720.0(46.6
—		
62)124		
124		
—		
		<i>Ans.</i> 46 $\frac{2}{3}$ feet.

Round timber having equal bases.

The customary way of measuring such timber is, to gird the tree about the middle with a small cord, and then take the fourth part of this girt for the side of a square, with which they proceed as in squared timber.

Example

Example 1. If a tree be 72 inches in circumference, or girt, and 24 feet long, how many feet of timber are in it?

A fourth of 72 is 18	F. in.
18	1 6
—	1 6
144	—
18	1 6
—	9
Area of the base 324	—
24	2 3
—	24
1296	—
648	48
— F.	6
144)7776(54 <i>Anf.</i>	—
	54

Example 2. If a piece of round timber be 54 inches in girt, and 22 feet 6 inches long, how many feet are contained in it?

$\frac{1}{4}$ of 54 is 13.5	F. in. p.
13.5	1 1 6
—	1 1 6
675	—
405	6 9 0
135	1 1 6
—	1 1 6
182.25	—
22.5	1 3 2 3
—	22 6
91125	—
36450	7 7 1 6
36450	27 10 1 6
— F.	—
144)4100.625(28.476 <i>Anf.</i>	28 5 8 7 6

This customary way is false and erroneous; for if the circumference of a circle be 1, the area will be .07958; now the fourth part of 1 is .25, the square whereof is .0625, which is considerably less than the true area .07958; and consequently the solidity thus computed will be considerably less than the true content.

But this customary way, false as it is, being established by long practice, is universally received, and merchants will not purchase timber at any other measure.

I shall here, however, show how the exact content may be found, by working the former example in a way that will give a true answer.

54	Brought up, 232.055
54	22.5
216	1160275
270	464110
	464110
2916	
.07958	Feet.
	144)5221.2375(36.25
23328	True content 36.25
14580	Falle content 28.47
26244	
20412	Error 7.78
Carried up, 232.05528	

Round timber having unequal bases, called tapering timber.

The customary way of measuring tapering timber is to divide the tree into parts of eight or ten feet long, and then take the girt at the middle of each part, by which, as taught above, is computed the solidity of each part, and their sum is the solid content of the whole tree.

Example. A tree 20 feet in length is divided into two parts, each 10 feet long, the middle girt of one of the parts being 64 inches, and of the other 36 inches: How many cubic feet of timber are in that tree?

$\frac{1}{4}$ of 64 is 16	$\frac{1}{4}$ of 36 is 9
16	9
96	81
16	10
256	810
10	
2560	
810	
144)3370(23.4 feet. <i>Anf.</i>	
288	
490	
432	
580	
576	
4	

This

This way of dividing the tree into parts is far more accurate than working by the middle girt of the whole tree ; and the shorter the parts are, the nearer the truth will the answer be.

But this way of measuring has been shown to be erroneous when the bases are equal ; and it is still more so in timber that tapers ; and the more tapering the tree is, the greater will the error be.

The truth is, a piece of tapering timber ought to be considered as the frustum of a cone, and the solidity computed by the directions given in Prob. 27.

MORE EXAMPLES.

<i>Length.</i> <i>Feet.</i>	$\frac{1}{4}$ <i>of the girt.</i> <i>Inches.</i>	<i>Content.</i> <i>Cubic feet.</i>
18	8	8
12	15	$18\frac{3}{4}$
22	$8\frac{1}{2}$	11
$27\frac{1}{2}$	19	68.9

VI. Gauging.

My design in this place is not to explain the principles or practice of gauging at any great length, but only to give the reader some general notion of that art, so as to enable him to find the content of any common cask or vessel, or of a cistern, or floor of malt.

P R O B. I.

To find divisors, multipliers, and gauge-points, with their uses.

282	cubic inches make an ale-gallon.
231	cubic inches make a wine-gallon.
268.8	cubic inches make a corn-gallon.
2150.42	cubic inches make a corn or malt bushel.

The numbers, then, 282, 231, 268.8, 2150.42, are the divisors ; and if 1 be divided by these, the quotients thence arising will be the equivalent multipliers, as exhibited in the following table :

TABLE I. For right-lined figures.			
<i>Divisors.</i>		<i>Multipliers.</i>	
282	A. G.	.003546	A. G.
231	W. G.	.004329	W. G.
268.8	C. G.	.0037202	C. G.
2150.42	M. B.	.00046502	M. B.

If the cubic inches in any vessel be divided or multiplied by these divisors

divisors or multipliers, the result will be ale-gallons, wine-gallons, &c.

Again, if the superficial inches or area of any right-lined figure be divided or multiplied by these divisors or multipliers, the result will be the area in ale or wine-gallons, &c.; that is, the number of ale or wine gallons contained in the same at 1 inch deep.

Ex. Suppose a plain figure, in form of a rectangle, 232 inches in length, and 64 inches in breadth, what is the area in ale, wine-gallons, &c.?

$$232 \times 64 = 14848 \text{ area in inches.}$$

By division.

$$\begin{array}{rcl} 282 &)14848 & (52.652 \text{ ale-gallons.} \\ 231 &)14848 & (64.277 \text{ wine-gallons.} \\ 268.8 &)14848 & (55.238 \text{ corn-gallons.} \\ 2150.42 &)14848 & (6.904 \text{ malt-bushels.} \end{array}$$

By multiplication.

$$\begin{array}{rcl} 14848 \times .003546 & = & 52.651 \text{ ale-gallons.} \\ 14848 \times .004329 & = & 64.277 \text{ wine-gallons.} \\ 14848 \times .0037202 & = & 55.238 \text{ corn-gallons.} \\ 14848 \times .00046502 & = & 6.904 \text{ malt-bushels.} \end{array}$$

But for circular areas another set of divisors and multipliers must be obtained, in the following manner, *viz.* .785398 is the area of a circle whose diameter is 1; by which, if the numbers 282, 231, 268.8, 2150.42, be divided, the quots will be a set of divisors. And if .785398 be divided by the same numbers, *viz.* 282, 231, &c. the quots will be a set of multipliers, as in the following table:

TABLE II. For circular areas.			
Divisors.		Multipliers.	
359.05	A. G.	.002785	A. G.
294.12	W. G.	.003399	W. G.
342.24	C. G.	.002922	C. G.
2738	M. B.	.00036523	M. B.

If the square of the diameter of any circle be divided or multiplied by these divisors or multipliers, the result will be the area in ale, wine, corn-gallons, or malt-bushels; that is, the number of ale, wine-gallons, &c. contained in the same at 1 inch deep.

Ex. Suppose the diameter of a circle to be 72 inches, what is the area in ale, wine-gallons, &c.?

$72 \times 72 = 5184$ the square of the diameter.

By Division.

359.05)5184(14.438 ale-gallons.
 294.12)5184(17.625 wine-gallons.
 342.24)5184(15.147 corn-gallons.
 2738)5184(1.893 malt-bushels.

By Multiplication.

5184 $\times$.002785 = 14.437 ale-gallons.
 5184 $\times$.003399 = 17.620 wine-gallons.
 5184 $\times$.002922 = 15.147 corn-gallons.
 5184 $\times$.00036523 = 1.893 malt-bushels.

If the area in ale, wine-gallons, &c. of an ellipse, be required, multiply the greater and lesser diameter together; and then divide or multiply their product by the divisors or multipliers for circular areas; and the result will be the area of the ellipse in ale, wine-gallons, &c.

Gauge-points are the sides of squares, or the diameters of circles, whose content, at 1 inch deep, is one gallon, one bushel, &c. and consequently are the square roots of the divisors exhibited in the two preceding tables. These gauge-points are marked on the sliding rule, for rendering the operation by that instrument more simple, and are as follow:

<i>Gauge-points for squares.</i>		<i>Gauge-points for circular areas.</i>	
16.79	A. G.	18.95	A. G.
15.19	W. G.	17.15	W. G.
16.39	C. G.	18.5	C. G.
46.37	M. B.	52.32	M. B.

The way of using these gauge-points will appear in what follows:

P R O B. II.

To find the content in ale, wine-gallons, &c. of any parallelopiped.

R U L E.

Find the solid content in inches, by multiplying the three dimensions continually; and this product, divided or multiplied by the divisors or multipliers in Table I. will give the content in ale, wine-gallons, &c.

Or, Find the area of the base in ale, wine-gallons, &c. by Prob. I.; and then multiply that area by the height or depth.

Ex.

Ex. Suppose a parallelopiped, trough or cistern, to be 81 inches long, 25 inches broad, and 26 inches deep : Required the content in ale, wine-gallons, &c.

81	282	)52650(186.7 ale-gallons.
25	231	)52650(227.92 wine-gallons.
—	2150.42	)52650(24.48 malt-bushels.
405		
162		
—		
Area 2025		
26		
—		
12150		
4050		
—		
52650		

Or thus :

$$282)2025(7.18$$

And $7.18 \times 26 = 186.68$ ale-gallons, &c.

By the sliding Rule.

In order to work by the sliding rule, you must find the side of a square equal to the area of the base, by extracting the square root of 2025; or, by the rule, thus : Set the lines C and D even at the ends ; and against any square number on C, you have the root on D, which in this example is 45 ; and accordingly,

Set the gauge-point 16.79 upon D, to the depth 26 upon C, and against the side of the square 45 upon D you have on C 186.7, the content in ale-gallons.

The operation for wine-gallons, malt-bushels, &c. is the same, only use the proper gauge-points.

P R O B. III.

To find the content in ale, wine-gallons, &c. of a vessel whose bases are similar and parallel, but unequal, being the frustum of a pyramid.

R U L E.

Find the area of each base, and a mean proportional between them, and multiply the sum of these three by one third of the depth or height, and the product is the content in cubic inches ; which, divided or multiplied by the divisors or multipliers in Table I. gives the content in ale, wine-gallons, &c.

Ex. Suppose a vessel, whose bases are rectangles, the length of the greater base being 100 inches, and its breadth 70 inches, the length of the lesser base 80, and its breadth 56, and the depth of the vessel 42 inches : Required the content in ale, wine-gallons, &c.

$$\begin{array}{l} 100 \times 70 = 7000 \\ 80 \times 56 = 4480 \\ \text{Mean proportional } 5600 \end{array} \left. \vphantom{\begin{array}{l} 100 \times 70 = 7000 \\ 80 \times 56 = 4480 \\ \text{Mean proportional } 5600 \end{array}} \right\} 7000 \times 4480 = 31360000, \text{ whose} \\ \text{square root is } 5600.$$

A third of the depth $\frac{17080}{14}$

$$\begin{array}{r} 6832 \\ 1708 \end{array}$$

282)239120(847.94 ale-gallons.

231)239120(1035.15 wine-gallons.

By the sliding Rule.

Set the gauge-point 16.79 upon D, to 14, one third of the depth on C; and against 130.7, the square root of the triple mean base 17080, upon D, you have on C 847.94, the content in ale-gallons.

P R O B. IV.

To find the content of a cylinder in ale, wine-gallons, &c.

R U L E.

Multiply the square of the diameter by the depth or height, and divide the product by the divisors for circular areas in Table II.

Ex. Suppose the diameter of a cylinder to be 56.5 inches, and the height 96 inches: Required its content in ale, wine-gallons, &c.

$$\begin{array}{l} 56.5 \times 56.5 \times 96 = 306456 \\ 359.05)306456(853.52 \text{ ale-gallons.} \\ 294.12)306456(1041.94 \text{ wine-gallons.} \\ 2738)306456(111.92 \text{ malt-bushels.} \end{array}$$

By the sliding Rule.

Set the proper gauge-point on D to the depth on C, and against the diameter on D you have the content on C.

P R O B. V.

To find the content of a vessel in the form of the frustum of a cone, in ale, wine-gallons, &c.

R U L E.

This may be done, as in Prob. 3. by multiplying the sum of the areas of the two bases and a mean proportional, by the third part of the depth, but more easily in the following manner:

To

To the product of the two diameters add one third part of the square of their difference; that sum is the square of a mean diameter; which multiply by the height or depth, and the product, divided by the divisors in Table II. gives the content in ale, wine-gallons, &c.

Ex. Suppose the greater diameter be 56.5 inches, the lesser diameter 19 inches, and the height 62 inches: Required the content in ale, wine-gallons, &c.

$$56.5 \times 19 = 1073.5 \text{ product of the diameters.}$$

19

$$\text{Diff. } 37.5 \times 37.5 = 1406.25 \text{ square of their difference.}$$

$$\begin{array}{r} 13)1406.25(468.75 \\ 1073.5 \\ \hline 1542.25 \text{ square of a mean diameter.} \end{array}$$

62

308450

925350

$$\begin{array}{l} 359.05)95619.50(266.31 \text{ ale-gallons.} \\ 294.12)95619.50(325.1 \text{ wine-gallons.} \\ 2738)95619.50(34.92 \text{ malt-bushels.} \end{array}$$

By the sliding Rule.

Set the proper gauge-point on D to the depth on C; and against 39.27, the mean diameter or square root of 1542.25 on D, you have the content on C.

Gaugers usually inch all such vessels; that is, they compute the content at every inch deep, and enter these contents in a table; and when they come to survey, they have only to take the depth, and by comparing the wet inches with the table, they have the content by inspection.

P R O B. VI.

To find the drip or fall of a tun.

The drip or fall of a tun is when the vessel is set a little reclining, for conveniency of cleansing, there being a plug hole at C for the wash to run off at.

By this position it will happen, that the liquor will rise to E on one side, when the bottom is but just covered at D on the other; and when the liquor rises to B, the part A B G will be empty.

Now,

Now, if the content of the empty part A B G be subtracted from the content of the whole vessel in a perpendicular position, the remainder will be the content of the vessel in this oblique position. It is also required to find how much liquor will cover the bottom, or fill the part D C E.

	Inches.
Suppose the diameter	A B = 48
the diameter	D C = 40
the height	m n = 60
the diameter	G H = 45.5
the diameter	F E = 42.1
the perpendicular	A L = 17.46
the height	n o = 15.53

The content of the whole tun in a perpendicular position, found by Prob. 5. is 324.2 ale-gallons.

To find the content of the part A B G, observe the following rule:

To the square of the top-diameter A B 48, add one half of the product of the said top-diameter A B and bottom-diameter G H 45.5; multiply this sum by the perpendicular A L 17.46; and divide the product by triple the divisors in Table II. viz. by 1077.15 for ale, and by 882.36 for wine.

Square of A B 48 = 2304, and A B 48 $\times$ G H 45.5 = 2184
Half of 2184 = 1092

$$3396 \times 17.46 = 58294.16$$

1077.15) 58294.16 (54.12 ale-gallons.

Tun's content 342.2 A. G.

Content of A B G 54.12 A. G.

Remains G B C D 288.08 A. G.

To find how much liquor will cover the bottom, or fill D C E, work as follows:

To the square of the bottom-diameter D C 40, add half the product of the said bottom-diameter and top-diameter F E 42.1; multiply this sum by the perpendicular height n o 15.53, and divide the product by triple the divisors in Table II. viz. by 1077.15 for A. G. and by 882.36 for W. G.

Q q

Square

Square of D C 40 = 1600, and D C 40 $\times$ F E 42.1 = 1684

Half of 1684 = 842

$$2442 \times 15.53 = 37924.26$$

1077.15)37924.26(35.207 ale-gallons.

If the frustum of a pyramid stand reclined, instead of the divisors assigned above, you must use the triple of the divisors in Table I. *viz.* 846 for ale, and 693 for wine, the rest of the work being the same.

P R O B. VII.

To gauge a copper with a rising crown.

Take the dimensions thus :

Extend a pack-thread over the middle from A to B.

Set one end of a rod in C and D, and move it forward and backward, till you find the shortest depth E C and G D; or, instead of a rod, you may hang a plummet.

Next, set the end of the rod on the top of the crown at o, and you have F o, which subtracted from E C, leaves m C the height of the crown.

E G is equal to C D, the diameter at the bottom of the crown, and the diameter m n may be measured.

Now, suppose the dimensions thus taken to be as follows :

<i>Inches.</i>	<i>Inches.</i>
E C = 47	A B = 99
F o = 42	C D = 64
m C = 5	m n = 65

Now to find the content of the copper from the crown upwards, the depth being 42 inches, take the diameter in the middle of every four, or of every six inches, (the more diameters you take the better), and insert these diameters in the second column of the following table. Then, by Prob. I. find the areas of the circles answering to these diameters in ale-gallons, and insert them in the third column; and next multiply each of these areas by 6, and insert the products in the fourth column; and their sum will be the content of A B m n; that is, so much as the copper will hold after the crown is covered.

The crown is to be considered as the frustum of a sphere, and its content, by Prob. 29. will be found to be 28.75 ale-gallons. But the content of the crown may more readily, and with sufficient accuracy, be found as follows:

The diameter C D is 64, and the area to this diameter is 11.408, which

which multiply by half the height of the crown, *viz.* 2.5, and the product is 28.52 for the content of the crown.

The content of the part *m n D C*, considered as the frustum of a cone, will be found to be 57.935 gallons; from which subtract the content of the crown 28.52, and the remainder is 29.415 gallons, the quantity of liquor that will just cover the crown.

<i>Parts of the depth.</i>	<i>Diameters.</i>	<i>Areas in ale-gal- lons.</i>	<i>Content of every 6 inches.</i>
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	80.	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	83.056
6	66.	12.1319	72.791
Sum	-	-	765.451
To just cover the crown			29.415
Content of the copper	-		794.866

P R O B. VIII.

To gauge a Cask.

The finding the content of casks is a difficult part in gauging, the shape of casks being so different, that not any one rule will serve for all. Gaugers therefore commonly distinguish casks into four forms or varieties, *viz.* 1. Such as resemble the middle frustum of a spheroid; 2. The middle frustum of a parabolic spindle; 3. The lower frustums of two equal parabolic conoids; 4. The lower frustums of two equal cones, as represented in fig. 1.

1. The utmost line in the figure, which exhibits the staves as very much curved, or arching, represents a cask of the first form; and is considered as the middle frustum of a spheroid, being the most capacious sort of cask, or what will hold more liquor than a cask of any other form.

2. The curved line next to the utmost line represents a cask of the second form; and is considered as the middle frustum of a parabolic spindle, being less capacious than the spheroid.

3. The third curved line represents a cask of the third form, which is considered as the lower frustums of two equal parabolic conoids abutting upon one common base; and is less capacious than any of the two former.

4. The innermost line, which is quite straight from bung to head,

Q q 2

head, represents a cask of the fourth form, and is considered as the lower frustums of two equal cones abutting upon one common base, being of all sorts of casks the least capacious.

Various rules are assigned in books of gauging for ascertaining the contents of these casks, but the most easy and practical way is, to find such a mean diameter as will reduce the proposed cask to a cylinder; and this may be done pretty accurately as follows:

Multiply the difference of the bung and head diameter by .7 for the spheroid, by .65 for the spindle, by .6 for the conoids, and by .55 for the cones, and add the product to the head diameter, and the sum is the mean diameter.

Example. Suppose the bung diameter be 32 inches, the head diameter 24 inches, and the length 40 inches; the content in each variety is required.

The difference between the bung and head diameter is 8, and

Mean diameter.

$$8 \times .7 = 5.6, \text{ and } 24 + 5.6 = 29.6 \text{ for the spheroid.}$$

$$8 \times .65 = 5.2, \text{ and } 24 + 5.2 = 29.2 \text{ for the spindle.}$$

$$8 \times .6 = 4.8, \text{ and } 24 + 4.8 = 28.8 \text{ for the conoids.}$$

$$8 \times .55 = 4.4, \text{ and } 24 + 4.4 = 28.4 \text{ for the cones.}$$

The content in ale-gallons, found by Prob. 4. follows:

A. G.

$$29.6 \times 29.6 \times .002785 \times 40 = 97.6 \text{ for the spheroid.}$$

$$29.2 \times 29.2 \times .002785 \times 40 = 94.98 \text{ for the spindle.}$$

$$28.8 \times 28.8 \times .002785 \times 40 = 92.4 \text{ for the conoids.}$$

$$28.4 \times 28.4 \times .002785 \times 40 = 89.85 \text{ for the cones.}$$

By the sliding Rule.

Set the gauge-point upon D to the length of the cask upon C; and against the mean diameter on D you have the content upon C.

It may not be improper to observe here, that however geometrical and accurate the rules for cask-gauging may be in themselves, yet the content found by them can only come near the truth, because casks have only some resemblance, but do not answer exactly in shape, to the form described above.

Hitherto we have considered casks as quite full; it now remains to point out a method of finding how much liquor is in casks that are not full. This is called *ullaging*; the quantity of liquor in the cask being the ullage; which, subtracted from the content of the whole cask, leaves the quantity that would fill the empty space or vacuity.

Now, the cask may either be supposed to lie with its axis parallel to the horizon, as represented in Pl. 4. fig. 2. or it may stand upright,

upright, having its axis perpendicular to the horizon. The former of these cases we shall consider in the first place, and the latter afterwards. And in both cases the rules shall be adapted to the spheroidal cask; because, in the practice of excise, most casks are considered as such.

It is usual to find the ullage by means of a table of segments suited to a cylindrical cask; but as every one may not be provided with such a table, we shall here assign a rule for finding the ullage, and that pretty accurately, without any table, as follows:

R U L E.

Divide the wet inches, taken at the bung, by the bung-diameter; and if the quot exceed .5, subtract .5 from it, and add one fourth part of the remainder to the former quot, and then multiply this sum by the content of the whole cask; the product is the ullage.

But if, upon dividing the wet inches by the bung-diameter, the quot be less than .5, subtract the quot from .5, and from the said quot subtract one fourth part of the remainder, and then multiply this last remainder by the content of the whole cask.

E X A M P L E.

	Inches.
Suppose the dimensions of the given cask to be	
Length	32.5
Bung-diameter	31
Wet	21
Dry	10
Content	75.37 A. G.

$$31 \overline{) 21.000000} \begin{matrix} .677419 \\ .5 \end{matrix}$$

$$4 \overline{) .177419} \begin{matrix} .044355 \\ .677419 \end{matrix}$$

.721774 sum.

$$\text{And } .721774 \times 75.37 = 54.40010638 \text{ ale-gallons.}$$

Subtract the ullage 54.4 from 75.37, the content of the cask, and the remainder will be the quantity of liquor that would fill the vacuity. Or, find the content of the vacuity thus: Divide the dry inches by the bung-diameter, and then proceed in all respects as you did in finding the ullage.

If the cask stand on end, as represented in Pl. 4. fig. 3. the quantity of liquor it contains may be found by the following rules:

1. Find the diameter DF at the surface of the liquor, by laying two rulers on different sides of the cask, and at the height of the liquor,

liquor, so as to be parallel; then measure the distance between the rulers, and from it subtract the thickness of the staves on both sides.

2. To twice the square of the diameter D F add the square of the head-diameter L M, multiply the sum by the wet inches, and divide the product by the triple of the divisors for circular parts, viz. by 1077.15 for ale-gallons, and by 882.36 for wine-gallons.

E X A M P L E.

		Inches.
Suppose the dimensions to be	Length B G	32.5
	Bung-diameter H I	27.
	Head-diameter L M	23.
	Wet E G	8.5
	Diameter D F	26.14
	Content of the cask	59.95 A. G.

$$D F \ 26.14 \times 26.14 \times 2 = 1366.6$$

$$\text{Add } 23 \times 23 = 529$$

$$\hline 1895.6$$

$$\text{Wet inches } 8.5$$

$$\hline 94780$$

$$151648$$

$$\begin{array}{r} 1077.15 \ 16112.60 \ (14.95 \text{ A. G.} \\ \text{Cask's content} \quad - \quad 59.95 \\ \hline \end{array}$$

Wanted to fill the cask 45.

If you work with the dry inches, the result will be the quantity of liquor that would fill the vacuity; which, subtracted from the content of the whole cask, will leave the quantity of liquor in the cask.

P R O B. IX.

To gauge Malt.

R U L E.

If the malt be in a cistern, or lie on a floor, in a rectangular form, find the area of the base in bushels, by multiplying the length into the breadth, and dividing the product by 2150.42; then multiply this quot by the mean depth; and the product is the content in bushels.

If the base be circular or elliptical, divide the square of the diameter, or product of the two elliptical diameters, by 2738, and multiply the quot by the mean depth.

Ex.

Ex. 1. There is a cistern, whose length is 84 inches, and breadth 54 inches, and the mean depth 43.6 inches: Required the content,

$$84 \times 54 = 4536 \quad \text{And } 2150.42)4536.00(2.1093 \\ \text{And } 2.1093 \times 43.6 = 91.965 \text{ bushels.}$$

Ex. 2. There is a malt-floor, whose length is 245 inches, and the breadth 184 inches, and the mean depth 5.6 inches: Required the content.

$$245 \times 184 = 45080 \quad \text{And } 2150.42)45080.00(20.963 \\ \text{And } 20.963 \times 5.6 = 117.3928 \text{ bushels.}$$

By the sliding Rule.

Set the depth on the line M D to the length on the line N upon the slider; and against the breadth on the line A you have the content on the line marked B upon the slider.

F I N I S.

Directions to the BOOKBINDER.

Place the plate of Napier's rods to front page 61.

Place the four copper-plates at the end, in successive order; and sew them, so as that every plate may, when opened out, be wholly without the book.

BOOKS written by JOHN MAIR, A. M. printed for and sold by
JOHN BELL, and WILLIAM CREECH, Edinburgh.

1. BOOK-KEEPING MODERNISED, or Merchants-accounts by Double Entry, according to the Italian Form; wherein the Theory of the Art is clearly explained, and reduced to Practice, in copious Sets of Books, exhibiting all the varieties that usually occur in Real business. To which is added a large APPENDIX. (Fourth Edition.) One Volume octavo, the same as the Arithmetic. Price bound 7s.

2. The TYRO'S DICTIONARY, Latin and English; comprehending the most usual Primitives of the Latin Tongue, digested alphabetically, in the Order of the Parts of Speech. To which are subjoined, in a smaller character, on the lower part of the pages, Lists or Catalogues of their Derivatives or Compounds; designed as an easy and speedy Method of introducing Youth to a general Acquaintance with the Structure of the Language, and preparing them for the use of a larger Dictionary. (Fourth Edition.) Price 4s.

3. An INTRODUCTION to LATIN SYNTAX; or, An Exemplification of the Rules of Construction, as delivered in Mr Ruddiman's Rudiments, without anticipating posterior Rules. To which is subjoined, An Epitome of Ancient History, from the Creation to the Birth of Christ. (Eighth Edition.) Price 2s.

4. A RADICAL VOCABULARY, Latin and English; comprehending the most usual Primitives of the Latin Tongue, digested alphabetically, in the Order of the Parts of Speech. To which is subjoined, An Appendix; containing Rules for the Gender of Nouns, and for the Preterites and Supines of Verbs, in English Prose; as also, An Explication of the Kalends, Nones, and Ides. (Fourth Edition.) Price 1s.

5. THE FIRST FOUR BOOKS OF C. JULIUS CÆSAR'S COMMENTARIES OF HIS WARS IN GAUL; with an English Translation, as literal as possible, and large Explanatory Notes, (Fourth Edition.) Price 2s.

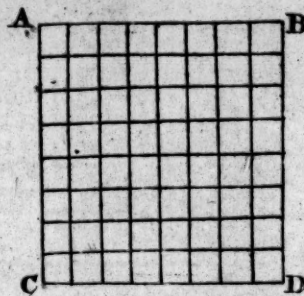
6. SALLUST'S HISTORY of CATILINE'S CONSPIRACY, and the WAR with JUGURTHA; with an English Translation, as literal as possible, and large Explanatory Notes. (Fourth Edition.) Price 2s. 6d.

7. A Select Century of CORDERY'S COLLOQUIES; with an English Translation, as literal as possible. (Third Edition.) Price 1s.

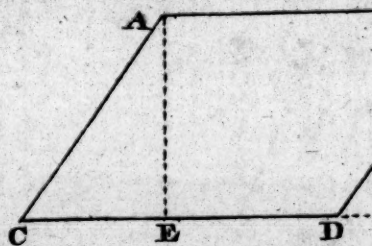
8. A Brief SURVEY of the TERRAQUEOUS GLOBE: Containing, 1. The Description and Use of the Globes; 2. The Construction and Use of Maps; 3. Geography, or a short View of the Earth's Surface, considered as inhabited by various Nations, &c. with Maps of the ancient Roman Empire and Modern World. (Third Edition.) Price 3s. 6d.



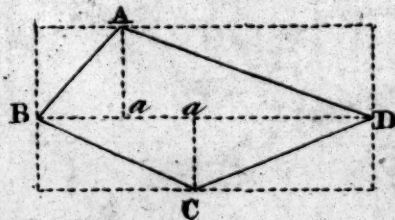
PROB. I.



PROB. IV.



P. VIII.



P. XI.

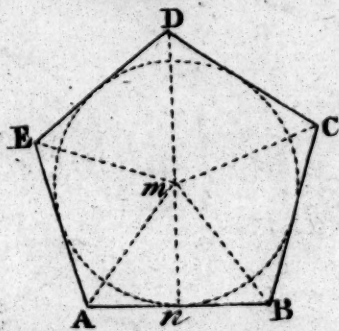
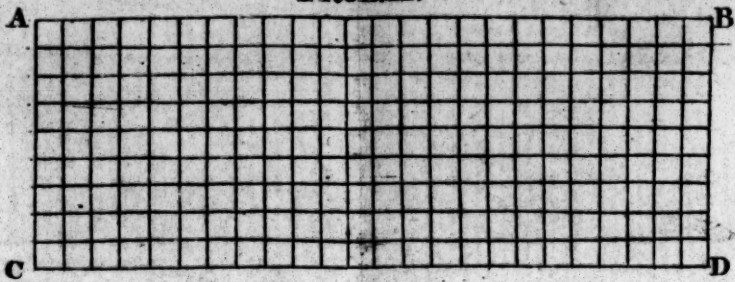
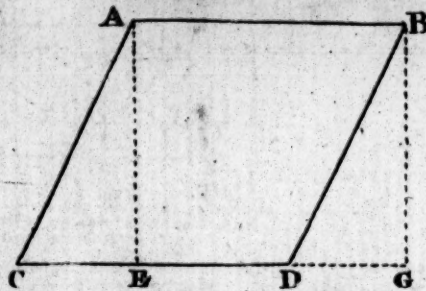


PLATE. I.

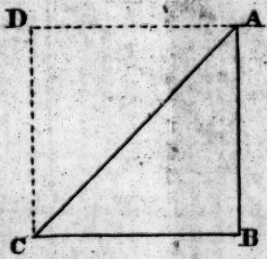
PROB. II.



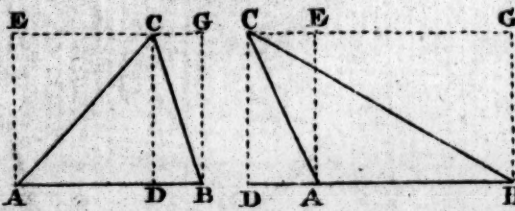
PROB. III.



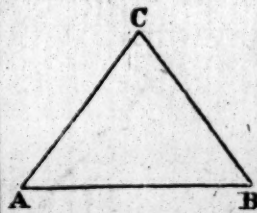
P. v.



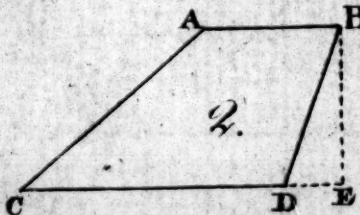
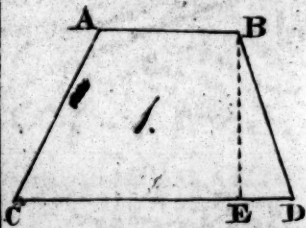
P. vi.



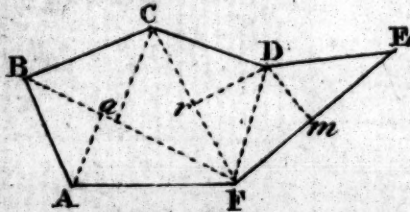
P. vii.



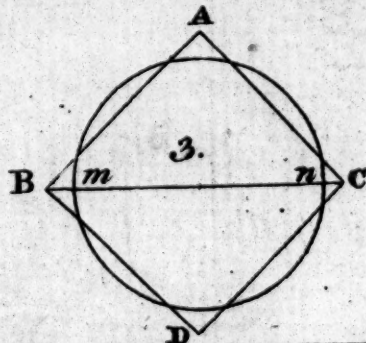
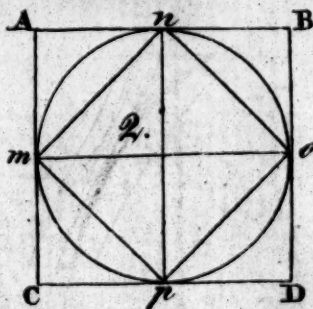
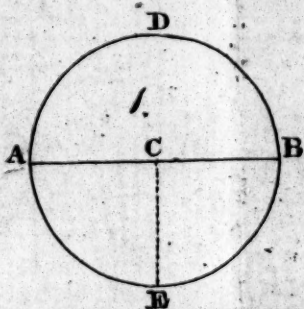
PROB IX

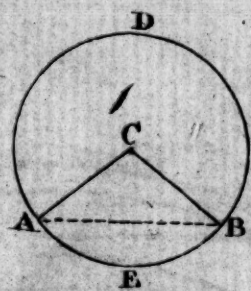


P. x.

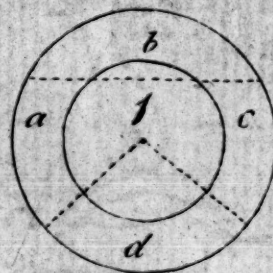


PROB. XII.

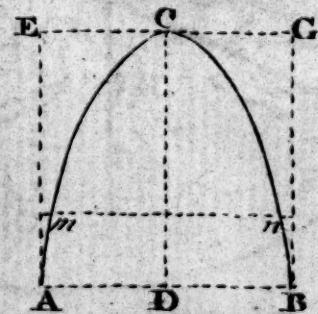




PRO.



PROB. XVII



PROB. XX

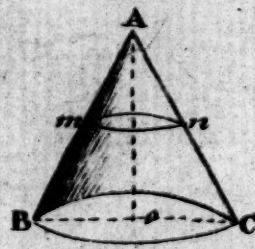
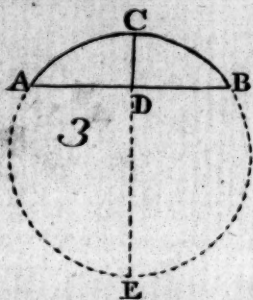
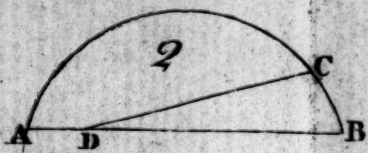
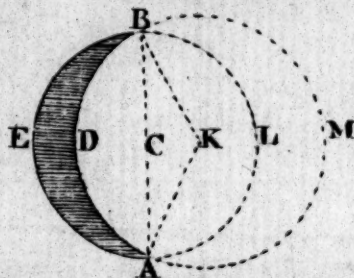


PLATE II

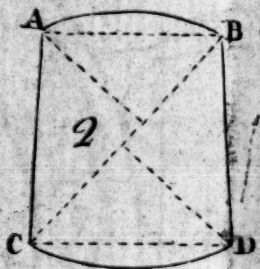
PROB. XIII



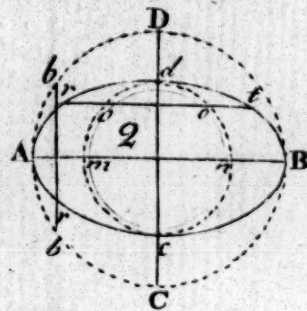
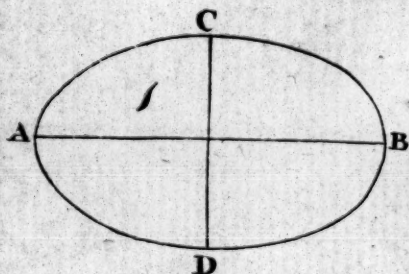
PROB. XIV



PROB. XV



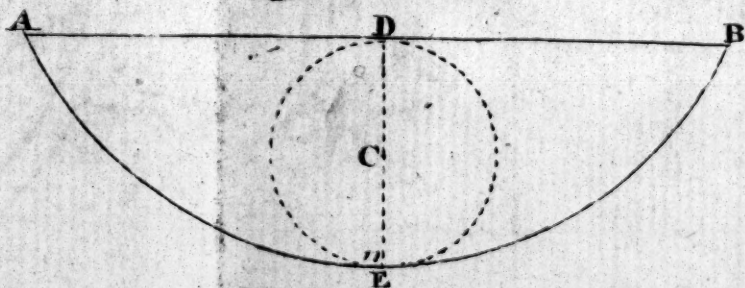
PROB. XVI



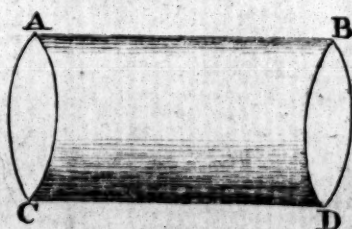
VII



PROB. XVIII



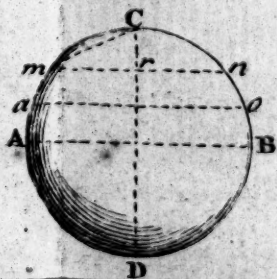
PROB. XIX



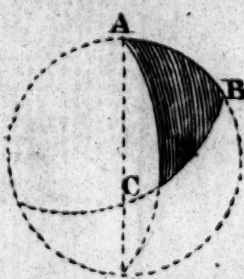
X



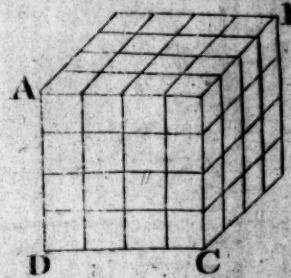
PROB. XXI



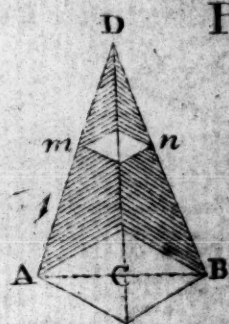
PROB. XXII



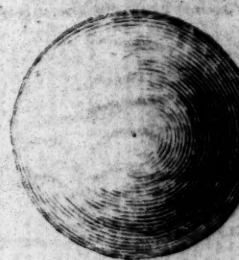
PROB. XXIII



PRO



PROB. XXVIII



PROB. XXXII

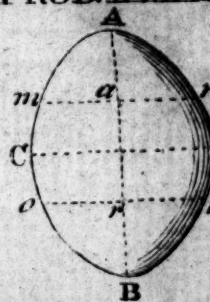
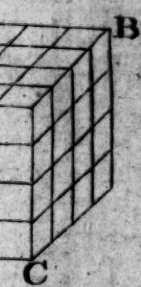


PLATE III

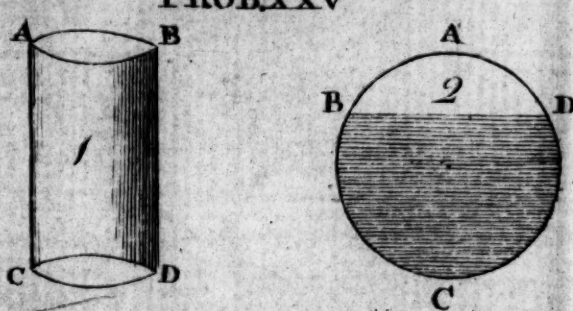
PROB. XXIII



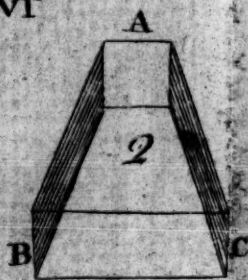
PROB. XXIV



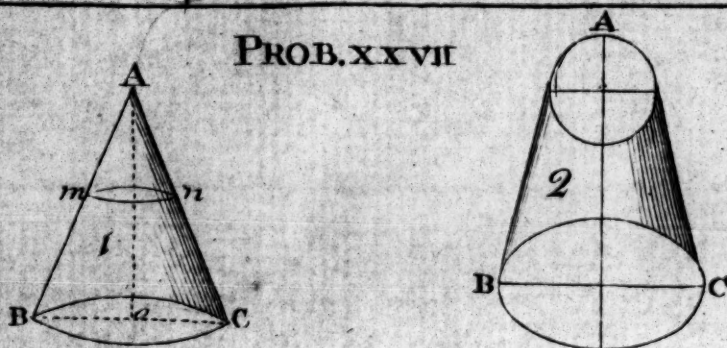
PROB. XXV



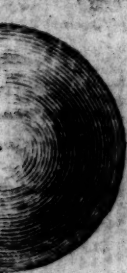
PROB. XXVI



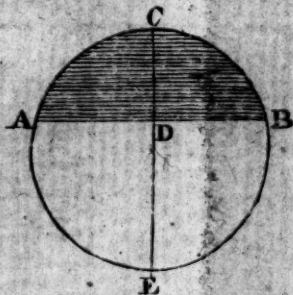
PROB. XXVII



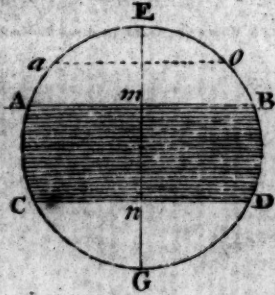
PROB. XXVIII



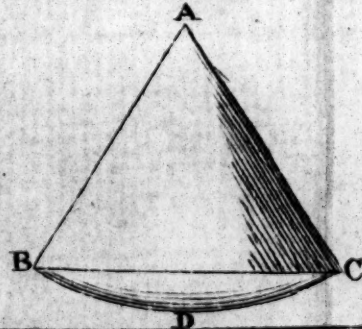
PROB. XXIX



PROB. XXX



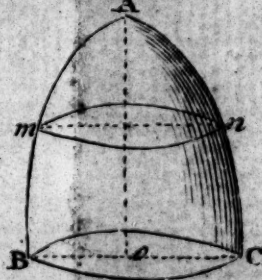
PROB. XXXI



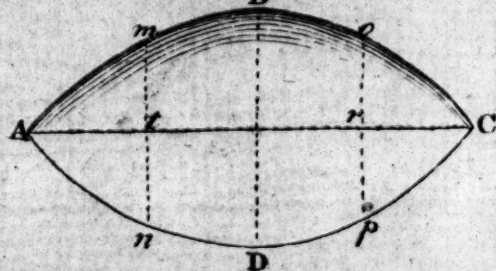
PROB. XXXII



PROB. XXXIII



PROB. XXXIV



Tetraedron

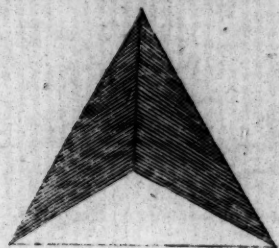


FIG. 1.



PROB. VI.

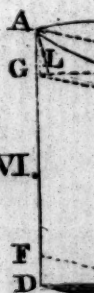


PLATE IV.

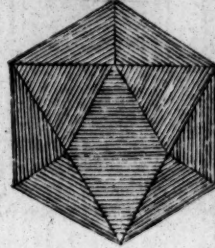
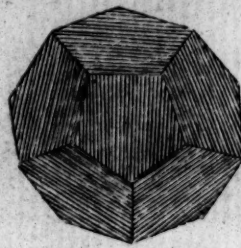
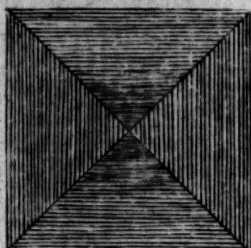
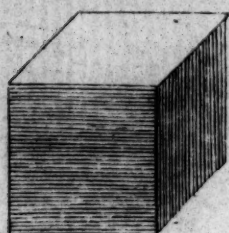
ron

Hexaedron

PROB. XXXV.
Octaedron

Dodecaedron

Icosaedron



SURVEYING

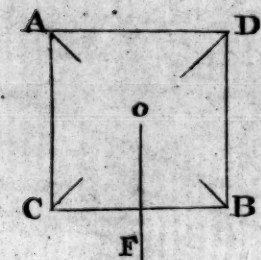
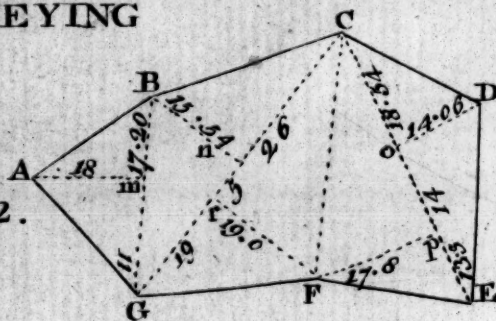
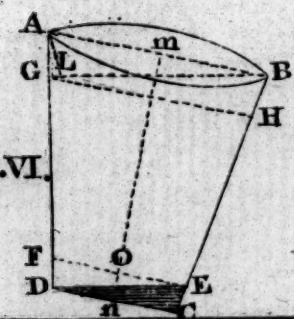


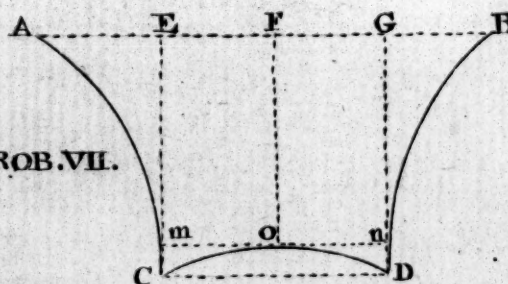
FIG. 2.



GAUGING



PROB. VII.



PROB. VIII.

